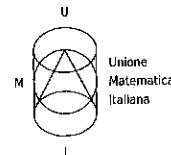


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Editors

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Between Culture and Mathematics

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Mathematical Fiction and the Prosaic Dangers of Salgarism

LEO CORRY

1. Salgarism

In his *Postscript to The Name of the Rose*, published in 1989, roughly nine years after the astounding success of his debut novel, Umberto Eco discussed the many difficulties and dilemmas he had to face while writing the book. The plot, as is well-known, takes place in a medieval convent whose daily routine is disturbed by a mysterious series of murders. The setting is fully immersed in Gothic imagery and the situations evolve around arcane scholastic debates among the monks. In fact, the key to solving the mystery arises from the awareness of the “detective” to a subtle nuance of a Latin declination in an obscure text that the culprit is fond of citing. Eco wanted his book to reach broad audiences, though one may guess that he never imagined the actual size of the crowds that the book would come to attract. He gave much thought to the question how to convey to the reader the considerable amounts of intellectual background material required to be able to carry on reading with interest. If he was to become too didactical in doing so, this would have a negative impact on the narrative qualities of the novel. Eco addressed this dilemma in terms of what he called “the risk of Salgarism”, and which he described as follows:

When the character in Emilio Salgari’s adventures escape through the forest, pursued by enemies, and stumble over a baobab root, the narrator suspends the action to give us a botany lesson on the baobab. Now this has become topos, charming, like the defects of those we have loved; **but it should not be done.** [3, pp. 557-558]

It is evident that anyone writing a fictional novel at the center of which there is some amount of mathematical ideas is likely to have to address—either implicitly or in full awareness—the same kind of problems discussed by Eco in connection to

his medieval fiction. It is quite likely, moreover, that the dilemmas arise more acutely in this case than in the case that Eco was involved with, if only because so many people around the world are daunted by the very sight of a mathematical symbol at the distance. The risks of Salgarism, indeed, continually lie in wait to raise their head and to become a burden and a challenge to the dramatic quality of much fiction involving mathematics. In this article I want to discuss the perils of Salgarism, as they arise in this specific genre of “mathematical fiction”.

2. An unlikely popular genre

I will use here the term “mathematical fiction” quite broadly, to refer to novels and short stories, films and plays, or any other kind of narrative, at the center of which we find mathematical ideas or whose main characters are mathematicians of any kind. It is somewhat surprising to realize, as a starter, that this genre is more popular than one could think on first sight. Alex Kasman, a mathematician at the College of Charleston, maintains a website that provides lots of updated information on the topic. At the time of writing these lines, Kasman lists 1183 works of various kinds, of which nearly 800 date from after 1990. He indicates a series of motifs which are quite recurrent in some of these works. Here are some examples:¹

- Anti-social mathematicians (109 entries)
- Autism (17 entries)
- Cool/Heroic mathematicians (45 entries)
- Evil mathematicians (45 entries)
- Female mathematicians (187 entries)
- Future prediction through math (47 entries)
- Genius (52 entries)
- Insanity (76 entries)
- Math as beautiful/exciting/useful (72 entries)
- Math as cold/dry/useless (37 entries)
- Prodigies (72 entries)
- Proving theorems (108 entries)
- Sherlock Holmes (16 entries)
- Time travel (53 entries)
- War (49 entries)

Many of these motifs are rather unsurprising, as they correspond to popular perceptions about mathematicians. A notable exception, perhaps, is the motive “Romance” (for which the site lists 207 entries), which corresponds to a recent trend intent on showing that, contrary to popular perceptions, mathematicians (like other, more pedestrian people) are just human beings who occasionally may even fall in love (e.g., *A Beautiful Mind*, 2001). In a different sense, it is also unsurprising to see that in the list of real mathematicians appearing in plots of mathematical fiction one finds, prominently, the names of Einstein, Turing, Gödel, Hilbert, Ramanujan, Hardy and Galois. Ramanujan, for example, is well-posed for fictionalization given the touching story of his childhood in a remote Indian town, his complex relationship with Hardy at Cambridge, and his rather sad end, back home in India. Einstein needs no further comment as he is the ultimate scientist-celebrity. Galois brings us back to romance. He is the only mathematician that can boast in his biography both prison for political reasons and death in a duel-at-dawn for the honor of a woman.

