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From Schelling to Spatially Explicit Modeling of Urban Ethnic and Economic Residential Dynamics

Itzhak Benenson
Tel-Aviv University
Erez Hatna
Wageningen University
Ehud Or
Tel-Aviv University

The robustness of outcomes to the parameterization of behavioral rules is a crucial property of any model aimed at simulating complex human systems. Schelling model of residential segregation satisfies this criterion. Based on the recently available high-resolution census GIS, we apply Schelling model for investigating urban population patterns at the resolution of individual buildings and families. First, we simulate ethnic residential dynamics in Yaffo (an area of Tel Aviv), and demonstrate good quantitative correspondence for a 40-year period. Second, we investigate income-based residential patterns in nine Israeli cities, reveal their high heterogeneity, and explain the latter by the presence of low fraction of wealthier householders who are tolerant of their poorer neighbors and reside in their proximity. We extend Schelling model in this direction and demonstrate qualitative correspondence between the model's outcomes and the observed income-based residential patterns.

Keywords: *Social simulation; Schelling model; High-resolution GIS; Urban residential dynamics*

Authors' Note: Please address correspondence to Prof. Itzhak Benenson, Department of Geography and Human Environment, University Tel Aviv, 69978, Ramat Aviv, Tel Aviv, e-mail: bennya@post.tau.ac.il. Color versions of the figures are available from http://www.tau.ac.il/~bennya/. The online appendix is available at http://smr.sagepub.com/supplemental.

Operational Modeling of Urban Social Systems: Is It at All Possible?

Social science does not differ from natural science in its acceptance of experimentation as the starting point of theory. However, the second part of the loop, from theory back to reality and experimentation, is inherently weaker. Social science does not demand theory falsification in Popper's sense. Instead, theory is considered an expression of the possible, used for explanation, not a formal description of reality that can and should be adjusted to each specific instance of the phenomenon in question and used for prediction. Put briefly, social science does not aim to make theory directly operational.

However, the hope of an operational social science continues to thrive. New interdisciplinary approaches repeatedly revive this hope, especially regarding the *urban systems* in which humans are tightly linked to their physical environment. The previous wave of optimism was triggered by the splash made by complex systems theory in the early 1960s (Prigogine 1967; Haken 1983). The latest has been driven by agent-based modeling, which has increasingly become the accepted tool for investigating social systems (Gilbert and Troitzsch 1999; Benenson and Torrens 2004).

Our article follows this recent line. We aim to tighten the connection between sociological theory, as captured by Schelling's (1978) model of residential dynamics and the dynamics of real cities. This connection became possible when the unique individual geographic information system (GIS) database constructed in the framework of Israel's 1995 population census was recently made available to researchers. We exploit the census database in two examples. The first relates to ethnic residential dynamics, in which the replacement of Schelling's idea of mutual avoidance in favor of asymmetric relationships resulted in successful simulation of Jewish-Arab residential dynamics in the Yaffo area of Tel-Aviv for a period of 40 years. The second is based on an analysis of income-based residential patterns in nine Israeli cities, in which high heterogeneity is observed in eight of the nine cities investigated. Conceptually, this heterogeneity can be explained by the fraction of wealthier householders who, being highly tolerant of their poorer neighbors, are able to reside in their proximity. Extension of the Schelling model made it possible to associate variations in individual preferences with the observed heterogeneity of the income-related residential distributions. Similar to the abstract Schelling model, the dynamics of both implementations are robust to a lack of knowledge about the parameters; we consider this robustness a main feature of models applicable in the social sciences.

The Experience of Urban Modeling in the 1970s and 1980s

Back in the 1970s, ideas of complexity originating in chemistry and physics immediately attracted students of social dynamics. A system's dynamics could now be considered as governed by a few ''order parameters'' and, depending on initial conditions, converging to one of a limited number of steady regimes, usually equilibriums. That is, no matter how complex the real system appeared to be, it was assumed that commonsense formalizations and rough estimates of model parameters would result in close to realistic dynamics, the latter to be understood by investigating model steady states and the system's convergence to those states (Forrester 1969; Haken 1993; Haken and Portugali 1995; Weidlich 2000).

Following the inspiring results of the late 1960s (Albin 1975; Chapin and Weiss 1965, 1968; Forrester 1969; Steinitz and Rogers 1970), numerous papers written during the 1970s and 1980s applied the complexity paradigm to urban social systems with the aim of comprehending and simulating their dynamics qualitatively as well as quantitatively. However, toward the 1990s, the excitement abated. Nowadays, we identify two reasons for this decline: While the first is a perpetual lack of data, the second is the ''stocks-and-flows'' analytical framework, which replicates equations of chemical dynamics and which was gradually, but uncritically, accepted by the majority of modelers (Allen 1997; Wilson 2002). In urban implementations, ''stocks'' are associated with the population, jobs, dwellings, offices, and so on, of the urban aggregates (regions), while ''flows'' describe interactions between the aggregates, usually on the basis of Newtonian ''gravitation flows,'' that is, flows assumed to be proportional to the regions' stocks and inversely proportional to the distance between those regions (Fotheringham and O'Kelly 1989). Yet, the formalization itself remains problematic: Different from chemical systems, in which the interaction laws are experimentally verified and reaction rates are stable in time, geographic aggregates are artificial, with flow rates inherently dependent on the stocks' state (i.e., the model structure per se demands verification for every implementation).

The number of parameters in stocks-and-flows models is very high. In cases of K stocks and N regions, the number of parameters is of an order of KN^2, an astronomical feat for the 3 to 5 stocks and 10 to 12 regions typically included in an urban model (Bertuglia et al. 1994). In addition, the parameters' estimates cannot be substituted with "likely" values: The models' dynamics are "too sensitive" to parameters (Lee 1973, 1994). In other words, either hundreds of parameters must be known and remain constant during the entire modeled period, or nothing definite about the system's dynamics can be stated.

The problems that arose in stocks-and-flows models reflect the more general problem of the *parameterization* of socioeconomic systems: As stocks-and-flows models demonstrate, in the absence of strict quantitative laws, "likely" analytical expressions of stocks' relationships are insufficient because the "natural" values of parameters result in overvariety of dynamics. Urban modeling of the 1980s could not free itself of this discouraging conclusion.

In the 2000s, two salient developments induced a new wave of optimism. The first was crucial improvements in data quality and availability: In many fields—vehicle and pedestrian traffic, demography, residential dynamics, property ownership, land-use development—the current accumulation of data began to exceed our ability to use these resources (Benenson and Omer 2003). The second development was a new view of modeling sociospatial systems, one based on infrastructure objects and human agents and aimed at simulating dynamics of very specific subsystems, like pedestrian flows on a street (Gilbert and Troitzsch 1999; Benenson and Torrens 2004). With respect to urban phenomena—land-use dynamics, vehicle and pedestrian traffic—this high-resolution approach to modeling has already resulted in successful and operational simulations (for a review, see Benenson and Torrens 2004). Such developments subsequently prompted reexamination of social science models in the hope that direct modeling at the resolution of social objects, in combination with a temporal scale appropriate for description of the behavior of objects, could cope with the parameterization problem and enable operational prediction. An excellent instance amenable to such reexamination is the renowned Schelling model of segregation (Schelling 1971, 1978), on which we focus in this article.

The Schelling Model as an Alternative to the Hedonic Model

Similar to the stocks-and-flows model, the highly popular Hedonic approach (Rosen 1974) to the modeling of socioeconomic behavior also suffers from the problem of parameterization. The Hedonic model was formulated with respect to property values, the idea being that value is a function of the property's attributes: physical size, floor, number of rooms, age, environmental characteristics such as location, accessibility, and so forth. This dependence can be captured in linear or nonlinear regressions of the property's value on these attributes (usually in their static form, without entering the previous state of property or the environment into the model).

Within the framework of urban geography, the Hedonic approach can be applied far beyond property value and prices; it has been used to explain neighborhood effects in general: population structure, migrations, individuals' opportunities, and so on (see, e.g., Aaronson 2001; Buck 2001; Dietz 2002; Herrin and Kern 1992; Ihlanfeldt and Scafidi 2002; Ioannides 2003; Lynch and Rasmussen 2004; Sampson, Morenoff, and Gannon-Rowley 2002). Comprehensive experimental studies have already confirmed numerous relationships between neighborhood characteristics on one hand and those of families and dwellings on the other.

As mentioned, Hedonic regressions are constructed primarily on the basis of data collected during one time interval. That is, there is usually no way to determine whether the regression model characterizes the phenomena at close to a steady state or far from it; if the latter is true, the revealed dependencies represent the instantaneous situation only, which might uncontrollably change in time. Recent panel studies have made possible the inclusion of neighborhood characteristics at the prior temporal moment (usually 5 to 10 years previously) into the regression; not surprisingly, significant relationships between neighborhood characteristics at the current and the prior moment have been revealed (Dawkins 2005; Iceland 2004; Ioannides 2002, 2003). Those translations of Hedonic regressions into dynamic models that we are aware of provide fairly good approximations of population growth in Amsterdam, divided into 15 regions during a 13-year period (Wissen and van Rima 1986).

This most recent progress manifests an important shift toward understanding neighborhood *dynamics*; however, it again raises the problem of parameterization in all its force. Indeed, following the Hedonic argument, one eventually reaches an analogue of stocks and flows, with linear or nonlinear Hedonic relationships substituting for the gravitation law. No rational arguments currently exist to substantiate inexorable failure when following this line of reasoning; however, the problems are evident. Just like the stocks-and-flows model, Hedonic regressions are based on uniform representation of interactions between system components. Yet, are the factors and dependencies meaningful for one set of the experimental data reproducible in others, and would the model be oversensitive to parameters?

