## Number Theory Homework \#6

1. Are there integers $x, y, z$ such that $3 x^{2}+2=y^{2}+6 z^{3}$ ?
2. Show that the congruence $x^{3} \equiv a(167)$ has solutions for all $a \in \mathbb{Z}$.
3. Find all solutions to each of the following congruences:
(a) $x^{2} \equiv 9(\bmod 256)$.
(b) $x^{2} \equiv-7(\bmod 128)$.
(c) $3 x^{2}+6 x+1 \equiv 0(\bmod 19)$.
(d) $x^{2}+3 x+7 \equiv 0(\bmod 37)$.
4. How many solutions does the congruence $x^{2} \equiv 121(\bmod 1800)$ have?
5. Prove that for each prime number $p$ there exist $a, b \in \mathbb{Z}$ such that

$$
-1 \equiv a^{2}+b^{2} \quad(\bmod p)
$$

(Hint: how many values in $\mathbb{F}_{p}$ do the expressions $a^{2}$ and $-1-b^{2}$ take?)
6. Evaluate each of the following symbols: $(8 / 11),(7 / 13),(5 / 19),(2 / 383)$, (-1/113), (-2/773), (71/73), (37/137), (30/199), (1711/1999), (-1/523).
7. Which prime numbers $p$ can divide integers of the form $x^{2}-5$ ?