But when we come to Turing, Gödel and Hilbert the case is somewhat different. Turing’s life is also fitting for being fictionalized, that’s true. But these three mathematicians appear prominently in mathematics in fiction, in my opinion, not just because the intrinsic importance of their achievements or, in the case of Turing, the dramatic (even tragic) aspects of his life. Rather, I believe that there is one specific, interesting reason related with their mathematics which strongly influences their prominent presence in fiction. I will return to this issue below. At this point I just suggest considering the case of Poincaré as a clear example of someone who is no less prominent as a mathematician than any of those three, but who is not the main hero of any piece of mathematical fiction. As far as I am aware, Poincaré has no more than some token appearances, such as for example in Alice Munro’s *Too Much Happiness* (2009), where we find him discussing the awarding of the Prix Bordin by the French Academy of Science to Sonia Kovalevskaya (herself another high-profile character of mathematics in fiction).

But besides these general comments, a main issue that needs to be considered by any would-be author of mathematical fiction is that of “poetic license”. This is the question about the extent to which it is legitimate or reasonable for a plot containing direct references to some historical reality to deviate from that reality as “known to historians”, or to introduce fictional elements (either on purpose or inadvertently) into a historically-based plot. This is a complex issue to consider for historical fiction in general, and not just for the case that we are considering here. I have discussed this in some detail in a previous article [2] and I will not rehearse that discussion here. Still, I do want to stress some points of particular interest.

In the first place, it should be noticed that the “reality” to which mathematical fiction may refer, and into which it needs to be embedded, is double: historical and

¹ <http://kasmana.people.cofc.edu/MATHFICT>. Accessed: November 15, 2105.

mathematical. Take for example Apostolos Doxiadis' 1992 short novel *Uncle Petros and the Goldbach Conjecture*. The unnamed narrator of this story tells us about the life of his admired uncle, a Greek mathematician named Petros Papachristos. Throughout his life Petros has been obsessed by the drive to prove a number-theoretical conjecture first formulated in 1742 by Christian Goldbach, and that asserts that every even number bigger than 2 is the sum of two prime numbers. After completing his own training as a mathematician, the nephew becomes obsessed with finding out the true story of the uncle, whom other members of the family generally consider a failure. Now, while the character of Petros is imaginary, real names of famous mathematicians, dates and places are incurred throughout the narration and they provide the historical setting which is quite accurate. The Goldbach conjecture, of course, is itself a famous instance of a real mathematical issue. If some particularly informed reader notices that a certain date is wrong, when compared against the historical record, that will probably not strongly affect the overall narrative quality of the plot. If, however, the *mathematics* discussed (e.g., the formulation of a certain theorem or the details of a proof that is described) were flawed, and even if only in a minor detail, this might typically lead to a strong disapproval on the side of the mathematically inclined reader, who would refuse to allow for any poetical license at this level. [2, p. 217]

The second point to be stressed is that, because of the very nature of the genre, even the most unbridled kind of poetic license than an author of mathematical fiction may undertake in his work is unlikely to spark reactions that come close in any possible manner to those that can arise from more politically explosive topics. Think, for example, of the strong controversy that arose on the wake of Mel Gibson's *Passion of Christ* (2004) or, to take a more recent case, with the depiction of either Lyndon B. Johnson or Martin Luther King and of their relationship in *Selma* (2014). Just compare this with *A Beautiful Mind* and the kind of emotional, yet mild, reactions it elicited in relation with its historical accuracy.²

3. The limits of power

As suggested right above, I think there is a significant, inherent reason why Turing, Gödel and Hilbert appear so prominently in mathematics in fiction. My conjecture is that the reason has to do with the attraction that large audiences may feel towards the more general topic of "The Limits of Power", or, to give it

² <http://www.theguardian.com/film/filmblog/2012/dec/19/a-beautiful-mind-john-nash>, Accessed: November 15, 2105.

another name, "The Inherent Weakness of the Powerful". Take again the example of *Selma*, a film that gave rise to heated debates about historical accuracy and poetic license. A main focus of debate was the depiction of Johnson's actual commitment to the cause of voting rights. Whatever opinion one may have concerning the historical accuracy of the events depicted, it is undeniable that an important issue addressed by the film (I think that this has not always been explicitly acknowledged), and that is at the center of its dramatic impact, is the way in which the arguably most powerful individual in the planet, the president of the USA, realizes that his power has inherent limitations and that he must yield to pressures coming from below, even from a part of the population that many would consider as devoid of any power whatsoever.

Selma and the civil rights movement is not exactly one of my topics of expertise and hence I bring up this topic here with some reticence. Still I do it because I think it sheds light on what I want to say about mathematical fiction, and what I am saying about that film is based just on a sample survey of what the experts are saying. Julian E. Zelizer, for example, is professor of American political history at Princeton University, and he has been frequently interviewed about the depiction of Johnson in the film. He has consistently stressed Johnson's full commitment to the bill he eventually passed, and has claimed, based on the historical record that he has closely inspected, that conflict arose between Johnson and King on the specific (if crucial) issue of the right time and process of implementation:

I do think [the film's] message -- Zelizer said in an interview³ -- is that politics comes from the bottom-up rather than top-down, so I think in some ways they went overboard in not wiping away, but downplaying how committed the top official, meaning the president of the United States, was to this. King and Johnson had a partnership by this time, not an adversarial relationship.