Natural science, even in its schoolroom version, presents a diametric view: Models with very few parameters can be estimated in "purified" experiments. When applied to realistic experiments, the models' deviations are explained by second-order effects. The demanded skepticism regarding the capacity of such a rigid and deductive approach to describe inherently loose social systems may be the reason why it is not especially popular in the social sciences.

Regarding urban residential distributions, our subject in this article, the deductive approach is represented by the classic Schelling model of segregation (Schelling 1971, 1978). The model considers residential segregation as a self-organizing outcome of interactions between householders; it is formulated in a rigid format with only one parameter. In what follows, we aim to investigate the potential of deductive socioeconomic modeling and use the Schelling model to modeling real-world residential distributions.

Robustness of the Schelling Model

To recall, the Schelling model considers an ensemble of *spatial* agents $\{\boldsymbol{g}\}$, each belonging to one of two mutually avoiding types, **B** and **W**, residing in cells of a finite square grid, with at most one agent per cell and able to change location at discrete moments of time. The agents' "residential behavior algorithm" $\boldsymbol{A}$ can be represented as follows:

- At every iteration, each agent **g** can change its location **c**.
- To decide whether to reside in a neighborhood, an agent **g** observes a *residential neighborhood* $\mathbf{N}(\mathbf{c})$ and calculates the fraction $\mathbf{S_c(t)}$ of the *strangers* (agents of not-**g** type) among the agents residing in $\mathbf{N}(\mathbf{c})$.
- If a fraction of strangers $\mathbf{S_c(t)}$ equal to or above a predefined level of tolerance $\mathbf{S_{Th}}$ is found in the original neighborhood (i.e., $\mathbf{S_c(t)} \geq \mathbf{S_{Th}}$), then **g** decides to relocate.
- To relocate, agent **g** constructs set $\mathbf{D} = \{\mathbf{d}\}$ of currently unoccupied cells within the *search neighborhood* $\mathbf{U}(\mathbf{c})$, for which the expected fraction of strangers is below the threshold (i.e., $\mathbf{S_d(t)} < \mathbf{S_{Th}}$).
- If set **D** is empty, **g** remains at **c**; otherwise, **g** selects one of the cells **d** from **D** according to a predefined choice rule $\boldsymbol{A}$ and relocates to that cell.
- Location **c**, previously occupied by **g**, immediately becomes available to other agents.

The above representation is quite generalized compared with Schelling's (1971) initial definition; nevertheless, it enables further variation in every component: in the size and form of neighborhoods $\mathbf{N}(\cdot)$ and $\mathbf{U}(\cdot)$, in algorithm $\boldsymbol{A}$ of new location choice, in considering in- and out-migration, and so forth.

In the late 1960s, Schelling investigated the basic version of the model by moving white and black pawns on a chessboard, assuming that $\mathbf{N}(\cdot)$ is a Moore 3×3 neighborhoods, $\mathbf{U}(\cdot)$ is an entire chessboard, and $\mathbf{S_{Th}} = \mathbf{2/3}$. His relocation algorithm ($\boldsymbol{A}$ rule) represented either random choice (Schelling 1969) of a vacant location in $\mathbf{U}(\cdot)$ or a version of Simon's (1982) satisficer rule (Schelling 1978), which can be paraphrased as follows: Construct a list of free locations within $\mathbf{U}(\cdot)$, randomize it, take the closest one satisfying condition $\mathbf{S_d}(t) < \mathbf{S_{Th}}$, and put the agent there. The result is well known: Irrespective of the initial state, the white and black pawns on the chessboard became completely segregated after a few moves; the system then stalls. According to Hegselmann and Flache (1998), Sakoda conducted similar games in the late 1940s, the results of which he published only as late as 1971 (Sakoda 1971).

Conceptually, the Schelling model is based on one parameter only, $\mathbf{S_{Th}}$, but its "disadvantage" lies in the relocation rule $\boldsymbol{A}$. $\boldsymbol{A}$ is a "rule," not an "equation"; this may explain why in-depth investigation of the Schelling model began only in 1990s and advanced relatively slowly, with simulations of various extensions remaining much more appealing than thorough mathematical investigation of the model's dynamics (Adamatski and Holland 1998; Bayer and Timmins 2005; Deutsch 2000; Deutsch and Lawniczak 1999; Frankel and Pauzner 2002; Morale 2001; Pollicott and Weiss 2001). In parallel, superficially similar models that assume pairs of agents exchanging places on the grid have also been investigated as variants of the Schelling model (Pollicott and Weiss 2001; Pancs and Vriend 2003; Zhang 2004) despite being qualitatively different mathematically.

For the simplest version of the Schelling model, two basic dynamics are possible, with $\mathbf{S_{Th}}$ varying within the [0, 1] interval and with the agents' density on the grid being very high (say, 99 percent), as would be characteristic of a real city. That is, no matter what the initial spatial distribution of the agents, at low levels of $\mathbf{S_{Th}}$ (high sensitivity to neighbors), the initial pattern always converges to a segregated state, displaying patches of **B** and **W** agents that are essentially larger than the residential neighborhood $\mathbf{N}(\cdot)$, with a size and form dependent on the model's fine details. After $\mathbf{S_{Th}}$ passes the level of about 0.7, almost no agents move after initialization, and initially the pattern remains random for an arbitrarily long time.

More than two types of agents, agents differing in a continuous attribute, specific threshold $\mathbf{S_{Th}}$ values for each group, different sizes and forms of neighborhoods $\mathbf{N}(\cdot)$ and $\mathbf{U}(\cdot)$, in addition to stochastic views of agent behavior, are among the variations currently being considered (Adamatski and Holland 1998; Deutsch 2000; Fossett and Waren 2005; Laurie and Jaggi 2003; Pancs and Vriend 2003). Yet, whatever the variations, they do not alter the essentials of Schelling agent behavior: They either tolerate neighbors or migrate; if they migrate, they choose available locations close to their current locations.

Investigations of the model's extensions have thus revealed an important fact: The Schelling model consistently manifests the same basic two-phase pattern. This result reminds us of Langton's (1986, 1992) characterization of cellular automata dynamics as well as the patterns revealed in other discrete models of local reactions and movements: cellular automata, random walker, and voting (Alves, Oliveira Neto, and Martins 2002; Galam 1997; Galam and Zucker 2000; Schweitzer and Zimmermann 2001). The two-phase pattern remains essentially valid even if we revoke some of the qualitative assumptions of the fundamental model and assume that (1) agents are capable of knowing about neighborhoods far from their current location and are able to migrate to any vacant location in the city, (2) the threshold $\mathbf{S_{Th}}$ evolves over time, and (3) agents differ in several characteristics (Benenson 1998; Flache and Hegselmann 2001; Fossett and Waren 2005; Laurie and Jaggi 2002, 2003; Portugali and Benenson 1994, 1995, 1997a, 1997b; Portugali, Benenson, and Omer 1994, 1997). Recent simulation research has, however, revealed the importance of the details of the specific residential choice algorithm. For example, if choices for which the probability per iteration is very low remain possible (as, for example, in the logit model), their cumulative probability over time can reach high values and prevent convergence of the residential pattern to segregation (Benenson and Torrens 2004; Bruch and Mare 2004).

The basic question remains: Can we apply the Schelling model to reality? The availability of spatially referenced data at the resolution of families and houses is crucial for the model's verification. The basic limitation here is that the verification cannot be based on samples; it demands knowledge about *all* the neighbors surrounding every agent. Recently, we obtained access to a database that makes this possible.

In what follows, we present two applications of Schelling's model. The first application involves a simulation of ethnic residential dynamics in Yaffo (an area of Tel-Aviv) for a 40-year period; the second simulates income-based residential dynamics in selected Israeli cities.

High-Resolution Residential Patterns in Israeli Cities

In 1999, the Israeli Central Bureau of Statistics (2000) released the world's first database of individual, spatially referenced data and made it available for supervised analysis, subject to Israel's privacy law. The database is the product of the 1995 population census; its core consists of two GIS layers, streets and building foundations, and of two nonspatial tables that contain family and individual attributes, such as country of origin, age, education, and number of children. Exceptional is the availability of data on individual annual income (for 1995), culled from the National Insurance Institute (social security) database; these data are related to census records based on individual identifiers. The building–family and family–individual relationships supported by the database make it possible to construct residential distributions according to selected characteristics at the resolution of separate buildings.

The analysis we present here is based on census data collected in nine cities located in the central part of Israel. Tel-Aviv, with approximately 350,000 residents, is the largest city; the populations of the others vary from 150,000 (Netanya) to fewer than 40,000 (Rosh HaAyin). The census database contains three characteristics—family income, parents' origin, and number of children—for at least 90 percent of the families in each city. The data on education level were available for about 15 percent of the households, while householders' estimates of the year of building construction were obtained for about 5 percent of the houses.

Test Case of the Schelling Model: Ethnic Residential Distribution in Yaffo

The first pattern studied was ethnic residential distribution in Yaffo, located in the south of Tel-Aviv. Yaffo is an area with a population of 30,000 Arabs and Jews; about 35 percent of its population was Arab in 2000. Irrespective of official pronouncements on relationships between Jews and Arabs in Tel-Aviv, members of one group do not like to reside in neighborhoods populated by members of the other group, a situation fitting Schelling's assumptions quite well. The residential processes to be observed in Yaffo were initiated in 1948 (a result of Israeli's War of Independence): According to Schelling's model, their outcome in 1995 should be stable segregation. We thus decided to simulate Yaffo's residential dynamics by constructing a model to cover the past 50 years (Benenson, Omer, and Hatna 2002).

