# Multi-Dimensional Reasoning in Competitive Resource Allocation Games: Evidence from Intra-Team Communication

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**Abstract**

We experimentally investigate behavior and reasoning in various resource allocation games with large strategy spaces: Blotto games, multi-object auctions with budget constraints, and all-pay multi-object auctions. In the experiment, a team of two players plays as one entity against other teams and team members are allowed to communicate with one another before choosing a strategy. The analysis of their communication reveals that they think in terms of dimensions or characteristics of strategies rather than in terms of individual elements of the strategy space. Furthermore, we find that the main dimensions considered by players are common to the various games studied and we detect a linkage in terms of the reasoning across these games in the dimensions' metric. We also identify commonly used decision rules within each dimension and study the effect of multi-dimensional reasoning on performance. Thus, we suggest that multi-dimensional reasoning is a frequently used decision procedure that connects the behavior observed in various resource allocation games.

**Keywords:** Blotto games, Bounded Rationality, Communication, Multi-dimensional reasoning, Multi-unit auctions, Text analysis.

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## 1. Introduction

Suppose that you manage a company that competes against another company in three separate product markets labeled A, B, and C. Each company employs 60 workers of similar potential productivity and has to assign each of them to the development of one of the three products. The company that assigns more workers to a particular product will complete the product development earlier and conquer the product market. How would you allocate your employees among the three products if conquering is equally profitable in all markets? Which strategy would you choose if product C is expected to yield a larger profit to the winning company compared to the other products?

The above type of resource allocation game is not unique to the case of R&D. A military version of this strategic situation, in which troops are allocated to a number of battlefields, is known as the Blotto game (Borel, 1921; Gross and Wagner, 1950). The game has received widespread attention due to its interpretation as a game between two presidential candidates who allocate their limited budgets among campaigns in "battlefield" states (see, for example, Brams, 1978). Other interpretations include a political vote-buying or promises game (e.g., Myerson, 1993; Laslier and Picard, 2002; Dekel et al., 2008) and allocation of security costs across various components of a computer system when defending against cyber attackers (Hausken, 2008; Hui and Chuang, 2011).

If a player can benefit from uninvested resources (e.g., by saving money for later or using human resources for other purposes) the situation above becomes theoretically equivalent to an all-pay multi-object first-price auction with limited endowment. Multi-object auctions of various types, such as those for oil leases and spectrum licenses, have naturally attracted the interest of economists (Krishna, 2002).

In this paper, we study the reasoning process of individuals in three types of resource allocation games: Colonel Blotto games, multi-object first-price auctions with budget constraints, and all-pay multi-object auctions with a limited endowment. In all these games, a strategy is a resource allocation in the form of an n-tuple, i.e., a division of a limited budget among a number of tasks, but the payoff functions differ across the games. Experimental behavior in these so-called "multi-battle contests" has been studied separately in the past (see Dechenaux et al., 2015, and Kagel and Levin, 2014, for a review). However, the vast majority of studies investigate deviations from the game equilibrium or conditions for convergence to equilibrium during the course of repeated play, rather than the reasoning and decision rules invoked by the players. We make a first attempt to *connect* the initial reasoning and behavior in different resource allocation games by identifying a common decision procedure.

The problem of allocating limited resources among a number of tasks is often a complex one because the space of possible allocations is large and the alternatives are not naturally ordered unless there are only two tasks. The problem becomes even more complicated in competitive environments in which each player wishes to have more resources assigned to a number of tasks than the other player.

Arad and Rubinstein (2012a) study experimentally a version of the Blotto game and suggest that the rich strategy space in such interactions triggers *multi-dimensional reasoning*, i.e., thinking in terms of dimensions or characteristics of strategies, rather than in terms of individual elements of the strategy space. In the above example, a prominent dimension of a strategy is the number of products to concentrate on. The decision in this dimension could be to focus the resources on two products, one product, or to invest equally in all products. Another important dimension involves the exact identity of the products to concentrate on. Arad and Rubinstein further suggest that within a dimension, players use level-k decision rules (Nagel, 1995; Stahl and Wilson, 1995), starting from an intuitive dimensional choice and carrying out an iterated best-response process within the dimension. For instance, the starting point in the first dimension in the above example is to invest equally in all products and hence a level-1 player will concentrate on two products, aiming to have more resources than the opponent on these two fronts.

Kohli et al. (2012) repeat the analysis of Arad and Rubinstein in a different parametric version of the Blotto game and find similar patterns in the players' choices. Selten et al. (2011) provide experimental evidence for multi-dimensional reasoning in a monopoly context, as participants focus on achieving a subset of goals rather than on maximizing the monopoly profit.

A number of models incorporate the idea of categorical beliefs (i.e., players' beliefs that are framed in terms of categories rather than strategies), including Piccione and Rubinstein (2003), Jehiel (2005), and Arad and Rubinstein (2017). The idea of categorizing alternatives has been studied in non-strategic contexts as well, such as in the simplification of a large set of alternatives in individual choice (e.g., Manzini and Mariotti, 2012) and in the categorization of sources of income and expenses in mental accounting (Thaler, 1999). Multi-dimensional reasoning is a natural extension of categorization as it involves simultaneous categorization of strategies along a number of dimensions.

In this paper, we investigate the relevance of multi-dimensional reasoning for various resource allocation games by looking at the players' reasoning process in addition to exploring their choices. In our experiment, a team of two players plays as one entity against other teams. Teammates are allowed to electronically communicate between themselves before choosing a strategy. We use a variation of the communication protocol introduced in Burchardi and Penczynski (2014), in which each team member can suggest a strategy and justify it in a written message. After the simultaneous exchange of the "suggested strategy" and the message, the team members decide individually on the action they wish the team to take. One of their two "final strategies" is randomly implemented as the team's action. Thus, participants have incentives to try to persuade their teammates, using a detailed message with arguments supporting their suggested strategy. Since the messages are exchanged simultaneously, the text reveals the individual reasoning before the players' exposure to their teammates' ideas.

The analysis of various forms of communication in experiments for the purpose of better understanding players' motives and thought processes has proved to be beneficial in different contexts (Schotter, 2003; Cooper and Kagel, 2005; Schotter and Sopher, 2007; Penczynski, 2016a, 2017). In our study, we use intra-team communication as a powerful diagnostic tool for multi-dimensional reasoning and for decision rules within dimensions. If multi-dimensional reasoning takes place, it likely shapes the team conversation and will be detected by this method.

Indeed, the analysis of communication between team members in all the resource allocation games studied provides evidence that players classify strategies in a number of dimensions and perform their strategic deliberation within the space of those dimensions. Furthermore, the main detected dimensions are common to the three types of games. Thus, analyzing written messages and chosen strategies along the lines of multi-dimensional reasoning allows for linkage in terms of the decision-making process across those different resource allocation games. The communication protocol does not change the types of strategies commonly used and can thus be seen as an unintrusive tool in this respect.

For each of the three games, we contribute to the understanding of prevalent reasoning processes and decision rules. We present the first direct evidence – to our knowledge – that multi-dimensional reasoning indeed structures participants' arguments in and beyond the well-studied Blotto game. We also investigate the decision rules that are used within dimensions to determine the preferred characteristic of the strategy. In addition to level-k decision rules (Lk) of iterated best responses within dimensions, we find evidence of non-belief-based decisions rules such as aiming to win the majority of battlefields in the Blotto game and aiming to achieve a minimally desired gain in auction games. The results are robust to changes in parameters in all three games. Interestingly, after the exchange of messages, the final strategies still reflect multi-dimensional reasoning to a similar extent.

Despite the differences between the three classes of resource allocation games, in all of them we find multi-dimensional reasoning, a similar consideration of dimensions in aggregate, and a within-subject correlation between classes of games in terms of the use of dimensions.

We further find that dimensions are important for successful play. We show that winning strategies feature reasoning in many dimensions and we highlight the more essential dimensions.