Elsewhere,⁴ Zelizer also explained that "while it is true that King wanted to move much more quickly on voting rights than LBJ and that the movement forced the president's hand through the marches, they were both on the same page in terms of objectives". Zelizer mentioned this as part of an attempt to provide a broader view of the public perception of Johnson's role as president and of Johnson's own view on his role in history. Remarkably for our purposes here is that Zelizer went on to stress the following point:

³ http://www.huffingtonpost.com/2015/01/15/selma-lyndon-johnson-julian-zelizer_n_6474214.html, Accessed: November 15, 2105.

⁴ http://www.huffingtonpost.com/2015/01/15/selma-lyndon-johnson-julian-zelizer_n_6474214.html, Accessed: November 15, 2105.

“Power. The only power I’ve got is nuclear and I can’t even use that!” LBJ once quipped. Although now thought of as a leader who saw no boundaries, Johnson was always aware of the limits of presidential power.

Now, before connecting the issue of limitations of power to mathematical fiction, there is another reason, more incidental in character, to bring up *Selma* as part of my discussion here. It is the fact that there is another successful film that opened in theaters around the world at roughly the same time and that falls squarely into my topic of discussion here. That is *The Imitation Game*, largely based on Andrew Hodges’ biography of Alan Turing. Both films were eventually nominated for several categories (including Best Picture) in the 87th Academy Awards for 2014. And like with *Selma*, also *Imitation Game* gave rise to many debates related with the poetic license indulged by the scriptwriter. Of course, the levels of the emotions involved in the two cases were quite different. One focus of debate concerning Turing was his personality and behavior, that the film depicted as those of a person in the autism spectrum, and for which there is no real historical evidence. More public resonance had the issue of Turing’s homosexuality (the film was accused of downplaying it), which by its nature is more explosive than all other aspects of the plot, and certainly more than any debate about Turing’s mathematical ideas.

Be that as it may, I think that Turing represents, perhaps more than any other mathematician that one may think of, the duality power-weakness and the inherent drama embodied in this duality. This is, I think, a main aspect of the fictionalized Turing that had so much appeal for the audiences of the film. On the one hand, the brilliant mind that single-handedly cracked the German code and thus crucially influenced the outcome of war. On the other hand, the fragile individual whose entire life, from childhood on, was plagued by bullying. This duality was even more manifest during the later years of his life when, after becoming an unacknowledged national hero, after having opened the way to the modern computer, and after having contributed seminal ideas to several important fields of mathematics, he found himself in a desperate situation that led him (apparently) to commit suicide. A similar kind of duality, I may add, is also a main dramatic ingredient in the plot of *A Beautiful Mind*.

But what is really remarkable in this respect is that not just Turing *the person* embodies so clearly the duality power-weakness, but also that the core of *his actual mathematical ideas* focus on the question of weakness and limitations of power—not of political, military, or personal power, but rather, of that most powerful and apparently indomitable of all human inventions, mathematics. This inherent weaknesses and limitations of the power of mathematics has been, I claim, at the focus of so many works of mathematical fiction, and I want to turn now to a brief discussion of this point.

4. The inherent weakness of mathematics

Let us then go back now to mathematical fiction. I want to suggest that one illuminating way to understand the relatively high level of attention paid in mathematical fiction to figures like Gödel and Turing and to motifs like logical incompleteness or physical indeterminacy (in the sense of Heisenberg) is by harking back to the more general topic of the weakness of the powerful just discussed above. Indeed, more than any other scientific discipline it is mathematics that is commonly associated (among educated as well as less educated audiences) with the idea of certainty and the power that derives from it. The certainty of mathematics and the ubiquitous reliance on mathematical evidence in our world, often as the ultimate criteria for decision, bestows the discipline with a peculiar and very visible kind of power and strength that is a clear source of both positive and negative feelings towards the entire field. But the fact that some truly important discoveries in the twentieth century imply that mathematics (and mathematical physics, for that matter) are affected by some deep inherent limitations that cannot be possible overcome have elicited and continue to elicit in readers and moviegoers, I surmise, the kind of reassuring and heartwarming feeling that films like *Selma* are prone to do. Let me discuss some specific examples related with this idea.