Our model directly complied with Schelling's basic assumptions: Agents representing Arab and Jewish families and the probability of leaving neighborhoods with too many strangers increased with rising fractions of strangers. We further assumed that Yaffo's householder agents had knowledge of the residential market and could resettle throughout the *entire* Yaffo area, not only within the neighborhoods of their current locations. Migrations into and out of Yaffo were also included in the model.

Modern GISs enable convenient definition of neighborhood relationships. In the Yaffo model, we based these relationships on the Voronoi coverage constructed for the layer of house foundations, with two houses considered neighbors if their Voronoi polygons share a common edge (Figure 1a). In addition, as Omer (1996) showed, a building's architectural style (Figure 1b) influences the residential decisions of Yaffo's population. We could thus account for the strong tendency of the Arab population to reside in oriental-style houses and the preference of the Jewish population for modern block buildings.

Because the street network in Yaffo has not changed since the 1950s, with new residential construction rather limited in scope, the area was quite convenient for the simulation of a 50-year process. Initial conditions were the size of the Arab and Jewish populations, by statistical area, in 1955 (Figure 2a), an overall 1:2 (Arabs to Jews) ratio in the areas having an Arab presence. We began simulations with Arab and Jewish agents randomly distributed over the buildings in these areas. The model succeeded in simulating the experimental data and produced the segregated spatial distribution observed in Yaffo in 1995 (Table 1, Figure 2b; for details, see Benenson et al. 2002).

Modeling Yaffo provided important experience. First, the best fit was obtained with the scenario in which Arabs and Jews maintained *asymmetric* relations: Jews avoided Arabs, while Arabs were almost neutral to Jews. Second, these preferences were held for the entire model period (i.e., 50 years). Third, similar to the basic Schelling model, the Yaffo model was robust to quantitative changes in parameters. For example, to obtain results that differed substantially from those in Figures 2b, we were required to *qualitatively* alter the scenario's assumptions, say, by introducing symmetry into relationships between Arab and Jewish agents. Moreover, the likelihood dynamics were obtained *despite* the fact that dwelling prices and family income *were not included* in the model. This insensitivity was caused by the algorithm of residential choice applied.

Figure 1
Yaffo Model: (a) Definition of the Neighborhood Relationships and (b) Map of Buildings' Architectural Styles

Source: Benenson et al. (2002).

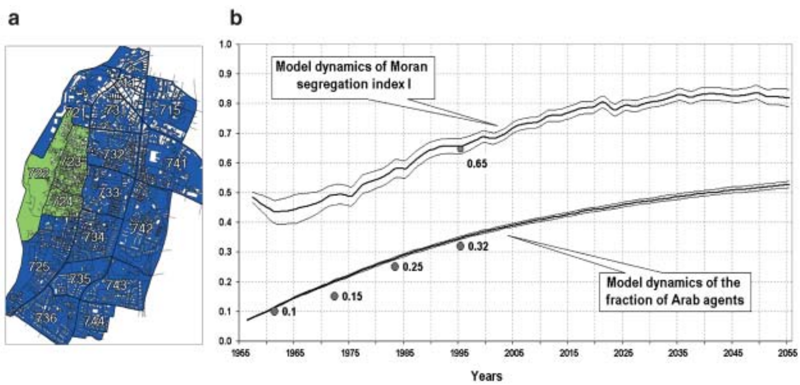
Table 1
Correspondence Between the Yaffo Model and Reality, 1995

Measure of Correspondence	Yaffo	Model
Overall percentage of Arab agents in the area	32.2	34.8
Moran index I for Arab agents	0.65	0.66
Percentage of Jewish agents in houses of oriental style	28.1	15.0
Percentage of Arab agents in houses of block style	18.5	8.0

Why Is the Yaffo Model so Insensitive to Parameters?

The reason for the Yaffo model's robustness was the "try the better" (**TRB**) algorithm of residential choice we formulated. An agent who uses **TRB** orders opportunities by their utilities *prior to making a choice*

Figure 2
Yaffo Model: (a) Initial Conditions and (b) Experimental and Model Dynamics of the Fraction of the Arab Population



Source: Benenson et al. (2002).

(Figure 3) and then tests the available opportunities in the preestablished order. Formally, the algorithm is constructed as follows:

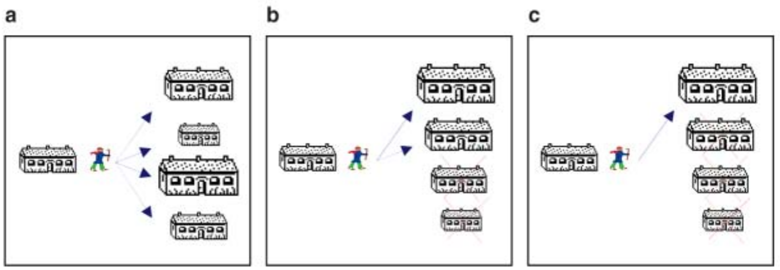
Given **K** opportunities with the aim of choosing one of them, an agent

1. estimates the utility $0 \leq \mathbf{u_i} \leq \mathbf{1}$ of each opportunity **i** ($\mathbf{i} = 1, 2, \ldots, \mathbf{K}$);
2. orders all **K** opportunities by their estimated utility value $\mathbf{u_i}$;
3. approaches opportunities according to an established order;
4. if an opportunity is still available and its utility ($\mathbf{u_i}$) exceeds the utility of the currently occupied location, accepts that opportunity with probability $\mathbf{p}_i = \mathbf{p}(\mathbf{u_i})$, where $\mathbf{p}(\mathbf{u})$ is a monotonous function of **c**, $\mathbf{p}(\mathbf{0}) = 0$, $\mathbf{p}(1) = 1$, and otherwise checks the next opportunity; and
5. quits after making a choice or reaching the end of the list.

TRB is conceptually close to the "take the best" (**TAB**) choice algorithm (Gigerenzer and Goldstein 1996) yet avoids the latter's determinism.

TRB and **TAB** both magnify the possibility that better opportunities will be chosen and that the worst opportunities will be ignored. This possibility is quite evident in **TAB**. To demonstrate the same principle for **TRB**, consider an agent who has to choose between **A** and **B**, with utilities

Figure 3
The Try the Better (TRB) and Take the Best (TAB) Choice Algorithms as Applied to Residential Choice: (a) The Set of the Opportunities, (b) the TRB Choice Algorithm, and (c) the TAB Choice Algorithm



$\mathbf{u_A} = 0.9$ and $\mathbf{u_B} = 0.8$, and assume that $\mathbf{p(u)} = \mathbf{u}$. The probability of choosing **A** when both options are available is $\mathbf{p_A} = \mathbf{u_A} = .9$, while the probability of choosing **B** is $\mathbf{p_B} = (\mathbf{1} - \mathbf{u_A}) \times \mathbf{u_B} = .08$; the probability of nonchoice is $\mathbf{p_ø} = (\mathbf{1} - \mathbf{u_A}) \times (\mathbf{1} - \mathbf{u_B}) = .02$. The $\mathbf{p_A}/\mathbf{p_B}$ ratio is thus $.9/.08 \approx 11$ in comparison with $\mathbf{u_A}/\mathbf{u_B} = 0.9/0.8 \approx 1.1$.

The most important aspect of **TRB**, like **TAB**, is the dynamic outcomes, which are robust to changes in model parameters as long as the order of preferences is maintained; when the order changes, the reaction is immediate (see "TRB choice probabilities" in Table 2).

The use of **TRB** also avoids problems associated with the logit model, which is a formalization of the idea of maximization of random utility under imperfect knowledge. To reiterate, the probability $\mathbf{p_i}$ to choose opportunity **i** is set in the logit model as equal to $\mathbf{p_i} = \mathbf{exp}(\alpha\mathbf{u_i}) / \Sigma_\mathbf{i}\mathbf{exp}(\alpha\mathbf{u_i})$ for choices with utilities $\mathbf{u_1}, \mathbf{u_2}, \ldots, \mathbf{u_K}$. Problems begin when one of the choices, say the first one, has very high utility $\mathbf{u_1}$ compared to the utilities $\mathbf{u_i}$ of the remaining $\mathbf{K} - 1$ choices. In this case, with an increase in **K** or with repeated applications in subsequent iterations, the probability that a low-utility choice, other than $\mathbf{u_1}$, will be selected tends to the unit, a bothersome possibility. Also note that **TAB** and **TRB** are defined by their utilities only, while the logit model depends on an additional parameter, α.

Table 2
Amplification of the Probabilities of Choice by TRB and TAB Heuristics

Opportunity	Utilities		
A	0.8	0.8	0.8
B	0.6	0.7	0.85
		TRB choice probabilities	
A	.80	.80	.12
B	.12	.14	.85
None	.08	.06	.03
		TAB choice probabilities	
A	1.0	1.0	.0
B	.0	.0	1.0
None	.0	.0	.0

Note: **TAB** = take the best; **TRB** = try the better.

Given this, the qualitative consequences of **TAB** and **TRB** choice algorithms should be viewed from a broader perspective. For ensembles of **TAB** or **TRB** agents, factors that do not overturn opportunities' utilities cannot, in effect, significantly influence outcomes. If the majority of householders follow the same set of criteria, the sensitivity of the outcomes of any "preference orderings" heuristic to the influence of those factors will be very low as long as the *order of the utilities* attached to the choices *remains the same*. If this holds, success in discovering the main factors determining human residential decisions, even if knowledge is incomplete, might enable the simulation of real-world dynamics; stated differently, complete knowledge will no longer be mandatory for successful modeling.