The paper is structured as follows. Section 2 describes the experimental design and the manner in which written messages are classified. In Sections 3–5, we report the analysis of both written messages and strategies in each of the three types of resource allocation games. In Section 6, we turn to discuss the association between the reasoning in the different types of games and we conclude in Section 7.

## 2. Method

### 2.1 Experimental design

The experiment was carried out in the Interactive Decision Making Lab at Tel Aviv University and the Experimental Economics Lab at Ben-Gurion University. The experiment was programmed in z-Tree (Fischbacher, 2007). It consisted of 16 sessions with a total of 249 subjects, about 50% of whom were women. (In three cases, research assistants filled in for subjects in order to complete a session. Their data is excluded from the analysis.) The participants were Tel Aviv University and Ben-Gurion University students in various fields of study. Recruitment of participants was done via ORSEE (Greiner, 2015).

Subjects played four games in the experiment. They were matched into teams in each game and played against other teams. Anonymity within a team and between teams was maintained both during and following the experiment. Payoffs in the games were stated in terms of points. Points accumulated during the session were converted to cash at the end of the experiment according to a fixed exchange rate: 5 points = NIS 1. Participants received NIS 35 (ca. USD 10) for participation and in addition were rewarded according to their performance, with the team's payoff being divided equally among its members. Sessions lasted about an hour.

We used a variation of the communication protocol introduced in Burchardi and Penczynski (2014), in which two anonymous participants play as a team and communicate electronically in the following manner: each member suggests a strategy and justifies it in a written message. After the simultaneous exchange of the suggested strategy and the message, the team members decide individually on the action they wish the team to take. One of the two final strategies is randomly implemented as the team's action.

In order to have the team partner's previous messages not influence subsequent suggested strategies, the experiment involved sequentially suggesting strategies for four games first, before the two teammates received each other's suggestions and made their final decisions (van Elten and Penczynski, 2017). Further, games 3 and 4 were introduced only after game 2 in order to prevent the behavior in the first block from being influenced by the description of the different type of games in the second block; see Table 1.

| ***Order*** | | **Games** | | | |
|---|---|---|---|---|---|
| | | **First Block** | | **Second Block** | |
| *1* | General instructions | | | | |
| *2* | Practice round of the communication interface | | | | |
| *3* | Instructions | 1 | 2 | | |
| *4* | Suggested strategies | 1 | 2 | | |
| *5* | Instructions | | | 3 | 4 |
| *6* | Suggested strategies | | | 3 | 4 |
| *7* | Final strategies | 1 | 2 | 3 | 4 |
| *8* | Feedback on outcomes & payoffs | | | | |

Table 1: Sequence of events in the experiment.

***Strategic games***

Following are the three classes of games that were played by teams in the experiments. The complete translated instructions are shown in Appendix B.

1. Colonel Blotto tournament

"You are playing the role of a colonel during wartime and other teams in the experiment are your opponents. Each team is allocated a given number of 'troops' that need to be allocated among a given number of separate 'fronts'.

"You win the battle in a particular front if you assign more troops than your opponent. In the case where you and your opponent both allocate the same number of troops to a particular front, both teams lose the battle.

"Your team will participate in a round-robin tournament, in which your team's deployment of troops will automatically face those of all other teams in the experiment. You cannot choose different deployments against different teams. Your team's total score will be the overall number of fronts you win against all other teams. The winner of the tournament will be the team with the highest score. If there is a tie in the total score, the winner will be determined randomly."

This class of games consisted of two games: "Blotto 6" with 120 troops allocated to 6 fronts and "Blotto 7" with 210 troops allocated to 7 fronts.

2. Multi-object (first-price) auction with budget constraints

"In this game, your team plays against two other teams. You can participate in up to three auctions: A, B, and C. If you win Auction A, you receive a prize of W; if you win Auction B, you receive a prize of X; and if you win Auction C, you receive a prize of Y. (In the second game, there is an additional Auction D that features prize Z.)

"You win a particular auction if your bid is the highest. In the case of a tie, a lottery will determine which of the teams wins the auction. If you win an auction, and only in that case, you will pay according to the bid you placed in that auction. You can bid in all three auctions, as long as the sum of your bids is at most M."

This class of games consisted of two games: “Auction 3” with W=X=Y=100 and “Auction 4” with W=X=Y=90 and Z=110. In both games, the budget was M=120.

3. All-pay multi-object auction with a limited endowment

“In this game, your team plays against two teams. You can participate in up to three auctions: A, B, and C. If you win Auction A, you receive a prize of W; if you win Auction B, you receive a prize of X; and if you win Auction C, you receive a prize of Y. (In the second game, there is an additional auction D that features prize Z.)

“You win a particular auction if your bid is the highest. In the case of a tie, a lottery will determine which of the teams wins the auction. You receive an endowment of M, which you may use for bidding in the auctions. In each auction, you pay your bid even if you do not win the auction. Unused points will remain in the possession of the team at the end of the game, and will be added to the team’s winnings.”

This class of games consisted of two games: “All-pay 3” with W=X=Y=90 and “All-pay 4” with W=X=Y=90 and Z=80. In both games, the endowment was M=60. In contrast to the limited budgets in the previous class of games, unused endowments contributed to the participants’ payoff.

Each session consisted of only two classes of games, with the two parameterizations of each particular class played sequentially. In order to avoid confusion, we never included the first-price and the all-pay multi-object auctions in the same session. Thus, the class of Blotto games was played in all sessions. The choice of classes and their order was altered between sessions. Table 2 describes the four types of sessions run in the experiment and the number of subjects in each type.

| ***n=249*** | **First Block (games 1+2)** | **Second Block (games 3+4)** |
|---|---|---|
| *54* | Blotto 6, Blotto 7 | Auction 3, Auction 4 |
| *69* | Blotto 6, Blotto 7 | All-pay 3, All-pay 4 |
| *54* | All-pay 3, All-pay 4 | Blotto 6, Blotto 7 |
| *72* | Auction 3, Auction 4 | Blotto 6, Blotto 7 |

Table 2: The number of subjects in each of the four types of experimental sessions.

Note that each subject played two types of games that, by design, differ in various aspects. In the Blotto game, the story is described in terms of a strategic war game, resources are troops that are allocated to 6 or 7 fronts, and the payoff is determined only by the number of battles won (i.e., conditional on winning the front, the assignment size has no effect on the payoff). In the multi-object auctions with budget constraints, the resource is money, there are 3 or 4 possible objects to bid on (i.e. 3 or 4 auctions), and the size of the bid in case of winning is relevant to the payoff, because the player obtains the value of the object minus the bid. In the all-pay multi-object auctions with limited endowment, the resource is money and there are 3 or 4 objects, as in the previous game, but unused resource contributes to the player's payoff. Another notable difference is that the Blotto game is defined as a round-robin tournament whereas both variations of the multi-object auctions are 3-player games. Thus, the differences between the three types of games are in the framing of the story, in the number of "fronts", in the budget, and in the score function. This design allows us to examine more cleanly, i.e., without triggering common reasoning by using similar stories or parameters, whether there are similarities between these games in the reasoning process.

**Comment on the Nash equilibria of the games**

The six games in the present study do not feature pure strategy equilibria. Their rich structure induces mixed-strategy equilibria that are often difficult to calculate, even for game theorists.[1] We examine reasoning and behavior in *one-shot* resource allocation games rather than in repeated play of these games. This leaves no room for the lengthy equilibration process that might occur in repeated interactions (Camerer et al., 2003; Holt and Roth, 2004). Therefore, we do not expect the subjects' behavior to fit the mixed strategy equilibrium and in the analysis of the data we focus on subjects' actual reasoning without referring to equilibrium behavior.

