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**Nicolas Bourbaki and the concept of mathematical structure. (English summary)**

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It is generally agreed that the most outstanding feature of contemporary mathematics is its structural character. It is also common to associate the structuralist trend in mathematics with Nicolas Bourbaki, the collective pseudonym adopted in the 1930s by a group of French mathematicians whose conception of mathematics centred on the general idea of structure. The author challenges this popular myth, arguing that Bourbaki's real impact on modern mathematics has virtually nothing to do with the role of the formal concept of structure in the Bourbakian oeuvre.

According to him, “structural mathematics” denotes primarily a way of doing mathematics. It crystallized first in van der Waerden's *Moderne algebra* of 1930, which offered a unified structural perspective on the whole of algebra, yet contained no formal explication of the concept of structure as such. “Structure” was an informal idea, belonging to what the author calls a “tacitly shared image” of mathematics. He holds that such images, which, unlike formal notions, are continually liable to criticism and change, decisively shape the developmental course of the discipline.

Bourbaki's theory of structure was an attempt to give a full explication of the structural image in formal mathematical language. The formal concept of structure was originally designed as the perfectly general basis for the unification of the various branches in the comprehensive *Éléments de mathématiques*. It turned out otherwise.

The author's close analyses of the ways in which the formal concept was introduced in Bourbaki's *Theory of sets*, and put to use in such books as *Algebra*, *General topology* and *Commutative algebra*, lead to the conclusion that structural concepts in fact play no significant unifying and generalizing role at all. The sources also strongly suggest that the relevant chapter of *Theory of sets* was published at a time when it was already apparent that the concept of structure could not live up to the initial expectations and that there was an alternative generalizing concept, the concept of category, which was at least as comprehensive and probably more fruitful.

Contemporary mathematics is structural only in the informal sense of a way of doing mathematics in accordance with a structural “image”. Bourbaki's structural concept was a rather superfluous attempt to give this image a formal underpinning. What it contributed to the development of structural images of mathematics was by no means the deductive consequence of the axiomatic introduction of the formal structural concept. The rise, expansion and consolidation of the structural approach to mathematics had nothing to do with any particular formal theory about structure.

Reviewed by *Eduard Glas*