Recent high-resolution cellular automata simulations of urban land-use dynamics provide indirect confirmation of the "*ordering*" perspective. We should recall that cellular automata models omit human decisions regarding land-use changes and simulate the latter as "depending on themselves." It is evident, however, that hypothetical or experimentally derived cellular automata rules reflect human decisions. Analysis of cellular automata simulations (Engelen, White, and Uljee 1997, 2002; Li and Yeh 2000; Turner 1988, 1989; White and Engelen 1997, 2000) reveals that their developers indirectly implement the principle we formulated above: They order potential land-use transitions for each unit by their likelihood, followed by attempts to implement a transition, beginning from the most likely one.

To summarize, the Yaffo model has taught us several important points:

1. Real-world ethnic residential dynamics can be successfully simulated with a Schelling-like model.
2. The "human-based" algorithm of residential choice implemented in the model results in the model's robustness to the incomplete knowledge regarding influencing factors and parameter variations.
3. Qualitative correspondence between the model and reality is achieved with agents who possess "human" properties—asymmetric relationships between group members and distance-independent migrations—despite the fact that traditional segregation-inducing quantitative variables, dwelling prices and family income, were not included in the model.

A View of Residential Distributions in Israeli Cities by Family Income

Yaffo's residential dynamics are quite special because they result from ethnic factors. Most Israeli cities are relatively homogeneous in terms of population ethnicity; consequently, the majority of residential choice decisions in Israel, just like in other areas in the Western world, are defined by the interplay between dwelling prices and family income. Our first step toward modeling this general case was mapping householders' incomes for nine cities. In what follows, we provide the maps resulting from monthly income transformed as $\mathbf{Income} \rightarrow \mathbf{log_2(Income)}$, which makes the (non-spatial) distribution of family income as close to normal. Note that the interval $[\mathbf{x}, \mathbf{x}+1]$ in a logarithmic scale corresponds to the interval of actual income $[2^{\mathbf{x}}, 2^{\mathbf{x}+1}]$, with the interval's upper bound two times greater than the lower bound. To make the patterns visually comprehensible, we base them on Voronoi coverage; to comply with privacy restrictions, the Voronoi polygon of the building represents the first-order average of the mean income in a building over a neighborhood, as shown in Figure 1a. This type of presentation creates a map smoother than the actual distribution; to overcome this bias, we present a map of Standard Deviation (SD) of the mean income in a building calculated over the same neighborhoods.

Analysis of the Census Data

Figure 4 presents the residential pattern for Ramat Hasharon, the most heterogeneous, and Figure 5 presents the pattern for Bat-Yam, the least heterogeneous, among the cities investigated (Figures 6a and 6b).

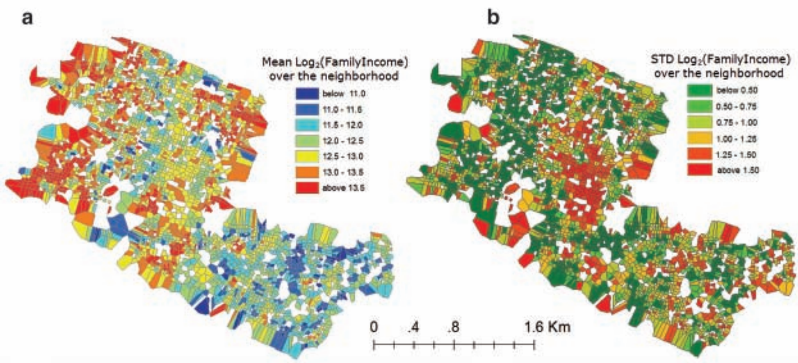
The mean **SD** and the **SD**'s 95th percentile for Bat-Yam (Figures 5 and 6b, Table 3) are the lowest of the nine cities; its map coincides most perfectly with the Schelling model for the case of agents characterized by a continuous property (Benenson 1999); that is, poorer and richer householders segregate. The situation in Ramat Hasharon is quite the opposite (Figures 4 and 6b, Table 3), with very high heterogeneity of residential distribution. The general situation in Israel is closer to that of Ramat Hasharon, a mix of homogeneous and heterogeneous areas (Table 3).

The nonsmooth **SD** maps characteristic of most of the cities studied altered our Schelling-induced view of urban residential distribution, that is, of areas populated by families significantly different in income covering significant portions of most Israeli cities. We rejected the hypothesis that these wealthier people might lack other residential opportunities, because the typical annual internal migration rate in Israel is about 5 percent.

Other reasons for the observed heterogeneity may be strictly personal or related to the local infrastructure; for example, newer dwellings constructed in poor areas are always more expensive than the surrounding structures and thus might attract wealthier householders. At the personal and family levels, the educated householders might be more tolerant of poorer neighbors, while poorer families with children may prefer to stay in wealthy neighborhoods to take advantage of better schools. Relations of this kind can be partially tested: As mentioned above, the census files include the year of building construction, years of education, and the number and ages of children. We could therefore estimate correlations among mean income of the households in the building and the building's age, the average education of the head of household, and the fraction of children of school age per family for the buildings over the areas where household incomes greatly varied.

To estimate correlations, we first defined which areas were to be considered "heterogeneous." We did so on the basis of "third-order" neighborhoods: those consisting of houses in "first-order" neighborhoods as defined in Figure 1a plus two additional rings of "neighbors of neighbors" of the house. The houses were included in the "heterogeneous" area if the **SD** of the householders' $\mathbf{log_2(Income)}$ over its third-order neighborhood was above 1.25 (in terms of income, $2^{1.25} \approx 2.4$). Basing on neighborhoods of the third order guarantees continuity of the "heterogeneous" area. The results do not change qualitatively if we define heterogeneity in terms of higher **SD** values over the third-order neighborhood, say 1.5.

Figure 4
Ramat HaSharon, the Most Heterogeneous of the Nine Cities Investigated: Distributions of (a) Family Income Averaged Over the Houses Within the Neighborhood and (b) Family Income SD Over the Houses Within the Neighborhood



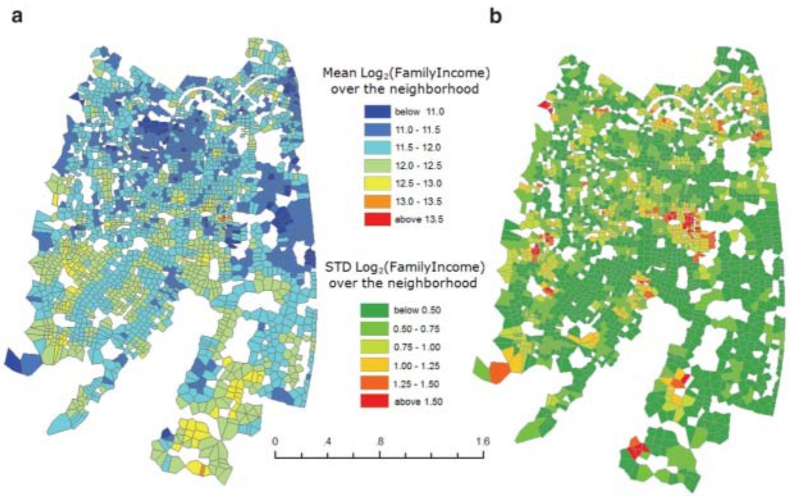
Note: Neighborhoods are defined as shown in Figure 1a.

As can be seen from Table 4, despite the exceptionally high resolution and sizable amount of data, the correlations approach standard values, characteristic of Hedonic regressions (Ioannides 2002, 2003; Ioannides and Seslen 2002). Correlations between family income and fraction of children fluctuate around zero, while those between income and education as well as income and building age remain positive; that is, richer householders in heterogeneous areas live in newer buildings and are more educated.

The strong correlation between education and income in Israel is a well-known fact (Israeli Central Bureau of Statistics 2000); thus, the only "new" information obtained from the data in Table 4 is that dwellings in more expensive newer buildings are more often populated by families with higher income. The basic question, why entrepreneurs should decide to build in poor areas and why wealthy agents should occupy these new buildings, remains unanswered.

In searching for an explanation, we decided to apply directly to the wealthier householders residing among poor neighbors and ask them why they remained in those neighborhoods. Census maps at the resolution of houses assisted in identifying neighborhoods with wealthier households.

Figure 5
Bat-Yam, the Most Homogeneous of the Nine Cities Investigated: Distributions of (a) Family Income Averaged Over the Houses Within the Neighborhood and (b) Family Income SD Over the Houses Within the Neighborhood



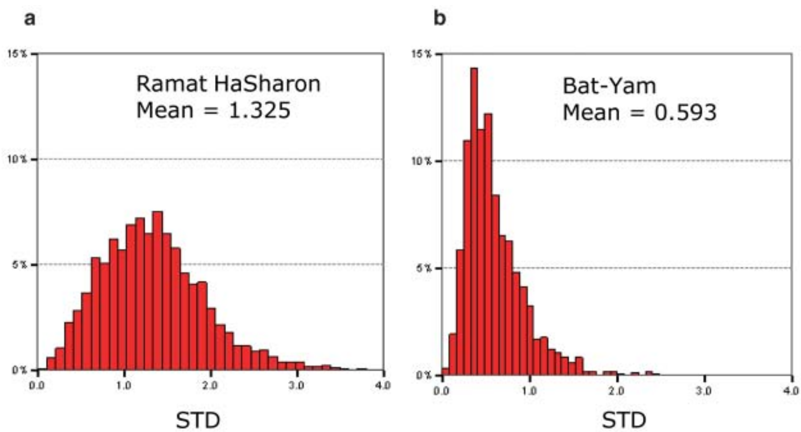
To estimate their attitudes to their neighbors, we approached the wealthier householders with simple questions.