[1] In the equilibrium of a two-player zero-sum Blotto game, the marginal distribution over each front is symmetric around the number of troops divided by the number of fronts (Roberson, 2006; Hart, 2008). However, we are not aware of a characterization of equilibrium for the case of a round-robin tournament of the Blotto game, which we study here. The characterization of a multi-object all-pay auction in the case where the entire budget is used is similar to the characterization of the Blotto game. However, we do not know the equilibrium of a three-player game, which is the version played in the experiment. Finally, to the best of our knowledge, the equilibrium of the first-price multi-object auction with budget constraints studied here has not been characterized, not even for a two-player game.

## 2.2 Classification of communication transcripts

We used a manual classification method in order to classify the written messages along the lines of multi-dimensional reasoning. For each game, two research assistants independently read the messages and classified them into two categories: multi-dimensional reasoning (i.e., thinking about dimensions of strategies) and other reasoning (e.g., responding to a distribution of strategies chosen by the other players or choosing a strategy randomly).

If a message was classified as reflecting multi-dimensional reasoning, the research assistant further classified the message according to two additional criteria: the *strategy dimensions* (i.e. features) mentioned in the text and the *dimensional decision rule* used within each of the dimensions. A list of possible dimensions and decision rules was composed by the authors after reading the messages in a pilot study. The list was given to the research assistants, but they were allowed to add new categories in both the dimensions and the dimensional decision rules. After deciding on their classifications independently and submitting them, the research assistants met to reconcile any disagreements between them and provided a joint classification for the cases on which they had disagreed. They were allowed to keep their two classifications in case of unresolved disagreement. Initially, the RAs agreed on 84% of the dimension classifications and on 61% of the more demanding decision rule classifications. While there were no remaining disagreements on the dimensions after the reconciliation, 35 out of 1655 decision rule classifications remained without agreement. Here, we only report on the agreed classifications. The full classification instructions appear in Appendix C.

The robustness and replicability of classifications of the type described above are examined with a large set of human classifiers recruited on MTurk in Eich and Penczynski (2017). Furthermore, the results of such classifications are shown to be replicable with machine-learning techniques (Penczynski, 2018).

Table 3 presents the exhaustive list of dimensions for all the six games in this study. Note that we use the general term “front” both in the Blotto game and when we refer to an individual auction in the multi-object auctions. Furthermore, a “reinforced” or “disregarded” front relates to a front with a high or a low assignment, respectively, relative to a uniform allocation of the resources. An elaboration on the meaning of each dimension will be done separately for each class of games in Sections 3–5.

| | Dimension |
|---|---|
| *D1* | Number of reinforced fronts |
| *D1A* | Asymmetric assignments to reinforced fronts |
| *D2L* | Type of assignment to disregarded fronts |
| *D2H* | Type of assignment to reinforced fronts |
| *D3* | Considerations of the identity of fronts (assignment order) |
| *D4* | How much of the budget to use |

Table 3: The dimensions considered by subjects in all six games in the experiment.

The differences between and within classes of games induce slight differences between dimensions and in their salience. Three of them are worth mentioning.

First, note that dimension D4 is relevant only in the all-pay auctions. Not using all resources is theoretically unreasonable in the Blotto games because no advantage is derived from withholding troops. Such choices are moreover very rare in first-price auctions, probably because a player does not benefit from an unused budget. The avoidance of a wastefully high bid when a much lower bid would have sufficed is only a secondary consideration.

Second, referring to dimension D3 is almost a necessity in the multi-object auctions in which the items' values are not the same, i.e., Auction 4 and All-pay 4, because a player needs to specify how the auction with a unique item value is treated. In the rest of the games, discussing the identity of the fronts to focus on is a response to the order of the fronts and not to a payoff-relevant aspect.

Third, in the Blotto games, messages discussing the type of assignment to reinforced fronts included mainly arguments about the unit digit and hence D2H was described in the instructions for the classifiers as "the specific assignment to reinforced fronts: the unit digit in such fronts". By contrast, in the first-price auctions the arguments focused on the rough magnitude of the bid in a particular auction. Accordingly, D2H was described for the classifiers as "values in reinforced (high-bid) auctions: considerations of the specific bid or an approximate value of the bid." This difference could be a result of both the different score functions and the difference in the number of fronts in the games.

We intentionally chose to study games that differ in these aspects, in order to examine whether multi-dimensional reasoning is a common decision procedure in a wide class of substantially different games.

## 3. Results: Blotto games

In each of the three Results sections 3–5, we start with an analysis of the messages, then turn to explore the relationship between the performance of a suggested strategy and the multi-dimensional reasoning expressed in its accompanying message, and finally report the characteristics of the suggested and final strategies.

The total number of participants in the two Blotto games was 249. The number of classified messages was 211 in Blotto 6 and 212 in Blotto 7. The remaining participants provided either blank messages or messages that lacked information on the strategy choice, such as "*Trust me, I know what I am doing.*"

### 3.1 Dimensions in Blotto games

Consider the following message written by subject #7 suggesting the strategy (0,38,41,41,0,0) in Blotto 6: "*In my opinion, we should focus on three fronts so that we will have a chance to win half of the fronts. The 41 is because usually people assign round numbers, such that if someone goes for the same method, he will probably assign 40. This way we will win in at least two fronts and in one we will have a high chance*" (direct translation from Hebrew). The message suggests that the subject had in mind two dimensions of a strategy: the number of reinforced fronts and the unit digit in reinforced fronts. Within the first dimension, he went for a choice that gives high chances for a modest score (winning in 2 or 3 fronts on average); within the second, he responded to the belief that others are intuitive in this dimension and allocate in multiples of ten. Thus, it appears that the subject did not best respond to a concrete belief on others' strategies but rather responded to his belief on their dimensional choices. His message was classified as reflecting multi-dimensional reasoning and, in particular, D1 and D2H.

By contrast, consider now the following message, written by subject #117 suggesting the strategy (20,20,20,20,20,20): "*I think that in terms of probability, we have the highest chances if the allocation is the same among all fronts, especially in light of the fact that there is no importance of one front over another.*" Such a message is classified as not reflecting multi-dimensional reasoning but rather a different type of reasoning. Note that it would be hard to guess this subject's reasoning process solely from the chosen strategy.

For all dimensions in the Blotto games, Table 4 provides an illustrating strategy and summarizes the proportion of messages that include each corresponding dimension. Figure 1 depicts the distribution of the number of dimensions per message.

| | **Dimension** | **Illustration in Blotto 6** | **% Blotto 6 (n=211)** | **% Blotto 7 (n=212)** |
|---|---|---|---|---|
| *D1* | *Number of reinforced (or disregarded) fronts* | (30,30,30,30,0,0): four reinforcements | 86% | 87% |
| *D1A* | *Asymmetric assignments to reinforced (or disregarded) fronts* | (35,35,25,25,0,0): four asymmetric reinforcements | 7% | 8% |
| *D2L* | *Unit digit in disregarded fronts* | (30,30,30,28,1,1): two disregarded fronts with unit digit 1 | 22% | 26% |
| *D2H* | *Unit digit in reinforced fronts* | (31,31,31,27,0,0): three reinforced fronts with unit digit 1 | 21% | 18% |
| *D3* | *Identity/location of reinforced fronts (order)* | (0,0,30,30,30,30): four reinforced fronts at the right | 39% | 38% |

Table 4: Frequency of dimensions in classified messages in the Blotto games.

The main findings from Table 4 are as follows. D1 is very common, mentioned by the vast majority of subjects, D1A is very rarely mentioned, and D2L, D2H, and D3 are somewhere in between, although D3 is more commonly mentioned. Although the proportion of D2L and D2H is similar, it turns out that a considerable group of subjects mention D2L but do not consider D2H and vice versa. In fact, only 5% of the subjects mention both dimensions. Figure 1 shows that about 60% of the subjects mention at least two dimensions in their message, which provides further support for multi-dimensional reasoning in the Blotto game.