I start with David Hilbert; Hilbert was arguably the most influential mathematician at the turn of the 20th century given the intrinsic importance of his own contributions. No less important than that, however, was the remarkable overall impact of the center of excellence that he was able to establish around him in Göttingen between 1895 and 1933. This was the Mecca of the exact sciences, which attracted hordes of young talents from the world over, intent of starting here what they hoped would become outstanding scientific careers. Audiences of popular science and of mathematical fiction alike are likely to be familiar with his name. It is quite common to allude to his figure (also in professional mathematical texts) as the champion of the “formalistic view” of mathematics, and to associate it with the attempt to definitely entrench, beyond doubt, the certainty of mathematics with the help of what is commonly known as “the Hilbert Program”. Hilbert’s stature as one of the giants of early 20th century science is never called into question in such accounts, of course, but his view of mathematics as the most powerful tool created by human thought and in which no problem will ever remain unsolved is often presented as overambitious and bound to fail. He is accused of a kind of evil intention to “ban intuition” from mathematics. But as it happens, when authors of mathematical fiction (and often of professional mathematical texts as well) bring up the figure of Hilbert under this kind of critical perspective, it is because, on the one hand, they don’t really know the details of Hilbert’s biography and the gist of his

approach to mathematics and science and, on the other hand, because they already know that there is a sequel to the story, and this is the sequel that involves Kurt Gödel. Gödel is the person who appeared out of the dark in 1930 and whose work “shattered into pieces” this kind of “absurd” and “misguided” ambitions which are commonly attributed to Hilbert.

As with any other work of historically-based fiction, depictions of Hilbert, of the so-called Hilbert program and of its shattering by Gödel, of the role accorded by Hilbert to “intuition” in mathematics, and so on, raise the dilemma of historical accuracy vs. poetic license. It is also common that, in the case of mathematics as in other cases, public perceptions about the topic are more strongly influenced by the fictional depiction than by the scholarly-oriented ones. And, finally, it is also common, and rather frustrating, that even works that purport to abide by scholarly standards prefer to rely on the fictional, rather than on the scholarly sources. I think this is particularly visible (as we will see now) in the case of mathematics, and it is even more strongly so, when the issue of the inherent weakness of mathematics is at stake (or apparently at stake).

An interesting example that illustrates my point appears in Rebecca Goldstein’s 2005 book *Incompleteness: The Proof and Paradox of Kurt Gödel*. Goldstein is a well-known figure in intellectual circles in the USA. A MacArthur Foundation fellow, she has been variously described as a philosopher and author of several novels and short stories, and she also wrote two biographical studies, one on Spinoza and one on Gödel, *Incompleteness*. Websites that sell this book (e.g., Amazon or Barnes and Noble) tell us that it “indelibly portrays the tortured genius whose vision rocked the stability of mathematical reasoning— and brought him to the edge of madness”. Harvard psychologist and laureate popular science author Steven Pinker is typically quoted along that description as saying that the book is “an unforgettable account of one of the great moments in the history of human thought”. This is how she presents the mathematical contribution of Hilbert [6, p. 136]:

The leading advocate of formalism was David Hilbert, who was the most important mathematician of his day. “Mathematics”, wrote Hilbert, “is a game played according to certain simple rules with meaningless marks on paper.” His proposal to formalize one branch of mathematics after the other, starting with the most basic branch of all, arithmetic, came to be called the Hilbert program. The successful completion of the Hilbert program would offer significant vindication of formalism, explaining the sui generis aprioricity of mathematics as derivative from the stipulation of rules.

Now, it seems that the simplest effort to check some recent scholarship on Hilbert (e.g., [1]) would show how wrong this picture is. What is Goldstein’s

source for “a game played according to certain simple rules with meaningless marks on paper”? *Mathematical Maxims and Minims*, a haphazard collection of incidental lore [8] (abundantly repeated over the Internet) which does not even claim to rely on some reliable source. Compare with the following, well-documented statement from around 1919, the time when Hilbert began to work out (in collaboration with Paul Bernays) the finitist program for proving the consistency of arithmetic:

We are not speaking here of arbitrariness in any sense. Mathematics is not like a game whose tasks are determined by arbitrarily stipulated rules. Rather, it is a conceptual system possessing internal necessity that can only be so and by no means otherwise. [5, p. 14]

If by “Hilbert program” we want to denote anything that can be directly connected to what eventually turned out to be Gödel’s theorem, then the program is far from being anything like “a proposal to formalize one branch of mathematics after the other, starting with the most basic branch of all, arithmetic”. While in the question of the actual infinite in the foundations of mathematics Hilbert found himself fighting against Brouwer’s “intuitionist” demand that only the potential infinite be allowed, to claim, as Goldstein does together with many other places where this is uncritically repeated, that Hilbert attempted “to ban intuition from mathematics” is to deeply misunderstand both Hilbert and Brouwer.⁵