Questionnaire on Householders' Attitudes to Their Neighbors

Our questionnaire contained three very simple, straightforward questions of the kind "Is it important for you that X among your neighbors is the same as yours?" where X referred to "economic status," "culture," or "education." Respondents were asked to indicate their answers on a Likert-type scale ranging from 1 (*very unimportant*) to 5 (*very important*).

Twenty heterogeneous and 20 homogeneous wealthy neighborhoods in Tel-Aviv were chosen on the basis of high-resolution maps, as shown in Figures 5 and 6. Apartments looking "wealthy" were selected during visits to heterogeneous neighborhoods, whereas apartments in homogeneous wealthy neighborhoods were randomly selected. The results of the

Figure 6
Distribution of Income SD over Neighborhoods in (a) Ramat HaSharon and (b) Bat-Yam



Note: Residential patterns in the two cities are presented in Figures 4 and 5, respectively.

interviews are presented in Table 5. As the table indicates, wealthier people in heterogeneous areas are more tolerant of their neighbors than are wealthier people in homogeneous neighborhoods.

The survey is very preliminary. The results nonetheless inspired the idea of *agent-specific* reactions to neighbors as a possible explanation of the residential heterogeneity found in Israeli cities: Tolerant agents remain among those who differ from them, while intolerant agents self-organize into more homogeneous areas.

Looking for confirmation of the idea, we found extensive data in favor of the presence of highly tolerant agents in a paper by Bruch and Mare (2004) in which they estimated householder preferences regarding the White-Black composition of the neighborhood. Bruch and Mare's research was based on stated residential preference data obtained from the Los Angeles and Boston modules of the Multi-City Study of Urban Inequality and the Detroit Area Studies for 1976 and 1992 (Farley, Fielding, and Krysan 1997). The 1992 Detroit data demonstrated that 28.1 percent of 711 White householders in the sample accepted the possibility of moving into a neighborhood where the fraction of Blacks is above 57 percent, whereas 27.7 percent of Black householders accepted the idea of

Table 3
Basic Characteristics of the 9 Cities Selected and of the City's Spatial Heterogeneity

City	Bat-Yam	Ashdod	Lod	Tel-Aviv	Ramla	Netanya	Rosh HaAyin	Kfar Saba	Ramat HaSharon
Population (×1,000)	140	130	52	350	40	150	40	70	40
Populated buildings	2,485	2,869	1,814	17,208	2,404	5,287	3,017	3,234	3,140
Mean value of **SD** over the neighborhoods	0.593	0.804	0.876	0.888	0.923	0.941	1.047	1.075	1.325
95th percentile of **SD** over the neighborhood	1.255	2.065	1.972	1.983	1.701	1.935	2.206	2.042	2.443

moving into a completely White neighborhood (see Bruch and Mare 2004). The Boston studies, with close to 250 Black and 250 White respondents (Farley et al. 1997), imply that it is possible to go even further with the idea of varying tolerance by demonstrating two peaks of residential choice probabilities as dependent on neighborhood composition (see Bruch and Mare 2004). We can interpret the first peak as characterizing typical intolerant householders who are ready to move into a neighborhood where at least of 90 percent of the residents belong to the same group and the second peak as characterizing tolerant householders, those who can accept up to 60 percent to 70 percent of the members of the other group within the neighborhood. Bruch and Mare used these data to construct continuous dependence of residential preferences on the fractions of strange and friendly neighbors, a dependence they subsequently used in simulations of the residential dynamics during long time intervals. The results bring them to the conclusion that local residential preferences cannot explain observed segregated global patterns. As pointed out above, the danger exists that low choice probabilities can critically influence results when applied many times.

Our idea was first to consider tolerance as a *property of a householder* (i.e., as a characteristic that does not change in time) and second to use choice algorithms that ignore opportunities having very low choice probabilities. The idea of tolerance varying between agents was first proposed by Portugali et al. (1997), who demonstrated that in a basic Schelling model of Black and White

Table 4
Correlations Between Mean Household Income in a Building and Three Available Characteristics of the House and Householders in Areas Satisfying Heterogeneity Conditions

City	Percentage of City Areas With **SD** > 1.25 Over Third-Order Neighborhoods	Fraction of Children in a Building	*n*	Fraction of Householders Who Graduated From High School	*n*	Year of Building's Construction	*n*
Bat-Yam	3.5	.035	88	.226	27	.233	5
Ashdod	22.8	–.269**	656	.187**	224	.224	29
Lod	20.0	–.124*	365	.354**	121	.055	26
Tel-Aviv	20.0	.158**	3,417	.221**	1,440	.581**	117
Ramla	18.0	.016	437	.202**	164	.390	15
Netanya	23.8	.165**	1,289	.262**	517	.140	30
Rosh HaAyin	34.9	.058	1,057	.291**	283	.702	6
Kfar Saba	44.2	.177**	1,435	.391**	650	.443**	98
Ramat HaSharon	68.8	.126**	2,166	.255**	831	.041	51

*$p < .05$. **$p < .01$.

Table 5
Mean of Answers to the Question "How Important Is It for You That Your Neighbors in [Your House/Neighboring Houses] Are Similar to You in [Characteristic]?"

	In Your House			In Neighboring Houses		
Characteristic	Rich Among Poor ($n = 18$)	Rich Among Rich ($n = 13$)	p	Rich Among Poor ($n = 20$)	Rich Among Rich ($n = 20$)	p
Economic status	2.56	3.31	$\sim$.85	2.20	3.10	$\sim$.10
Cultural level	2.72	4.00	$\sim$.01	2.35	3.75	<.01
Level of education	2.21	3.38	<.01	1.80	3.10	<.01

agents, residential segregation is maintained despite the 30 percent presence of tolerant agents in each of the two groups. Their implementation did not, however, account for the economic advantage of poor areas: lower housing prices, which can increase the attractiveness of dwellings there for tolerant householders. O'Sullivan (2002) also built on wealthy householders entering poorer areas in his cellular automata model of gentrification; however, we propose a reason of this behavior. In our model, we aim at explaining heterogeneity exclusively; we therefore intentionally ignore the possibility of global decreases or increases in the real estate value. Our goal is to demonstrate that low fractions of householders tolerant to poorer neighbors can be sufficient to obtain a mix of homogeneous and heterogeneous areas in a city as well as to test the plausibility of this hypothesis and its robustness to the limited knowledge on householders' behavior.

The Model of Residential Heterogeneity in Israeli Cities

Formally, our aim was to explain the income residential pattern typical of Israeli cities: the mix of homogeneous and heterogeneous areas on the base of varying agents' tolerance to poorer neighbors. We thus considered a Schelling-like model in which householder agents occupy and migrate between dwellings, represented by cells of a grid. As we show below, model residential patterns in scenarios of all intolerant householders shortly self-organize toward clear income gradients and low heterogeneity. Low fractions of householders tolerant to poorer neighbors result in essential increases in the model city's heterogeneity; the heterogeneity does not grow much when the fraction of tolerant householders is further increased.

Some details of this explanation are warranted here.

The Model's Basic Principles

The model incorporates all "experimentally induced" principles that have worked well until now:

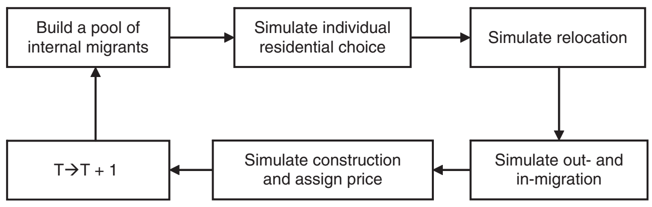
- **TAB** as the residential choice heuristic
- Knowledge about vacancies does not depend on the distance between an agent's current location and the location of the vacancy
- Asymmetry of relationships between rich and poor: rich agents avoid poor agents but not vice versa

A householder agent **g** is characterized by "income" **E**(**g**), and "intolerance" **INT**(**g**), whereas a dwelling **c** is characterized by "price" **P**(**c**). **E**(**g**) and **INT**(**g**) are assigned to an agent **g** independently of each other. At each time step, city residents make decisions as to whether to stay at their current locations or move to other dwellings. An immigrant agent decides whether to occupy the location in the city. The decisions are based on the disutility of the current location as well as on the utilities of the vacancies. The utility of location **c** for an agent **g** is defined by its price **P**(**c**) and by the mean income of the neighbors located within the Moore 5×5 neighborhood **N**(**c**) of **c**.

To maintain a realistic line of argument, we account for an aspect ignored in the basic Schelling model, namely, the relationship between dwelling price and householder's income. We assume, first, that an agent must have sufficient income to locate in the selected house and, second, that an agent is willing to spend only part of his or her income for the dwelling. This part is high for poor agents and decreases with the increase in an agent's income.

Formally, we assume that a residential agent is willing to spend a certain fraction **k**(**E**(**g**)) of income **E**(**g**) on a dwelling. If an agent **g** decides to occupy the "cheap" house **c**, for which **P**(**c**) < **k**(**E**(**g**)), the reminder **k**(**E**(**g**)) – **P**(**c**) will nonetheless be spent at the moment of occupation on upgrading **c** "to **g**'s level," an approach similar to that used in O'sullivan's (2002) model of gentrification. Alternatively, overspending in the case of a dwelling of **k**(**E**(**g**)) < **P**(**c**) results in an additional penalty (say, accumulated mortgage interest) on utility, which is also applied only once,

Figure 7
Flow of Model Events



when an agent enters the dwelling. A formal description of the model is provided in the Appendix.