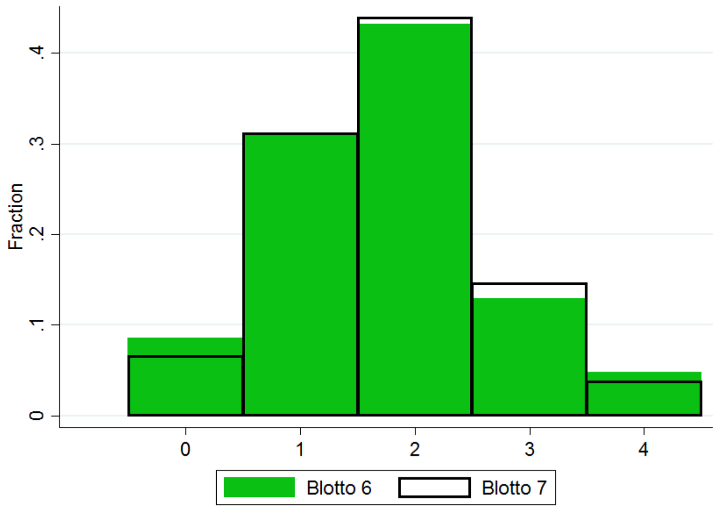


Figure 1: Number of dimensions per message in classified messages in the Blotto games (n=211, n=212).

### 3.2 Dimensional decision rules in Blotto games

As the message of subject #7 illustrates, one has to decide on the dimensional choice within each dimension. For example, when subject #7 considered the unit digit in the reinforced fronts (D2H), the subject responded to the belief that others were intuitive in this dimension and allocated in multiples of ten. We apply the concept of level-k decision rules for the strategy space to the space of characteristics of a strategy, and accordingly, the dimensional decision rule of the subject in D2H is classified as L1, i.e., a response to a belief that the opponent is not strategic in this dimension but is attracted to salient choices (Crawford and Iriberri, 2007a, 2007b; Arad and Rubinstein, 2012b). Table 5 reports the type of decision rules that were detected in the written messages and Table 6 provides some examples of arguments written by subjects that are consistent with those decision rules.

Note in particular the collection of decision rules R, i.e., a reasonable argument without a specific belief. For example, when subject #7 considered the number of reinforcements, D1, the subject chose to reinforce three fronts assuming that this would guarantee a minimal score. There might have been an underlying assumption that others reinforce four or five fronts or allocate troops equally between fronts, but this is not mentioned in the message. Similarly, the following message contains a reasonable argument although it is not based on a concrete belief on others' choices within D1: "*No point in allocating the troops equally to six fronts because we will lose for sure. I prefer betting on two strong fronts and hope that others won't reinforce these particular fronts.*"

In D2L, for example, the idea of assigning 0 to a front because assigning more would be wasteful is not based on a concrete dimensional belief either. In D3, the idea of being unpredictable in the ordering of the assignments so that others cannot exploit a systematic ordering is not a response to others' ordering. Thus, the R classification includes all decision rules that do not reflect a response to a particular categorical belief but do reflect some strategic reasoning and hence are not purely intuitive.

Of course, due to the absence of an *articulated* belief, R can alternatively be thought of as a reasonable argument on the basis of an implicit, ambiguous belief that is not articulated.

| | **Decision rule** |
|---|---|
| *L0* | Intuitive or random |
| *L1* | Response to a specified (categorical) belief |
| *L2* | Response to a belief that others are L1 (or L1 & L0) |
| *R* | Reasonable argument without a specified belief (e.g., an attempt to guarantee a particular minimal score) |
| *N* | No explanation |

Table 5: Common decision rules (within dimensions) in classified messages.

| | **Argument consistent with the decision rule** |
|---|---|
| *L0* | *Which fronts? Let's say 1,3,5, and 7 just because they look nice and symmetric.* (D3) |
| *L1* | *I assigned 1 to certain fronts because there is a chance that the other team assigned 0, so we can win them*. (D2L) |
| *L2* | *I think others will allocate equally to win more fronts, but I also think that others regard us as likely to do the same.* (D1) |
| *R* | - *It is better to win in half of the fronts with certainty than to gamble on the outcome in all*. (D1)<br>- *Assigning a few troops to a front is equivalent to giving up the front entirely.* (D2L) |

Table 6: Examples taken from written messages for arguments consistent with the various decision rules.

The complete distribution of decision rules in the different dimensions appear in Table S1 in the Appendix. The main findings are that L1 and R are the most frequently used dimensional decision rules. In D2L, D2H, and D3, L1 is the dominant decision rule. However, in D1, R is particularly prominent. Interestingly, a significant portion of decisions in D1 are based on an attempt to win a particular number of fronts, given a reason for this goal.

### 3.3 Multi-dimensional reasoning and performance in Blotto games

We calculated the expected score of the suggested strategies when playing against the suggested strategies of all subjects in the experiment. This measure eliminates the random aspect of the score in any session with few opponents. Table 7 presents the top three scores for each Blotto game.

| *Rank* | **Blotto 6** | **Blotto 7** |
|---|---|---|
| *1* | (2,3,41,41,31,2) [3.61] | (1,1,54,55,53,45,1) [4.07] |
| *2* | (3,31,31,31,21,3) [3.56] | (4,1,1,51,51,51,51) [4.06] |
| *3* | (2,38,38,2,38,2) [3.47] | (2,51,2,51,51,51,2) [4.05] |
| | & (1,1,31,31,31,24) [3.47] | |

Table 7: Winning suggested strategies and their expected score [in brackets] in the Blotto games.

The results show that the best performing strategies reinforce three or four fronts, use the unit digits 1, 2, and 3, and assign a small number of troops to the first front, and often to the last one as well, thus suggesting the use of dimensions D1, D2L, D2H, and D3. These properties are similar to those of the best performing strategies in Arad and Rubinstein's (2012a) tournament with thousands of participants. It is natural to inquire whether considering a larger number of dimensions improves, on average, the strategy's performance. Table 8 provides some indications of the relationship between dimensional thinking in classified messages and the suggested strategy's expected score.

| *# of dimensions* | n | Average score | n | Average score |
|---|---|---|---|---|
| | **Blotto 6** | | **Blotto 7** | |
| *1* | 65 | 2.49 (0.65) | 66 | 3.12 (0.53) |
| *2* | 91 | 2.71 (0.51) | 93 | 3.23 (0.50) |
| *3* | 27 | 2.88 (0.40) | 31 | 3.33 (0.76) |
| *4* | 10 | 3.28 (0.23) | 8 | 3.41 (0.70) |

Table 8: Average score (standard deviation in parentheses) for each number of dimensions in the Blotto games.

The table suggests that increasing the number of considered dimensions generally improves average performance. We found strong support for this result using a ranksum Mann–Whitney test to compare the scores of strategies with the different numbers of dimensions.[2] Furthermore, for subjects who mentioned at least one dimension, we studied the effect of particular dimensions on the score. In a linear regression explaining the score in Blotto 6, the coefficients for the dummy variable for the dimensions D2L, D2H, and D3 were positive and significant. In Blotto 7, D2L was positive and significant and D1 was found to be negative and significant (see Table S6 in the Appendix). Note that merely considering D1 was not necessarily beneficial since almost all subjects considered this dimension. Moreover, note that the decision rule within the dimension was a highly important determinant of the score. To summarize, while different dimensions may have made different contributions to the score, overall, the use of more dimensions was beneficial.

[2] In Blotto 6, we found that the score of strategies that correspond to one *mentioned dimension* is significantly lower than that of strategies with three dimensions ($p<0.01$) and four dimensions ($p<0.001$) and marginally significantly lower than that of strategies with two dimensions (0.082). In addition, two dimensions correspond to a significantly lower score than four dimensions ($p<0.001$) and a marginally significantly lower score than three dimensions ($p=0.083$). Finally, three dimensions correspond to a significantly lower score than four dimensions ($p<0.01$). In Blotto 7, one dimension corresponds to a significantly lower score than three dimensions ($p<0.01$) and four dimensions ($p<0.05$), and two dimensions correspond to a significantly lower score than three dimensions ($p<0.05$).