In the last sentence of Goldstein’s chapter on Hilbert, we understand the logic of her narrative buildup: “Enter Kurt Gödel”. Typically, the man that is supposed to have destroyed the ultimate dream of the most powerful mathematician on Earth, Hilbert, is described here as in many fiction and non-fiction texts, as a person with great mathematical powers but overall physically and emotionally weak. Goldstein tells us that in the famous 1930 conference at Königsberg where Gödel first presented his incompleteness result, he was not “one of the big fish”, rather, he was scheduled, “together with other small fry, to give a 20-minute talk”. (p. 147) The account of the low-profile manner in which Gödel presented his result is essentially correct in the book, but what interests me here to focus on is the contrast between the powerful and the weak and how, in the final account, the latter was able to shatter the ambitious plans of the former (p. 157):

⁵ On this point, see a critical review by Juliette Kennedy in the *Notices of the AMS*, April 2006 (p. 451): “The author’s lengthy discussion of the Hilbert Program, which she characterizes as being dedicated to *eliminating intuitions*, is not exactly erroneous, but does violence to the spirit of that program, in the opinion of this reviewer, and will justifiably perplex mathematicians.”

We have not first-hand accounts of the manner of Gödel's presentation that October day in 1930 ... But we know enough about the emphatically anti-charismatic Gödel, with his aversion to external drama ... to be able to imagine how it went. The somber and uninflected statement of the crux of the matter, with no rhetorical flourishes, no hyped-up context to help his listeners grasp the importance of what was being said. No Strum und Drang, only zipped-up genius ...

Goldstein also describes in her book another one of Gödel's seminal mathematical contributions that showed Hilbert's unbridled optimism was completely baseless. In order to do so, she brings us back to Hilbert's famous speech of 1900 where he presented a list of twenty-three problems that should be high in the agenda of mathematicians in the century that was about to begin. He expressed his full confidence in the unlimited ability of mathematics to solve any problem and famously declared that "in mathematics there is no *ignorabimus*". The first problem in the list is the requirement to prove Cantor's Continuum Hypothesis. As it happened, an important breakthrough of 1963 by Paul Cohen, combined with a previous contribution by Gödel dating from a few years earlier, amounted to the surprising result that, within what is considered to be the standard way to present set theory (i.e., the axiomatic system of Zermelo-Fraenkel), the hypothesis is undecidable.⁶

Now, Goldstein summarized this situation by alluding to an often repeated claim to the effect that Hilbert would not have welcome this outcome, because it represented what he "claimed could not be: an *ignorabimus*—a claim that can neither be confirmed nor discredited, a claim about which we remain ignorant" (p. 139). This description involves a coarse misunderstanding not just of Hilbert's views, but also of the mathematical meaning of the result. In my view, nothing could count as stronger support for Hilbert's optimistic view about mathematics than the Gödel-Cohen proof. Not that he would not be surprised on the direction in which it took the question (and perhaps somewhat disappointed, but only as a first reaction, I assume), but in the wake of it, we know now with full mathematical certainty that a certain mathematical statement, either CH or its negation, can be added to the axiomatic system ZF without thereby leading to any contradiction. If one can call this *ignorabimus*, then I don't know what is knowledge, much less certain knowledge. To add a touch of poetical justice to this matter, it so happens that it is easy to trace a genealogical line of ideas leading precisely from Hilbert's early works on the foundations of geometry to those developed by Gödel, and especially by Cohen in order to complete their proofs.

⁶ On this point, see a critical review by Gregory Moore in *American Scientist*: <http://www.americanscientist.org/bookshelf/pub/the-incomplete-g-del>, Accessed: November 15, 2015.

My main point in saying all of this is to stress how the adoption of the motif of the powerful facing the weak is appealing also in the case of mathematical fiction and that it can become so strong as to obscure the plain historical facts, and the figure of Gödel is perfectly suited to the task. Another example that illustrates this point appears in the novel *Turing*, published in 2003 by the distinguished Greek computer scientist working at Berkeley, Christos Papadimitriou. Here is how he incorporates a narrative similar to that of Goldstein into his fiction, while taking it into unexpected directions [6, p. 123]:

Hilbert ... was a great mathematician who lived about a hundred years ago. He had this dream: to build a computer that would prove for you any theorem you submit to it. This way he was hoping to make the ultimate discovery in mathematics, the theorem that would prove all theorems.