We model the city as an open system; to simplify, we do not distinguish between births and immigration and between mortality and emigration. The number of householder agents added to the city's population at a given time step is proportional to the current number of residents. A constant fraction of agents leaves the city at every time step. In response to demand, new dwellings are "constructed" in cells adjacent to the occupied ones; the price of a new dwelling is assigned as proportional to the income of its neighbors.

When the simulation begins, the city consists of a small 3×3 patch of occupied buildings, located in the center of the grid. New householder agents are added to the city population; their income **E**(**g**) is assigned according to the income distribution typical of Israeli cities; the intolerance **INT**(**g**) of immigrant agent **g** is assigned independently of income. All model dependencies are intentionally rough and represented by piecewise linear functions. The basic flow of events in the model is presented in Figure 7.

The model dynamics for various distributions of **INT**(**g**) are the subject of our study.

Model Results

To verify our hypothesis that the heterogeneity of the residential distribution in Israeli cities is caused by householders who are tolerant of poorer neighbors, we investigated the qualitative correspondence between residential patterns in Israeli cities and model patterns. For every scenario, we examined the correspondence in two ways: First, we considered maps of average income and the associated **SD**s in terms of $\mathbf{log_2(E[g])}$ over the 3×3 neighborhoods: The maps are constructed in the same way as those of other Israeli cities in Figures 4 and 5. Second, we constructed nonspatial distributions of the **SD**s and their 95th percentile: $\mathbf{SD}_{95}$ (Figure 8), which can be compared with the data in Figure 6 and Table 3.

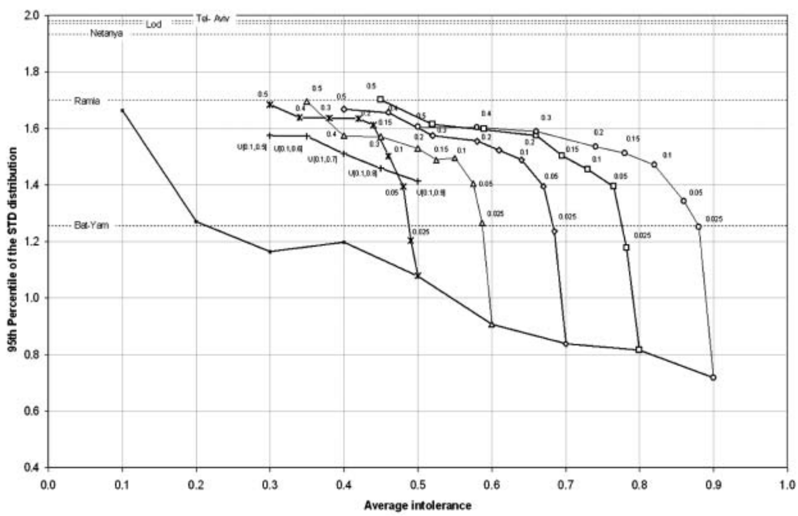
As stated above, the scenarios we investigated differed with respect to the distribution of $\mathbf{INT(g)}$ among immigrants to the city. In the basic scenarios, all agents have the same intolerance. In the second group of scenarios, the values of $\mathbf{INT(g)}$ is set either 0.1 or $\mathbf{INT_{Max}}$, while we vary $\mathbf{INT_{Max}}$ and the probability $\mathbf{p}_{0.1}$ that an agent's intolerance equals 0.1 between the scenarios. In addition, we consider the third group of scenarios in which agent intolerance is uniformly distributed on the interval [0.1, $\mathbf{INT_{Max}}$] and where $\mathbf{INT_{Max}}$ varies between the scenarios. Depending on the scenario, the city requires up to 400 to 500 iterations to sprawl over the 100×100 grid and then stabilize. After this period, the city pattern varies very slowly; visually, nothing changes during next 1,000 time steps. The level of heterogeneity is stabilizing after first 50 to 100 iterations. We used the Bat-Yam pattern as an anchor for distinguishing between the homogeneous and nonhomogeneous patterns; the $\mathbf{SD}_{95}$ for Bat-Yam equals 1.255 (Table 3).

Residential patterns in a city of agents of identical intolerance. In the first series of scenarios, we assumed that the same value of $\mathbf{INT(g)} = \mathbf{INT_{Max}}$ (i.e., that the variance of $\mathbf{INT[g]}$ equals zero) is attached to all immigrants. The values of $\mathbf{SD}_{95}$ corresponding to $\mathbf{INT_{Max}}$ varying from 0.1/0.9 are represented by the lowest curve in Figure 8. As can be seen from Figure 8 and, in addition, from the maps of the average and **SD** of $\mathbf{log_2(E[g])}$ for the case of $\mathbf{INT_{Max}} = 0.5$ (Figure 9, column a), the residential pattern remains highly segregated until very low levels of intolerance. To pass the $\mathbf{SD}_{95} = 1.25$ criterion, characteristic of Bat-Yam's residential pattern, the value of $\mathbf{INT_{Max}}$ should decrease to below 0.2.

Let us consider this series of scenarios, in which agents share the same intolerance level as the basic scenario and proceed to those scenarios in which the city population contains both intolerant and tolerant agents.

The consequences of introducing tolerant agents into the city. To verify the idea that a low fraction of tolerant agents is sufficient to obtain an essentially heterogeneous pattern, we investigated second and third series of scenarios. The second series consisted of a subseries of two-value distributions of $\mathbf{INT(g)}$: $\mathbf{INT(g)} = \mathbf{INT_{Max}}$ and $\mathbf{INT(g)} = 0.1$. We considered

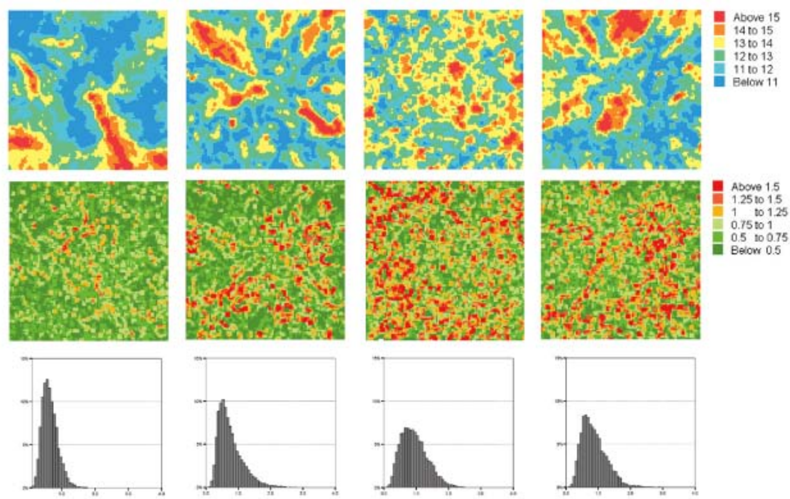
Figure 8
95th Percentile of the SD of Log_2 (Income) Over the 3×3 Moore Neighborhood for Various Scenarios



Note: The bottom curve represents scenarios in which all agents have the same **INT**(**g**). Each branch that originates at intolerance value X and raises left-top represents a subseries in which the fraction $\mathbf{p}_{0.1}$ of immigrants is assigned **INT**(**g**) = 0.1, while the rest **INT**(**g**) = X. The labels show the value of $\mathbf{p}_{0.1}$. Separate curve denotes scenarios in which immigrants' **INT**(**g**) is uniformly distributed on [0.1, X], X = 0.5, 0.6, 0.7, 0.8, 0.9.

five subseries (branches above the bottom line in Figure 8), with $\mathbf{INT_{Max}} = 0.5$, 0.6, 0.7, 0.8, and 0.9. The scenarios of the subseries differ in the fraction of the highly tolerant agents (i.e. those with **INT**[**g**] = 0.1) in the population. Formally, the subseries scenarios differ in the probability $\mathbf{p}_{0.1}$ that an immigrant agent's intolerance equals 0.1 (the fraction of agents whose intolerance equals $\mathbf{INT_{Max}}$ equals, respectively, to $1 - \mathbf{p}_{0.1}$). We consider the values of $\mathbf{p}_{0.1} = .025$, .05, .10, .15, .20, .30, .40, and .50 in each series. For example, the subseries that starts with the scenario of $\mathbf{INT(g)} = \mathbf{INT_{Max}} = 0.9$ shared by all immigrants (the rightmost curve in Figure 8) proceeds with the scenario in which the probability $\mathbf{p}_{0.1}$ that immigrant's **INT**(**g**) = 0.1 equals .025, while the probability that

Figure 9
Maps of $\log_2$(Income) Average (top), SD (middle), and Nonspatial Distribution (bottom) of the SD Over the 3×3 Moore Neighborhood for Four Distributions of Immigrants' Intolerance INT(g): (a) INT(g) = 0.5 for 100 Percent of Immigrants; (b) INT(g) = 0.5 for 90 Percent and INT(g) = 0.1 for 10 Percent of Immigrants; (c) INT(g) = 0.5 for 50 Percent and INT(g) = 0.1 for 50 Percent of Immigrants; and (d) INT(g) of the Immigrants Is Distributed Uniformly on [0.1, 0.5]



INT(**g**) = 0.9 equals .975; in the next scenario of the subseries, $\mathbf{p}_{0.1} = .05$, and so on. In the last scenario of the subseries, $\mathbf{p}_{0.1} = .5$.

The third series contains five scenarios only, in which **INT**(**g**) of the immigrants is distributed uniformly on [0.1, $\mathbf{INT_{Max}}$], with $\mathbf{INT_{Max}} = 0.5$, 0.6, 0.7, 0.8, and 0.9, respectively.