### 3.4 Suggested and final strategies in Blotto games

In this section, the introduced dimensions are analyzed solely on the basis of the data on the suggested strategies and the final strategies. Table 9 presents the proportion of strategies that corresponds to each number of reinforced fronts (D1).

Table 9 indicates that strategies with one or no reinforced fronts are very rare. Hence, the proportions in the table are consistent with the findings that almost all subjects considered D1 in their messages. The reinforcement of 0, 1, or 2 fronts, which is not successful, is slightly less frequent in the final strategies than in the suggested strategies.[3]

| ***Reinforced Fronts*** | **Blotto 6 (n=249)** | | **Blotto 7 (n=249)** | |
|---|---|---|---|---|
| | **suggested** | **final** | **suggested** | **final** |
| *0* | 5% | 3% | 4% | 2% |
| *1* | 7% | 2% | 4% | 2% |
| *2* | 25% | 23% | 10% | 10% |
| *3* | 31% | 35% | 26% | 25% |
| *4* | 27% | 33% | 31% | 39% |
| *5* | 5% | 4% | 20% | 18% |
| *6* | | | 6% | 3% |

Table 9: Fraction of strategies corresponding to number of reinforced fronts in the Blotto games. Reinforcement is defined as allocating more than 20 or 30 troops, respectively.

Table 10, which summarizes the use of unit digits in the strategies (D2L and D2H), shows a moderate use of unit digits 1, 2, and 3. Focusing on the suggested strategies, we found that the unit digit 0 is more frequently used in disregarded fronts than in reinforced fronts. In Blotto 6, 52% of the assignments to the reinforced fronts (n=829) and 65% of the assignments to neglected fronts (n=665) have the unit digit 0. In Blotto 7, 37% of the

[3] The final strategies give an idea of which strategies teammates found persuasive in their communication. A deeper analysis of persuasion in the spirit of Penczynski (2016b) goes beyond the main objective of our study.

assignments to the reinforced fronts (n=1115) and 68% of the assignments to the neglected fronts (n=628) entail the unit digit 0.

| ***Strategies*** | **Blotto 6 (n=249)** | | **Blotto 7 (n=249)** | |
|---|---|---|---|---|
| | **suggested** | **final** | **suggested** | **final** |
| *All assignments have the unit digit 0* | 43% | 33% | 22% | 18% |
| *Some assignments have the unit digit 1,2, or 3* | 40% | 54% | 52% | 60% |
| *The rest of the strategies* | 18% | 13% | 26% | 22% |

Table 10: Type of troops assignment in terms of the unit digit in strategies in the Blotto games.

A comparison between unit digit patterns in the suggested strategies and the final strategies suggests that subjects' sophistication increased as a result of communication. Table 10 shows that the incidence of the unit digit 0 decreases overall. The data further shows that this is true for both reinforced and disregarded fronts in Blotto 6 and Blotto 7.

As for dimension D3, Figures 2 and 3 suggest that, in both games, fewer resources are assigned to the first and last fronts than to the intermediate ones. Furthermore, 55–63% of the subjects assigned few or no troops to Front 1 in their suggested strategy. The final strategies are very similar to the suggested strategies in this aspect, with slightly fewer resources assigned to Front 1 and 2 in the final strategies and more to the intermediate fronts.

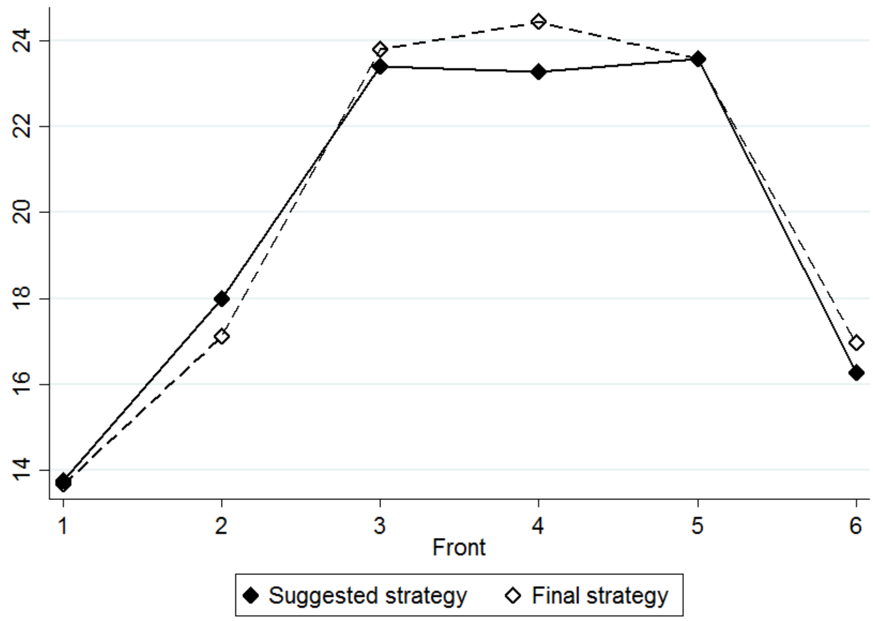


Figure 2: Average assignment to each front in Blotto 6.

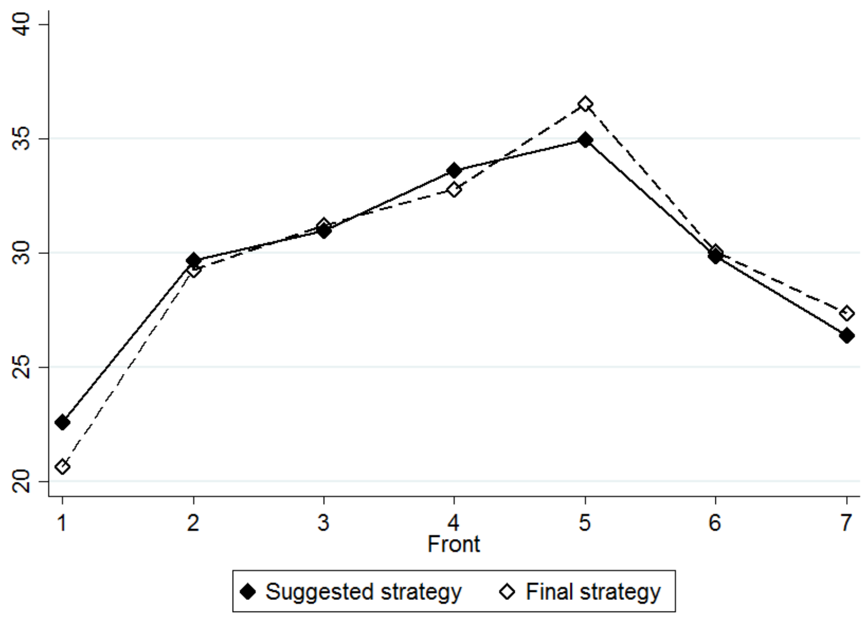


Figure 3: Average assignment to each front in Blotto 7.

## 4. Results: Multi-object first-price auctions

The total number of participants in the first-price multi-object auctions was 126. The number of classified messages was 101 in Auction 3 and 105 in Auction 4.

### 4.1 Dimensions in first-price multi-object auctions

In order to see multi-dimensional reasoning at work in multi-object auctions, consider the message written by subject #21 suggesting the strategy (0,0,42,78) for Auction 4, in which item D is more valuable than the other items: "*I think we should bid only in two games. Specifically, we should participate in Auction D – because everybody thinks there is no point in participating in it because everybody else will, and hence in practice it will have low bids or no bids at all"* (direct translation). This message was classified as reflecting two dimensions: the dimension of how many auctions to focus on (D1) and the dimension of the identity of those auctions (D3). While the subject did not explain her dimensional decision rule in D1, the message suggests that she practiced two steps of reasoning (L2) in D3. The starting point of her iterative reasoning process was the intuitive rule *to participate in Auction D* because it entails a higher prize and hence the first step of her reasoning was to avoid Auction D and the second step was to participate in the auction after all.