Now, speaking of Hilbert's ideas in terms of computers may be a nice motivational idea in an introductory course in computer science and of course poetic license may allow for it in a fictional novel, yet it is important to keep in mind that it has nothing to do with historical reality. Much less the idea of a computer that could prove any theorem. One way or another, in the novel, the powerful program promoted by the powerful Hilbert would be beaten soon by the work of a twenty-four years old student at Cambridge, Alan Turing. Turing, in this story, introduced his ingenious machines to be able to speak of "computable numbers", which are "an unlikely slayer of Hilbert's dream". Nevertheless, his intention was precisely to do that, to prove that Hilbert's project (the one described in the novel in terms of computers) is impossible. In fact, Turing's result "implies something devastating about maths: It must forever be incomplete" (p. 128). At any rate, an attempt to stick more closely to the historical record appears in the following pages of the novel, when Papadimitriou indicates that some of the ideas on which Turing based his achievement—and in particular the very idea of the possible incompleteness of mathematical systems—dated back some years earlier to the work of Gödel.

Now, while I do not want to put constraints to Papadimitriou's full poetic license to tell whatever he likes about a putative Hilbert's program involving computers, and its connection to Turing machines, I think that an interesting point here is that his conclusion about the significance of the putative result of Turing vis-à-vis Hilbert is wrong. Personally, I have never understood why people, including leading scientist, think that incompleteness results, can be seen as devastating for mathematics. For me, they are evidence to the contrary, namely, to the strength of mathematics, because they stress the extent to which this discipline of knowledge is able to delineate the exact deductive power of any well-defined system within it. More importantly, I see absolutely no reason to believe that Hilbert himself would have seen Gödel's or Turing's results as devastating for

mathematics in any sense. Gödel's results may have been a crucial obstacle to Hilbert's specific attempt to prove the consistency of arithmetic with finitary methods using a formalistic approach. He may have been surprised by the idea of incompleteness and the fact that it can be proved. But I conjecture that, contrary to what many people believe, Hilbert would have very soon come to appreciate the value of Gödel's innovative techniques for analyzing axiomatic systems and he would have enthusiastically joined the efforts himself, had he had still the physical and mental forces to do it. I am just conjecturing here because at the time Hilbert was ill and his mental capacities had considerably weakened. But, I think that, imagined stories about Hilbert aside, be they fictional texts such as Papadimitriou's or supposedly non-fictional ones as Goldstein's, there is nothing in the historical record to make us believe otherwise, and there is much to support this conjecture.

Contrary to frequent depictions that texts of mathematical fiction are prone to adopt, Hilbert was no formalist when it came to an overall view of what is mathematics about. What is historically true is that, in the context of solving a specific problem (to prove the consistency of arithmetic by finitary methods) and in a very specific historical and mathematical context, Hilbert suggested that for the sake of finding solution we might look at arithmetic as a system of meaningless signs whose results are derived by blind manipulation according to rules stipulated in advance. This is not for Hilbert what arithmetic (much less mathematics at large) is but rather an approach to possibly solve a problem, and one that proved to be enormously successful at that, precisely because it opened the door to results such as Gödel's.

5. Mathematical Fiction, Salgarism and Weakness

I want to focus now on a specific text of mathematical fiction that will help connect all the threads opened above. What I have in mind is the 1999 novel *In Search of Klingsor*, by the Mexican writer Jorge Volpi. Volpi, born in 1968, is a leading figure of his generation. He participated in the 1990s in the so-called "Crack Manifesto" published by a group of young Latin-American writers intent of distancing themselves from the traditions associated with magical realism. His works focus on characters and events that are far removed from the daily life in his immediate cultural context. Rather, the topics developed in his plots relate very often to European history and to science. He has been widely translated and he has been recognized with important awards, such as Premio Biblioteca Breve and Premio Casa de las Américas, and also with a grant from the Guggenheim Foundation. He has also been visiting professor in institutions like Princeton University. In other words, Volpi is a prominent and well-known writer, rather than any kind of marginal figure.

In Search of Klingsor tells the story of two fictional characters: Francis Bacon, a young American physicist, and Gustav Links, a German mathematician who is also the narrator of the story. After meeting amidst the ruins of post-war Germany, Bacon and Links ally in the search for Klingsor. Klingsor is tenuously described as a main figure in Hitler's scientific programs, ranging from phrenology to the attempt to build a nuclear bomb. The name Klingsor is taken from the magician in Wagner's *Parsifal* and the novel itself is likewise interspersed with the plot of the opera. But the list of characters also includes Heisenberg, Schrödinger, Bohr and many other famous scientists. Accordingly, there are continuous references to scientific ideas such as the mathematical infinity. Also Heisenberg's uncertainty principle is strongly alluded to in the plot, as a way to suggest the unavoidable influence of the observer over what is observed. "Although the notion of subjective truth certainly occurred to the Sophists in ancient Greece and to Henry James in the nineteenth century – we read in the text – it was our good friend Erwin [Schrödinger] who established the scientific foundations of such a theory." [9, p. 24]

Also Hilbert and Gödel play central roles in the plot of this novel, and many of the standard (and typically wrong) images of the two are repeated here together with the typical hints to the power-weakness tension associated to them. What is particularly amusing in my view, however, is the way in which Volpi repeatedly incurs in mathematical Salgarism. An interesting example is found in a scene which is reminiscent of a famous passage of Thomas Mann's *Doktor Faustus*, when Adrian Leverkühn's childhood friend and narrator of the story, Serenus Zeitblom, tells us of Leverkühn's first experience with women. In *Klingsor* it reads as follows (p. 112):

When we were seventeen years old, we visited our first whorehouse together, and even asked the prostitute to service us at the same time. We wanted her to touch us simultaneously, so we could observe each other's ridiculous expressions of sexual excitement.

