Number Theory Homework #4

1. Compute Euler's phi-function $\varphi(n)$ for $141 \leq n \leq 150$.

2. Show that the Euler function $\varphi(n)$ is even for $n > 2$ using the fact that a is coprime to n if and only if $n - a$ is coprime to n.

3. Find the last two digits of the decimal expansion of 3^{1123}. (For example the last two digits of 1729 are 29.)

4. Show that 4 is not a primitive root modulo p for any prime p.

5. Let p be an odd prime. Show that for any a coprime to p, either $a^{(p-1)/2} \equiv 1\ (p)$ or $a^{(p-1)/2} \equiv -1\ (p)$.