In contrast to subject #21's message, a message suggesting to try winning against a particular strategy assumed to be chosen by others would not be classified as multi-dimensional reasoning. Subject #155's message in Auction 4 illustrates such reasoning: "*If people assign the points uniformly, I think that the best chance of winning is as follows: Auction D - 40, Auction C - 15, Auction B - 33, Auction A - 32.*"

For all dimensions in the first-price multi-object auctions, Table 11 provides an illustrating strategy and summarizes the proportion of messages that included each corresponding dimension. Figure 4 presents the distribution of the number of dimensions per message.

| | *Dimension* | **Illustration in Auction 3** | **Auction 3 (n=101)** | **Auction 4 (n=105)** |
|---|---|---|---|---|
| *D1* | *Number of auctions with high bids (or disregarded)* | (60,60,0): two high bids | 62% | 78% |
| *D1A* | *Asymmetric assignments to auctions with high bids* | (70,50,0): two asymmetric high bids | 13% | 10% |
| *D2L* | *Type of assignment to disregarded auctions* | (55,55,10): one disregarded front in the 10s | 21% | 20% |
| *D2H* | *Type of assignment to auctions with high bids* | (53,52,15): two non-round high bids in the 50s | 63% | 43% |
| *D3* | *Considerations of the identity of auctions* | (0,60,60): two high bids in B and C | 51% | 88% |
| *D4* | *How much of the budget to use for bids* | (30,30,0): Using 60 of a budget of 120 | 5% | 4% |

Table 11: Frequency of dimensions in classified messages in the auctions.

The main findings from Table 11 are as follows. D1, D2H, and D3 are used by the majority of subjects, where D1 and D3 are naturally more common in Auction 4 due to the asymmetry in the auctions' prizes.[4] The proportions of D3 and D2H are higher than in the Blotto games, while dimension D2L is mentioned by about 20% of the subjects as in the Blotto games. D1A and D4 are rarely mentioned in the auctions. Figure 4 shows that 67%–85% of the subjects mention at least two dimensions in their messages and 37%–47% mention at least three dimensions, which provides support for multi-dimensional reasoning in the multi-object auctions.

[4] This is in line with Chowdhury et al.'s (2017) results that the effect of value salience (asymmetry in values) on deviations from Nash equilibrium in a multi-battle contest is higher than that of label salience.

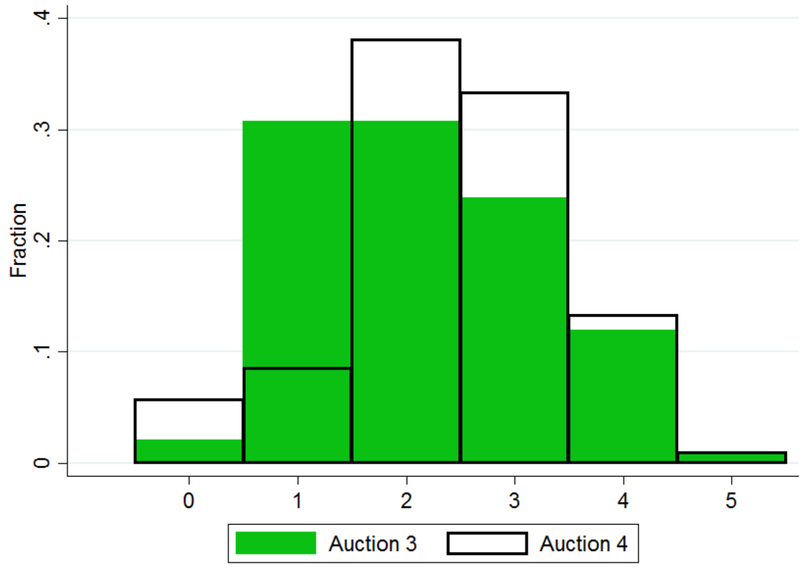


Figure 4: Number of dimensions per message in classified messages in the auctions (n=101, n=105).

### 4.2 Dimensional decision rules in first-price multi-object auctions

The types of dimensional decision rules in the auctions are similar to those in the Blotto games and are summarized in Table 5. Table 12 provides examples of arguments which are consistent with these rules, written by subjects in the multi-object auctions.

| | **Argument consistent with the decision rule** |
|---|---|
| *L0* | *I just chose two particular auctions randomly*. (D1) |
| *L1* | *I think others will focus on the higher-value auction and hence I focused on the other auctions.* (D3) |
| *L2* | *The rest of the people will not participate in Auction D, because they think everybody else will, so I am assigning an average amount there.* (D3) |
| *R* | *- No point in assigning too many points on an auction because then we will get a low payoff.* (D2H)<br>*- I think we should assign a lot to one auction so that we will win it, at least.* (D1) |

Table 12: Examples taken from messages for arguments consistent with the various decision rules in the auctions.

The most frequent decision rules were L1 and R; see Table S2 in the Appendix for the complete distribution. In contrast to the findings in the Blotto games, in the auctions L1 was the most frequent rule in D1. However, in dimensions D2L and D2H it was only the second most frequent rule, after the collection of rules classified as R, whereas in the Blotto games L1 was the most frequent rule in these dimensions.

Within the R category, a prominent decision rule observed within D2H, which did not appear in the Blotto games, refers to a choice of bid magnitude that maximizes the likelihood of winning conditional on gaining at least a sum X in the case of winning. Subject #93 suggests the strategy (10,10,40,60) and writes in Auction 4: *"I believe that we should bid a lot of money in two auctions, for example in Auction D, so that we will guarantee winning it. In fact, bidding 60 in D leaves us with 50 points or 5 shekels each, which is quite awesome"* (direct translation).

Like subject #93 in this example, a considerable number of other subjects described goals that differ from maximizing profit, such as aiming to win a particular number of auctions or achieving a minimal desired profit.[5] Thus, despite the similarities in the considered dimensions, the reasoning within dimensions differs between the auctions and the Blotto games.

### 4.3 Multi-dimensional reasoning and performance in first-price multi-object auctions

Table 13 presents the top three suggested strategies in Auctions 3 and 4, where the expected score is calculated by matching each player with any pair of competitors from the whole experiment, thereby eliminating the random aspect of the session score among the small set of players in a session.

| ***Rank*** | **Auction 3** | **Auction 4** |
|---|---|---|
| *1* | (59,0,61) [58.72] | (45,45,23,7) [88.83] |
| *2* | (60,0,60) [58.49] | (41,41,37,1) [88.59] |
| *3* | (61,0,59) [58.26] | (50,50,10,10) [88.04] |

Table 13: Winning suggested strategies and their expected scores [in brackets] in the auctions.

[5] Branas-Garza et al. (2011) report various motivations in a traveler's dilemma, including the "aspiration" for a given amount of money, which resembles the goal of achieving a minimal desired profit described here.

In both games, neglecting one auction proves to be beneficial. In Auction 3, neglecting Auction B turns out to be successful, while in Auction 4, “almost neglecting” the auction with the higher prize, Auction D, is the key. Therefore, it is not enough to understand that one needs to decide on which auctions to focus. The optimal choice of focus is sensitive to the details of the game. The fact that some winning strategies use bids that are multiples of 10 suggests that the unit digit factor is less important in the auction games than in the Blotto games.