But very soon we move away from that promising story and, instead, the narrator starts to tell us about his university studies (p. 113):

My chosen field—and I say this with a certain pride—was mathematical logic, specifically the theory of infinite sets developed by George Cantor at the end of the nineteenth century.

And from here on, the next pages are devoted to a learned account of the rise of modern set theory. We are told, as part of the plot of what purports to be a kind of thriller, that "Georg Ferdinand Ludwig Philipp Cantor was born on March 3, 1845", and from here we go on to hear who were Dedekind and Kronecker and

what did they do, what is the Continuum Hypothesis, and so on. After a while, like with Salgari's baobab, we are taken back to the main line of the story, hoping that we can now follow the intricacies of this arcane knowledge (and leaving the whorehouse story well behind). My guess is that whoever knew some of this mathematics before reading Volpi would skip this part as wholly uninteresting and possibly not very accurate, whereas whoever did not know could hardly have learnt what is needed (or may have felt a kind of boredom and also skipped as his more knowledgeable reader friend would do). In any case, Volpi's condensed account of the history of the foundations of mathematics is marred with errors of historical fact. That's not terribly problematic, if we are willing to accord full poetic freedom. Still, I surmise that Volpi was trying his best to be accurate. The real problem remains, however, that all of this learned explanation cannot be said to be great literature (that's my humble opinion, anyway) and that it does very little to enhance the narrative quality of the story.

Let me nevertheless give you a taste of Volpi's narrative style by quoting here some highlights (pp. 81-82):

Mathematics was the most objective and evolved scientific instrument known to mankind ... But ... nobody knew for sure if mathematics might contain, somewhere within itself, a germ in decomposition, a fungus or a virus capable of refuting its own results. ... At the dawn of the twentieth century, the situation was more bewildering than ever. Conscious of Cantor's theories and the aberrations they produced, the English mathematicians Bertrand Russell and Alfred North Whitehead joined forces in an effort to reduce the entire scope of mathematics to a few basic principles, just as Euclid had done two thousand years before ... Unfortunately, the work was so vast and complex that in the end, nobody was truly convinced that all mathematical statements could be reduced to their theories without falling into contradictions at some point or another.

The difficulties and challenges posed by Cantor's theory are transformed here into "aberrations". OK, let it be for the sake of narrative license. But here Volpi arrives (unavoidably, I guess) to Hilbert's 1900 Paris lecture (which actually predates Russell and Whitehead's work) and he tells us that Hilbert "explained a theory that would thereafter be known as Hilbert's Program". Well no, the program has nothing to do with the problems of 1900. It was put forward about two decades later. But, OK, poetic license again. Volpi goes on to explain that in "this treatise" [which treatise are we talking about? The speech?] one of the problems described by Hilbert was "the so-called axiom of completeness, which questioned whether the system later described in the *Principia* – or any axiomatic system, for that matter – was comprehensive, complete and free of contradiction." This statement is so completely wrong and nonsensical that I don't know exactly what to say about it if I don't want to set limitations to poetic license. But still, time travel seems not

to be foreign to a realistic novel like *Klingsor*: how could Hilbert say anything about a system that was to be described in a book published about a decade later?

Another remarkable point concerning the narrative qualities of the novel is the chapter called "The Quest for the Holy Grail". I have not said this yet, but the theme of the "Holy Grail" is a recurrent one in popular books and works of mathematical fiction. Look for example at the dust jacket of Simon Singh famous book on Fermat's Last Theorem. Or even better, just google "holy grail mathematics" and check the number of results. But in *Klingsor*, there is an additional justification to rely on the metaphor, given that in the plot of Wagner's *Parsifal* the Quest for the Grail plays an important role. Also on this matter Volpi follows the Salgarist strategy in order to make sure that his readers understand the plot by explaining in some detail the legend of the grail.

And then, at some point, Gödel makes his sudden and unavoidable appearance in the plot. Unsurprisingly he is described as follows (pp. 80-81):

Professor Gödel was a short, taciturn man with the body of a flagpole; his general appearance called to mind an opossum or a field mouse. ... yet it was true ... ten years earlier he had completely destroyed, with a single result, the entire edifice of modern mathematics.