As one can see in Figure 8, a low fraction of tolerant agents induces an essential increase in residential heterogeneity of the model pattern. The scenarios of $\mathbf{p}_{0.1} = .025$ (i.e., with only 2.5 percent of highly tolerant agents) resulted in an $\mathbf{SD}_{95}$ of about 1.25 (Bat-Yam's $\mathbf{SD}_{95}$) no matter what is the $\mathbf{INT_{Max}}$ intolerance value of the remaining agents in the city.

The next scenario, with $\mathbf{p}_{0.1} = .05$, resulted in an increase in the $\mathbf{SD}_{95}$ from about 1.2 to close to 1.4. This sensitive reaction of $\mathbf{SD}_{95}$ to the fraction of the highly tolerant agents halts, however, when $\mathbf{p}_{0.1}$ reaches the values of .15 to .20, with the value of $\mathbf{SD}_{95}$ reaching about 1.5. The value of $\mathbf{SD}_{95}$ for the scenario of an unrealistically high (50 percent) fraction of tolerant agents resulted in an $\mathbf{SD}_{95}$ between 1.6 and 1.7 (Figure 8). Figure 9, column b, presents the residential patterns and nonspatial distribution of **SD** for the scenario in which $\mathbf{INT_{Max}} = 0.5$, $\mathbf{p}_{0.1} = .1$, while Figure 9, column c, does likewise for the unrealistic scenario of $\mathbf{INT_{Max}} = 0.5$, $\mathbf{p}_{0.1} = .5$ (i.e., with 50 percent tolerant agents).

Finally, separate five-point curves on the graph in Figure 8 present the values of $\mathbf{SD}_{95}$ for the scenarios in which immigrants' intolerance is assigned according to uniform distributions. The $\mathbf{SD}_{95}$ values vary between 1.4 and 1.6. Figure 9, column d, represents residential patterns and nonspatial distributions of **SD** for $\mathbf{INT}(\mathbf{g})$, uniformly distributed on $\mathbf{U}[0.1, 0.5]$.

Our conclusion, therefore, is that the idea does work and that a low fraction of tolerant householders makes the city pattern more heterogeneous compared with a city having only intolerant agents. The result does not depend on immeasurable parameters, such as the numeric values of intolerance assigned to "intolerant" and "tolerant" agents: As long as the fraction of tolerant agents passes the 10 percent to 15 percent level, the city is "heterogeneous," with this heterogeneity quite close to the maximally possible. The model is thus robust to the perpetual lack of arguments in favor of specific formalizations of "human tolerance": The notion's redefinition in every study says nothing about possible variation in the householder's tolerance over time, in response to personal and nonpersonal factors, and so on. Taking the model result literally, one can say that to preserve urban residential heterogeneity at a level comparable with that observed in Israeli cities, we need 10 percent to 15 percent of householders tolerant of poorer neighbors and aiming at economic advantages when remaining in poorer neighborhoods.

Despite basing our simulations on real-world estimates of model parameters, including the mean and **SD** of the income distribution used for assigning agents' $\mathbf{E}(\mathbf{g})$ as well as migration rates, we do not dare state that our abstract model reproduces real cities. This is particularly so because the heterogeneity of the model patterns is characterized by $\mathbf{SD}_{95} \sim 1.5$: essentially lower than that in real cities, for which the modal value of $\mathbf{SD}_{95}$ is about 2.0 (note the marks on the *y*-axes in Figure 8). However, the discrepancy can be easily explained by the oversimplification of the pricing rule for new dwellings, which simply averages willingness to pay (see equation 8 in the appendix). This rule imposes an essential constraint on the variability of dwelling prices in the model and, consequently, reduces the heterogeneity of the residential pattern.

The above presentation of the model results aimed at testing the hypothesis only; many aspects of the model remain beyond the current framework and will be published subsequently. But what about making the model operational, our exalted goal, declared at the beginning of this article? We see two steps as necessary to achieve it. First, data on infrastructure development during the modeling period should be introduced. The awkward rule of pricing used in the model should be substituted by externally given infrastructure dynamics. Such a step will enable investigation of householders' behavior per se. Second, householders' behavior in the model is defined by reactions to neighbors and the dwelling's price; we investigated the former in detail while accepting the simplest of assumptions regarding the latter. The study of the economic aspects of householders' behavior should be thus extended, within the model, in laboratory experiments, and in the field.

Discussion

We began this article by asking whether the operational modeling of social systems is possible. After presenting results supporting this possibility, are we prepared to respond to that question regarding the urban residential distributions at the focus of this article? All our arguments were aimed at convincing the reader that Schelling's view of residential patterns as outcomes of householders' choices in "friendly" neighborhoods may be the correct answer. We began with the critique of the stocks-and-flows model, which was good for chemical dynamics but did not fit cities. Along the way, we made major stops at robustness to variations in "immeasurable" parameters as a mandatory property of social science models, the good fit between Schelling-like models and Jewish-Arab residential dynamics in Yaffo, mapping of personal income data in nine Israeli cities and the idea of the heterogeneity as caused by the tolerant householders. Yet, the model of income residential patterns did not allow us to approach representation of real cities.

Considered qualitatively, the question is whether the above results are characteristic of the investigated implementations of the Schelling model or whether we have captured background properties of householders' behavior, necessary for modeling urban residential dynamics as socioeconomic phenomena. The agent-based models translate this question into another: whether residential choice behavior can be understood as a function of a few basic properties belonging to the dwelling and the householder. If the same model could be applied to a sufficient number of examples, their qualitative analytical expressions can be revealed on the basis of experimental data.

We are optimistic and do believe that in ‘‘simple’’ and tightly infrastructure-related cases, such as urban residential dynamics or vehicle and pedestrian traffic, laws of agents' behavior—householders, drivers, or pedestrians—do exist and that researchers will (eventually) be able to recognize and implement them in models, just as students of physical, chemical, or engineering systems already do.

Our view is that the frameworks borrowed from the other disciplines, such as the stocks-and-flows approach, were useful at the initial stage of socioeconomic modeling only, whereas the principles that govern social systems are derivatives of individual behavior and have yet to be ‘‘refined.’’ We further argue that the distillation cannot be accomplished without direct links between behavioral models and experiments; for example, exceptional data on urban residential distributions that we were fortunate enough to acquire resulted in four principles: asymmetry of avoidance relationships between agents, ‘‘frugal’’ choice heuristics (Gigerenzer and Goldstein 1996), distant migrations, and, finally, agent-specific tolerance.

Humans are not molecules, and whatever the behavioral laws of model agents, they will always contain loosely, if measurable, characteristics, such as the intolerance to neighbors used in this article. To avoid the problem, we require robustness of the model outcome to the formal expression of the respective laws. Our belief is that human systems that can be represented by robust models *are operationally comprehensible* just because rough knowledge about their parameters is sufficient for understanding their dynamics. Our hope is that in robust cases, we will be able to proceed operationally beyond the extreme case of Jewish-Arab residential dynamics in an area where infrastructure has remained relatively constant for 40 years.

This requires a qualification: Our view is that robustness is a necessary but not at all sufficient property of social models. We would mention a few cases that remain beyond our discussion: the model's robustness to the lack of knowledge on the distribution of tolerance among householders demands experimental support; it might be that in reality, the fraction of tolerant householders is always very low and thus insufficient to explain the observed heterogeneity. The model likewise assumes that householders' income does not change over time, an assumption that is far from realistic. The only response we offer to these and other questions is further investigation; in this respect, we consider our deductive and agent-based approach as a yet underinvestigated alternative that should be directly tested in close association with high-resolution GIS databases.

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Itzhak Benenson is a professor and head of the Department of Geography and Human Environment, Tel Aviv University, and a head of Spatial Modeling and GIS laboratory there. His main areas of interest are spatially explicit modeling of the urban and regional processes, transportation modeling, accessibility analysis, high-resolution spatial population dynamics, GIS and spatial analysis. He is an author of "Geosimulation: Automata-Based Modeling of Urban Phenomena" (Wiley, 2004), written together with Paul M. Torrens.

Erez Hatna holds a PhD in Geography from the Department of Geography and Human Environment of Tel Aviv University. His main field of research is spatially explicit modeling of urban residential and land-use dynamics, with a particular focus on human decision making as a driving force of urban and regional development. Presently, he is a fellow researcher at the Department of Environmental Sciences of Wageningen University.

Ehud Or holds a MA in Geography from the Department of Geography and Human Environment of Tel Aviv University. Presently he is a head of the Application Department of the Israeli National Road Company.

APPENDIX: FORMAL DESCRIPTION OF A MODEL

Following is the model that appears after the flowchart in Figure 8. The equations in which **t** is not explicitly indicated represent variables at the same time step.

Main model variables:

- Let **g** denotes a householder agent, $\mathbf{E(g) > 0}$ be **g**'s income and $\mathbf{INT(g) \in [0, 1]}$ be **g**'s intolerance to poorer neighbors (an agent **g** with $\mathbf{INT(g) = 0}$ does not react to poorer neighbors at all).
- Let **c** denote a dwelling and **P(c)** the price of **c**.
- Let **N(c)** denote a Moore 5x5 neighborhood of **c** and **V(c)** the set of occupied dwellings within **N(c)**.
- An agent **g**, located or considering location at **c**, is influenced by the price **P(c)** of **c** and a weighted average $\mathbf{E_{N(c)}(g)}$ of **g**'s actual or potential neighbors in **N(c)**

$$\mathbf{E_{N(c)}(g) = \Sigma_{h \in V(c)} E(h) * w_d(h) / \Sigma_{h \in V(c)} w_d(h)} \qquad (1)$$

where $\mathbf{w_d(h)}$ denotes the influence on **g** of neighbor **h** located at block distance **d** from the center of **N(c)**. In what follows we employ the values $\mathbf{w_1 = 1}$ and $\mathbf{w_2 = 0.5}$.