We now turn to examine whether considering more dimensions improves the performance of the suggested strategies with which they are associated. Table 14 presents the average score of suggested strategies as a function of the number of dimensions mentioned in the accompanying message.

| ***# of dimensions*** | **n** | **Average score** | **n** | **Average score** |
|---|---|---|---|---|
| | **Auction 3** | | **Auction 4** | |
| *1* | 30 | 42.03 (9.55) | 8 | 66.27 (11.43) |
| *2* | 30 | 44.51 (9.25) | 40 | 62.6 (17.98) |
| *3* | 24 | 41.98 (11.19) | 35 | 63.43 (10.4) |
| *4* | 12 | 40.40 (12.98) | 14 | 62.28 (19.47) |
| *5* | 1 | 39.94 | 1 | 70.48 |

Table 14: Average score (standard deviation in parentheses) for each number of dimensions in the auctions.

The table suggests that increasing the number of considered dimensions does not improve average performance. Ranksum Mann–Whitney tests that compare the scores of strategies with different numbers of dimensions indicate no significant differences in performance. Using a linear regression to study the effect of particular dimensions on the scores, we find that none of the dimensions significantly improves the scores; see Table S6 in the Appendix. It may well be that in the auctions the particular choices within dimensions are the crucial element in determining the scores. For example, considering D3 is beneficial only if the player eventually neglects Auction B in Auction 3 and almost neglects Auction D in Auction 4.

### 4.4 Suggested and final strategies in first-price multi-object auctions

When we analyze dimensions by looking only at the strategies, we find that 92% of the subjects use all 120 points in the two auction games in their suggested strategies. The average spending is about 118 points, suggesting that dimension D4 regarding the budget use is hardly relevant to strategy choice in the auction games. This is consistent with the rare appearance of this notion in the messages. The final strategies show a similar pattern with 94%–98% of subjects using all 120 points.

Turning to dimension D1, Table 15 presents the proportion of strategies that correspond to each number of high-bid auctions. The numbers are consistent with the findings that the vast majority of subjects consider D1. In Auction 3, the majority decides to have two reinforcements and neglect one auction. The second most popular choice is to neglect two auctions. In Auction 4, the majority of subjects neglect two auctions and the second most popular choice is to neglect only one. In the final strategies, the reinforcement of one or zero auctions is plausibly observed in slightly fewer occasions than in the suggested strategies.

| ***High bids in*** | **Auction 3 (n=123)** | | **Auction 4 (n=126)** | |
|---|---|---|---|---|
| | **suggested** | **final** | **suggested** | **final** |
| *0 auctions* | 11% | 5% | 4% | 1% |
| *1 auction* | 33% | 27% | 14% | 12% |
| *2 auctions* | 56% | 68% | 57% | 63% |
| *3 auctions* | | | 25% | 26% |

Table 15: Number of high bids in the auctions. High bid is defined as a bid higher than 120/3 or 120/4, respectively.[6]

[6] The strategies of three subjects in Auction 3 were eliminated since they used more than 120 points for bids.

Table 16 summarizes the use of different unit digits in the strategies, which corresponds to an aspect of dimensions D2L and D2H (but does not capture these dimensions entirely). The table shows that most of the subjects do not consider the unit digit aspect of their bids and choose all their bids as multiples of tens or zero in their suggested strategies. We can further distinguish between high-bid auctions and neglected auctions in our analysis. Aggregating over all subjects and all auctions, we find that the unit digit of 0 is more frequent in neglected auctions than in reinforced ones. In Auction 3, 68% of the bids in high-bid auctions (n=232) and 68% in neglected auctions (n=146) have the unit digit of 0. In Auction 4, 64% of the bids in high-bid auctions (n=288) and 76% in neglected auctions (n=216) have the unit digit of 0. Thus, the above pattern in the first-price auctions is similar to that observed in the Blotto games, albeit weaker.

Table 16 also shows the final strategies and indicates that, as in the Blotto games, the sophistication with respect to the unit digit increases as a result of communication.

| ***Strategies*** | **Auction 3 (n=126)** | | **Auction 4 (n=126)** | |
|---|---|---|---|---|
| | **suggested** | **final** | **suggested** | **final** |
| *All bids have the unit digit 0* | 58% | 49% | 54% | 43% |
| *Some bids have the unit digit 1,2, or 3* | 21% | 30% | 25% | 36% |
| *The rest of the strategies* | 21% | 21% | 21% | 21% |

Table 16: Type of bids in terms of the unit digit in strategies in the auctions.

As for dimension D3, Figures 5 and 6 show the average bids for each front and suggest that the first and last auctions are less frequently reinforced than the intermediate ones in both Auctions 3 and Auction 4. The pattern of focusing on the intermediate auctions is similar to the observation in the Blotto games. The differences between the average bids in the final strategies and those in the suggested strategies are very small in Auction 4. In Auction 3, however, we find that in the final strategies, the average assignment to Front A is smaller and the assignment to Front C is larger, compared to the suggested strategies.

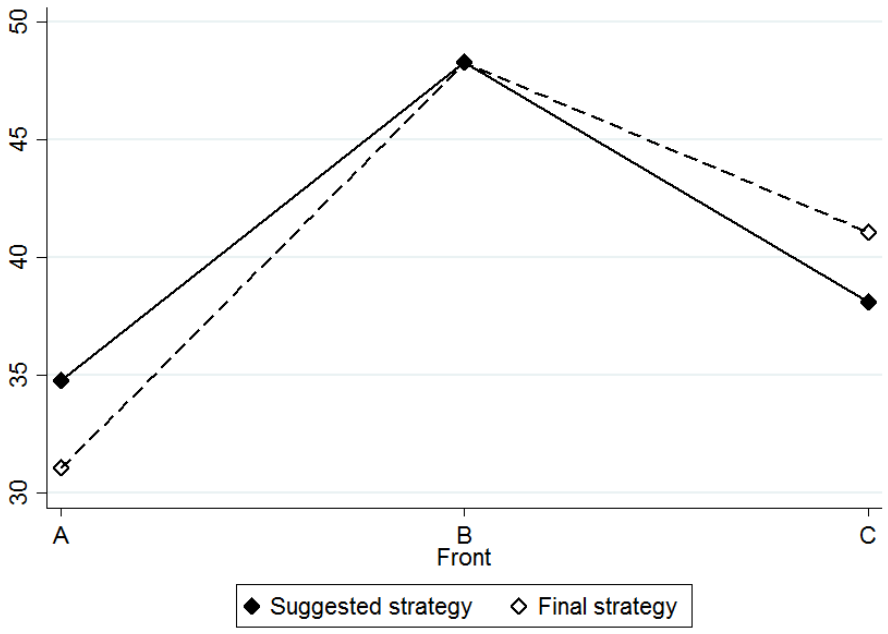


Figure 5: Average bids for each front in Auction 3.

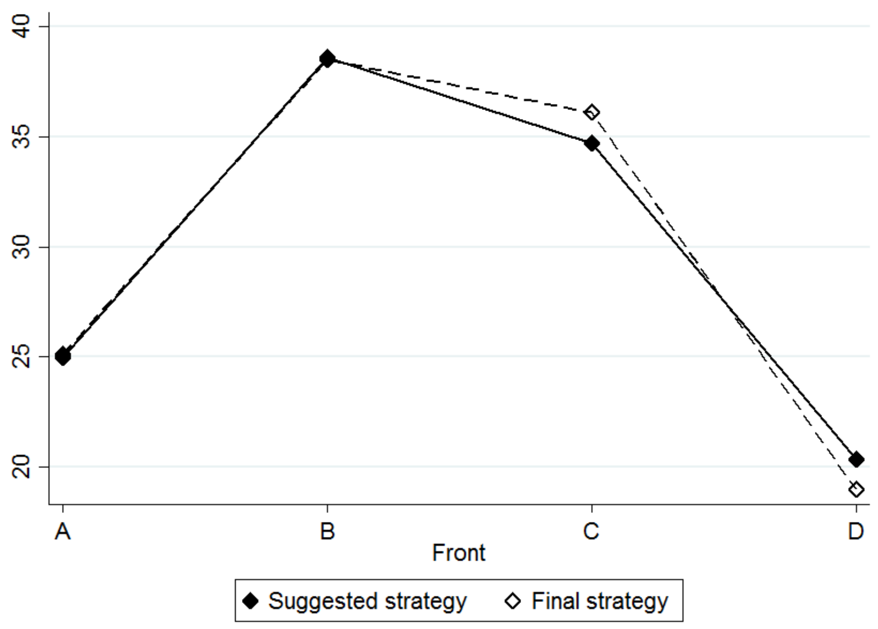


Figure 6: Average bids for each front in Auction 4.