I hope I don't need to explain again why I find this kind of description so problematic in terms of what it says about mathematics. One would wish that the addition of this new character to the plot might at least add some dramatic qualities to the plot. However, for me as a reader, it only added a sense of artificiality about the main (fictional) character of the story. Let me briefly explain why.

Bacon's encounter with Gödel takes place in the context of wartime scientific activity at the Institute of Advanced Study in Princeton. The IAS had succeeded Göttingen as the new hub of scientific excellence. At the time, one would meet here the likes of Einstein, Gödel, Weyl, and Oppenheimer. Perhaps one of the most remarkable figures in this unmatched gallery was John von Neumann, who also started his glorious career under Hilbert at Göttingen. In the plot of *Klingsor*, von Neumann informs Bacon about the forthcoming visit of Gödel at the institute. Unfortunately for him, however, on the day of the planned visit he had promised his fiancée Elizabeth to travel to Philadelphia together. This was, as it happened, at an all-time low in their relationship (p. 80):

When Bacon told her, he explained the importance of the event and assured her that they would make the trip the following month. Elizabeth, however, simply told him to go straight to hell. ... Bacon resolved not to give in this time. He was too determined to meet Gödel to let one of his fiancée's idle threats get in the way. In fact, he thought, this might just be the perfect excuse to take a rest from her for a few weeks, to be alone and to think about his future.

Amidst the almost unanimous choir of praise that followed the publication of *Klingsor* and its translation into so many languages, one of the few dissenting voices was that of science journalist Oliver Morton. Against the background of what I have written here, it becomes now clear why I cannot fail to agree with his assessment of the book:⁷

For a book dressed up as a thriller, a lack of thrills is a disadvantage. Among other things, it makes it harder for Volpi to make the ideas that he is intent on communicating matter. The publishers seek to compare *In Search of Klingsor* to *The Name of the Rose*; but in Eco's book, the theological disputations and details of the medieval mindset are deeply implicated in the murders to be solved and the dangers faced by characters that we care about.

6. Concluding Remarks: Eco, Fiction and Mathematical Fiction

In the foregoing pages I have explained why I think that Eco's remarks on Salgarism are enlightening when trying to make sense of mathematical fiction. I want to conclude this article by relying once again on his ideas, this time by looking at the published version of the Charles Eliot Norton lectures which he delivered at Harvard University in 1993. While discussing the meaning of the very idea of narrative fiction and the role it plays in our lives, he wrote the following [4, p. 116]:

We read fictional texts since they come to the aid of our metaphysical narrowmindedness and offer an illusion of order within a world whose total structure we are unable to grasp and to describe. Since we know that fictional universes are created by an "authorial entity", we know that there is a "message" behind them.

Because of the double reality to which mathematical fiction is related, mathematical and historical, this interesting idea of Eco takes an interesting turn when it comes to mathematical fiction. Mathematics, no doubt, is the ultimate paradigm of an orderly world. In mathematics we do look for a "total structure", as Eco says here for fiction, but we start from the assumption that we will be able to grasp that order. That's, at the bottom, the aim of mathematical research. Indeed, as already indicated above, Hilbert was the all-time champion of mathematical optimism, as embodied in the quotation "wir müssen wissen, wir werden wissen". Apparently,

⁷ Oliver Morton, *The Guardian* May 10, 2003: <http://www.theguardian.com/books/2003/may/10/featuresreviews.guardianreview13>, Accessed: November 15, 2015.

we need no external fictional superstructure (in the form of fiction) that will help us impose order on the apparent chaos of mathematics (reality) because the work of the mathematician is to carve out this order from within.

One may question whether or not there exists an "authorial entity" behind the structure embodied in this "actual world" of mathematics. But no one will deny that the kind of comfort that Eco attributes to our experience with fictional worlds is manifest in a very remarkable and direct way in our encounters with mathematics. True, some people experience difficulties in technically mastering the world of mathematics. But once mastered, it provides perhaps the utmost example of a fictional (or fictional-like) world where the certainty of an underlying message is strongly felt and where, indeed, progress is continually and consistently made on the way to elucidating that message.

It seems to me, nevertheless, that what attracts the attention of writer of mathematical fiction is not so much mathematics itself, as that enigmatic figure, the mathematician. And it is the riddle of the mathematician, much less of mathematics, that this kind of fiction tries to shed light upon by means of the power of narrative fiction as explained by Eco. The drama of the weakness of the powerful is embodied very often in the mathematician, but as we saw above, some authors also try to make it appear in mathematical knowledge itself, and not always in a very successful manner. And above all, one of the main challenges facing an author intent on undertaking this kind of fiction is how to avoid the ever-present dangers of mathematical Salgarism.

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