## Number Theory Homework #6

- 1. Are there integers x, y, z such that  $3x^2 + 2 = y^2 + 6z^3$ ?
- **2.** Show that the congruence  $x^3 \equiv a$  (167) has solutions for all  $a \in \mathbb{Z}$ .

3. Find all solutions to each of the following congruences:

- (a)  $x^2 \equiv 9 \pmod{256}$ . (b)  $x^2 \equiv -7 \pmod{128}$ .
- (c)  $3x^2 + 6x + 1 \equiv 0 \pmod{19}$ . (d)  $x^2 + 3x + 7 \equiv 0 \pmod{37}$ .
- 4. How many solutions does the congruence  $x^2 \equiv 121 \pmod{1800}$  have?
- **5.** Prove that for each prime number p there exist  $a, b \in \mathbb{Z}$  such that  $-1 \equiv a^2 + b^2 \pmod{p}.$

(Hint: how many values in  $\mathbb{F}_p$  do the expressions  $a^2$  and  $-1 - b^2$  take?)

6. Evaluate each of the following symbols: (8/11), (7/13), (5/19), (2/383), (-1/113), (-2/773), (71/73), (37/137), (30/199), (1711/1999), (-1/523).

7. Which prime numbers p can divide integers of the form  $x^2 - 5$ ?