Building a pool of internal migrants:

- An agent **g** decides whether to stay at a current dwelling **c** or migrate based on the *disutility* **DU(c, g)**.
- **DU(c, g)** is determined by the income $\mathbf{E_{N(c)}(g)}$ of **g**'s neighbors; **g** reacts to neighbors if they were, on average, poorer than **g;** the strength of the reaction depends on agent's intolerance **INT(g)**:

$$\mathbf{DU(c, g) = [E(g) - E_{N(c)}(g)] * INT(g)} \quad \textbf{if} \quad \mathbf{E(g) > E_{N(c)}(g)} \qquad (2)$$

$$\mathbf{DU(c, g) = 0} \quad \textbf{if} \quad \mathbf{E(g) \le E_{N(c)}(g)}$$

- Disutility is piecewise linearly converted into probability $\mathbf{P_L(c, g)}$ of entering a pool of internal migrants:

$$\mathbf{P_L(c, g) = P_{Leave} * DU(c, g) / (0.5 * P(c))} \quad \textbf{if} \quad \mathbf{0 \le DU(c, g) \le 0.5 * P(c)} \qquad (3)$$

$$\mathbf{P_L(c, g) = P_{Leave}} \quad \textbf{if} \quad \mathbf{0.5 * P(c) < DU(c, g)}$$

where $\mathbf{P_{Leave}}$ is the probability of random exit from the city. In all scenarios, $\mathbf{P_{Leave} = 0.004}$. The latter is a rough estimate of the monthly out-migration rate in Tel-Aviv.

- The pool of internal migrants unites with the pool of new immigrants; each member of the unified pool searches for a vacant location in which to reside.

Utility of the vacant dwelling:

- The total utility $\mathbf{U(c, g)}$ of the vacancy $\mathbf{c}$ for $\mathbf{g}$ is the sum of two components:
  - *Economic* – $\mathbf{U_E(c, g)}$ with respect to the price $\mathbf{P(c)}$ of $\mathbf{c}$
  - *Social* – $\mathbf{U_S(c, g)}$ with respect to the average income $\mathbf{E_{N(c)}(g)}$ of $\mathbf{g}$'s potential neighbors at $\mathbf{c}$

i.e.
$$\mathbf{U(c, g) = U_E(c, g) + U_N(c, g)} \qquad (4)$$

Economic component of utility:

- A residential agent is willing to spend a certain fraction $\mathbf{k}$ of income $\mathbf{E(g)}$ for a dwelling. This housing budget $\mathbf{k(E(g))}$ is assumed to equal the entire income $\mathbf{E(g)}$ of poor agents although only a portion of income for wealthy agents. We assume that $\mathbf{k(E(g)) = k_0 * E(g)}$, where:

$$\begin{aligned} &\mathbf{k_0 = 1} && \mathbf{if} && \mathbf{E(g) \le E_{avg}} \\ &\mathbf{k_0 = 2/3} && \mathbf{if} && \mathbf{E(g) \ge E_{avg} + 2 * E_{STD}} \end{aligned} \qquad (5)$$

and that $\mathbf{k_0}$ linearly decreases with increasing $\mathbf{E(g)}$ within the interval ($\mathbf{E_{avg}, E_{avg} + 2*E_{STD}}$), where $\mathbf{E_{avg}}$ is a global average of agents' income in the city.

- If $\mathbf{g}$ locates in a "cheap" house $\mathbf{c}$, for which $\mathbf{P(c) < k(E(g))}$, then the reminder $\mathbf{k(E(g)) - P(c)}$ is nonetheless spent on upgrading $\mathbf{c}$ "to $\mathbf{g}$'s level."

- We assume that it is preferable to locate in a cheaper house and upgrade it than to directly invest the entire dwelling budget into the residence. That is, in cases where $\mathbf{P(c) \le k(E(g))}$, a portion of the dwelling budget $\mathbf{D_E} = \mathbf{k(E(g)) - P(c)}$ is invested in upgrading, with the resulting utility higher than that of purchase of dwelling of a price $\mathbf{k(E(g))}$.

- Alternatively, over-spending in the case of a dwelling of $\mathbf{k(E(g)) < P(c)}$, results in an additional penalty (say, mortgage interest) on utility.

- The economic component of utility $\mathbf{U_E(c, g)}$ of cell $\mathbf{c}$ for occupation by $\mathbf{g}$ is thus defined as follows:

$$\begin{aligned} &\mathbf{U_E(c, g) = E(g) - k(E(g)) + m*(k(E(g)) - P(c))} && \mathbf{if} && \mathbf{P(c) \le k(E(g))} \\ &\mathbf{U_E(c, g) = E(g) - P(c) - m*(P(c) - k(E(g)))} && \mathbf{if} && \mathbf{k(E(g)) < P(c)} \end{aligned} \qquad (6)$$

where **m** > 0 defines the positive benefit of upgrading or the negative penalty for taking a mortgage.

Social component of utility:

- The social utility $\mathbf{U_N(c, g)}$ of a vacancy **c** for occupation by **g** is defined as follows:

$$\mathbf{U_N(c, g) = E_{N(c)}(g) - E(g)} \quad \textbf{if} \quad \mathbf{E_{N(c)}(g) > E(g)} \tag{7}$$

$$\mathbf{U_N(c, g) = [E_{N(c)}(g) - E(g)]*INT(g)} \quad \textbf{if} \quad \mathbf{E_{N(c)}(g) \leq E(g)}$$

It is important to note that the social component of utility and thus of total utility in the model can be negative. Vacancies having negative utility are ignored by agents when searching for a location to occupy.

Individual residential search and relocation:

- The agents included in the search pool are considered in random order.

- Each agent **g** in the search pool constructs a list of **q** vacancies **D(g**) = **{d}**. To construct **D(g**), **g** randomly selects **q** locations from the set of all dwellings vacant at **t**; a vacancy **d** is included into **D(g)** if **U(d, g)** is higher than the utility of **g**'s current dwelling **c** (this condition is valid for internal migrants only) and **g**'s income **E(g)** is higher than **P(d)**. Agent **g** then ranks the dwellings in **D(g)** according to utility **U(d, g)**. We thereby construct the ordered lists **D(g)** for all agents **g** in the search pool.

- Each agent **g** attempts to occupy the best dwelling among those listed on **D(g)**. If the dwelling is still vacant, **g** is removed from the pool and relocates to the new dwelling. If it is already occupied by one of the agents from the pool who considered it before **g**, **g** remains in the pool. After the first pass, some agents are relocated into the best-for-them dwellings. The remaining agents try to occupy the second-best vacancy in their **D(g**) according to a new random order. This process continues until all agents find another location or all **q** vacancies are tested but not taken. In the latter case, **g** stays at the current location.

Out-migration:

- Agents leave the city for random reasons with probability $\mathbf{P_{Leave}}$ per time step.

In-migration:

- The number of immigrants entering the city at any given time step is calculated as **n(t)/100 + 10**, where **n(t)** is the number of occupied dwellings in the city at iteration **t**

- Based on the experimental data, the income **E(g)** of immigrant **g** is assigned according to a truncated log-normal distribution. First we consider $\mathbf{Log_2(E(g))}$ as normally distributed with the average equal to **12**, and **STD** to **2.2**, and then we truncate this distribution by assuming that $\mathbf{Log_2(E)}$ must be above $\mathbf{Log_2(E_{min}) = 9}$ (that is, $\mathbf{E_{min} = 2^9}$ = NIS **512**, taken as the minimal possible income in 1995, the census year).

- Intolerance **INT(g)** of immigrant agent **g** is assigned independently of **E(g)**.

The study of the model dynamics for various distributions of **INT(g)** is the subject of our investigation.

Dwelling construction and price:

- The maximum number of dwellings that can be activated at each time step **t** is equal to the number of immigrants entering the city whereas a dwelling cell is activated if at least **3** of the neighboring cells within its 3x3 Moore neighborhood are occupied.

- The price of a new dwelling **c** activated at time step **t** is assigned according to the weighted average of the neighbors' willingness to pay within the 5x5 Moore neighborhood **N(c)**:

$$\mathbf{P_{t+1}(c)} = \mathbf{\Sigma_{h\in V(c)}k(E_t(h_d))*w_d(h)/\Sigma_{h\in V(c)}w_d(h)} \qquad (8)$$

Just as in (1), $\mathbf{w_d(h)}$ denotes the influence on **c** of neighbor **h** located at block distance **d** from the center of **N(c)**; in what follows we employ the values $\mathbf{w_1 = 1}$ and $\mathbf{w_2 = 0.5}$. The price **P(c)** of an activated dwelling remains unchanged throughout the simulation.

Initial conditions:

- Initially, **9** agents are assigned income **E(g)** and intolerance **INT(g)** values according to the chosen scenario.

- These agents are randomly located in cells of the **3x3** neighborhood in the center of a grid; the price of dwelling **c** occupied by agent **g** is set equal to **k(E(g)).**

The list of numeric model parameters that are kept constant between scenarios:

- Model time step: **one month.**

- Weights employed for calculating neighborhood status: $\mathbf{w_1 = 1, w_2 = 0.5}$.

- Maximum probability of leaving location $\mathbf{P_{Leave}}$ (per month): $\mathbf{P_{Leave} = 0.004}$

- Agent's minimal willingness to pay: $\mathbf{k_{min}(E(g))} = \mathbf{2/3*E(g)}$.

- Profit of upgrading/loss from mortgage coefficient: **m = 0.2**.

- Mean and standard deviation of the Log_2(immigrant's status): **12** and **2.2**, respectively.

Number of dwellings an agent evaluates during residential search: **q = 30.**