Number Theory Homework #2

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1. (a) Determine all solutions in integers of the following Diophantine equation:

$$24x + 138y = 18.$$

(b) Determine all solutions in *positive* integers of the following Diophantine equation:

$$54x + 21y = 906.$$

2. Show that $\sqrt{5}$ is an irrational number, that is, there is no rational number $\frac{a}{b}$ such that $(\frac{a}{b})^2 = 5$.

3. (a) The Fibonacci numbers are defined by the recursion $F_{n+2} = F_{n+1} + F_n$ with initial conditions $F_0 = 0,\ F_1 = 1$. Show that

$$F_n = \frac{(\frac{1+\sqrt{5}}{2})^n - (\frac{1-\sqrt{5}}{2})^n}{\sqrt{5}}.$$

(b) Show that the number of steps in the Euclidean algorithm to compute $\gcd(F_{n+1}, F_n)$ is at least $C \log_2 F_n$ for n sufficiently large, where C is a constant. (Hint: Show that the number of steps is $n - 1$.)

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Course homepage:
`http://www.math.tau.ac.il/~borovoi/courses/NumberTheory/NT.html`