## 5. Results: All-pay multi-object auctions

The total number of participants in the multi-object all-pay auctions was 123. The number of classified messages was 108 in All-pay 3 and 109 in All-pay 4.

### 5.1 Dimensions in all-pay multi-object auctions

In order to see the use of multi-dimensional reasoning in the all-pay multi-object auctions, consider the message written by subject #55, who suggested the strategy (29,0,0,31) in All-pay 4: "*I think we should split the points between two auctions. We should bet half the points specifically on auction D, in which the rest of the teams are likely to invest less since its prize is lower. If we split 30-30 between two auctions, we have a good chance of winning. In addition, I suggest not to assign round numbers but rather 31/41 because usually people tend to use round numbers and one point of difference could make us win the auctions*" (direct translation). This message is classified as reflecting the three dimensions of how many auctions to focus on (D1), the identity of those auctions (D3), and the type of assignment to auctions with high-bids (D2H). The dimensional decision rule within D1 is classified as N and the decision rules within D2H and D3 are classified as L1.

The type of reasoning expressed by subject #55 resembles that observed in the messages in the Blotto games and the first-price auctions. By contrast, the message of subject #59 in All-pay 4 suggesting (1,1,1,1) illustrates a slightly different type of thinking, which is unique to the all-pay auctions: "*In order to win an auction, one should assign it all points, but it is possible that another team will assign its points to the same auction and then the winner will be selected randomly. This is an undesired risk for a potential gain of 20-30 points. Therefore, we better keep the points that we have received. It is worth assigning one point to each auction for the case that in one of the auctions the other teams did not assign any point. Such a situation is very likely and it is a shame to miss a chance to win an auction cheaply.*" This message was classified as reflecting D2L, the type of assignment to disregarded fronts (decision rule L1), and, notably, dimension D4, how much of the endowment to use for bids (decision rule R).

For all dimensions in the all-pay multi-object auctions, Table 17 provides an illustrating strategy and summarizes the proportion of messages that included each corresponding dimension. Figure 7 presents the distribution of the number of dimensions per message.

| | ***Dimension*** | **Illustration in Allpay 3** | **All-pay 3 (n=108)** | **All-pay 4 (n=109)** |
|---|---|---|---|---|
| *D1* | *Number of auctions with high bids (or "disregarded")* | (30,30,0): two high bids | 64% | 56% |
| *D1A* | *Asymmetric assignments to auctions with high bids* | (40,20,0): two asymmetric high bids | 2% | 1% |
| *D2L* | *Type of assignment to "disregarded" auctions* | (25,25,10): one disregarded front in the 10s | 23% | 34% |
| *D2H* | *Type of assignment to auctions with high bids* | (23,22,10): two non-round high bids in the 20s | 19% | 16% |
| *D3* | *Considerations of the identity of auctions* | (0,30,30): two high bids in B and C | 38% | 63% |
| *D4* | *How much of the endowment to use for bids* | (20,20,0): Using 40 of an endowment of 60 | 28% | 26% |

Table 17: Frequency of dimensions in classified messages in the all-pay auctions.

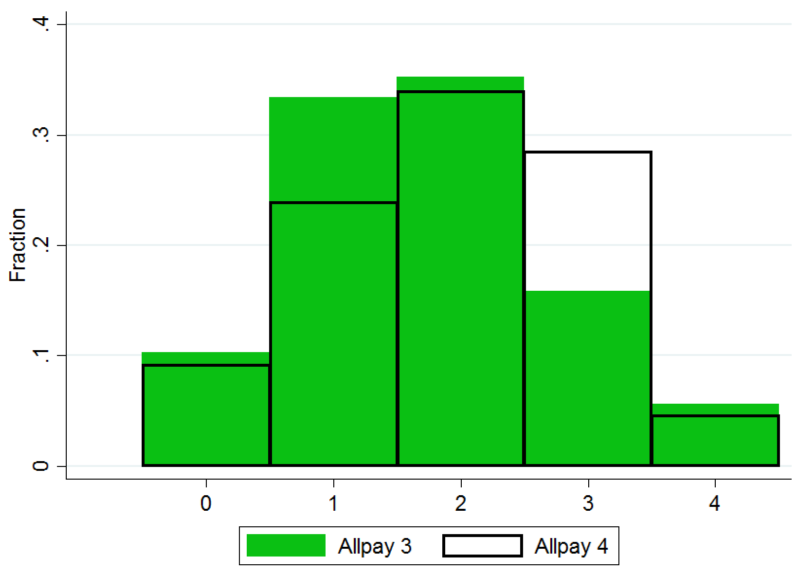


Figure 7: Number of dimensions per message in classified messages in the all-pay auctions (n=108 and n=109).

The main findings of Table 17 are as follows. D1 is used by the majority of subjects. Due to the asymmetry in the auctions' prizes, D3 is the most frequently mentioned dimension in All-pay 4. It is the second most popular in All-pay 3. D4 is mentioned by slightly more than 25% of the subjects, and D1A is very rarely mentioned as in the previous games. Figure 7 shows that 57%–67% of the subjects mention at least two dimensions in their messages and 22%–33% mention at least three dimensions, providing support for multi-dimensional reasoning in the multi-object auctions.

### 5.2 Dimensional decision rules in all-pay multi-object auctions

The dimensional decision rules in the all-pay games are similar to those in the Blotto games and the first-price multi-object auctions (Table 5 contains a summary of the decision rules; see Table S3 in the Appendix for the complete distribution of decision rules). Overall, the most frequent decision rule in the all-pay auctions is L1, with a similar pattern of usage to that found in the Blotto games, while R is the most prominent decision rule in D1 and D4. We note that, as in the first-price auctions, a considerable group of subjects described goals that differ from maximizing profit.

### 5.3 Multi-dimensional reasoning and performance in all-pay multi-object auctions

Table 18 presents the top three suggested strategies in All-pay 3 and 4, where the expected score is calculated by matching each player with any pair of competitors from the experiment, as in Auctions 3 and 4.

| ***Rank*** | **All-pay 3** | **All-pay 4** |
|---|---|---|
| *1* | (10,5,45) [121.8] | (19,18,17,6) [162.7] |
| *2* | (6,48,6) [120.3] | (21,31,7,1) [162.4] |
| *3* | (6,20,34) [120.2] | (30,4,4,22) [160.7] |

Table 18: Winning suggested strategies and their expected scores [in brackets] in the all-pay auctions.

Recall that in these games, any unused endowment is counted as part of the score and a considerable group of subjects indeed kept a portion of their endowment of 60 points. It is noticeable that in both games, the winning strategies used all 60 points in the bids. In All-pay 3 they focused on one auction (B or C) and made small bids in the others. In All-pay 4, they focused on two or three auctions. Some of the winning strategies used bids that were multiples of ten in one of the auctions, but most of their bids were not multiples of ten.

We now turn to examine whether considering more dimensions improves the performance of the suggested strategies with which they are associated. Table 19 presents the average score of suggested strategies as a function of number of dimensions mentioned in the accompanying message.

| ***# of dimensions*** | **n** | **Average score** | **n** | **Average score** |
|---|---|---|---|---|
| | **All-pay 3** | | **All-pay 4** | |
| *1* | 36 | 94.93 (15.70) | 26 | 108.43 (30.27) |
| *2* | 38 | 100.28 (10.17) | 37 | 128.91 (22.22) |
| *3* | 17 | 99.97 (11.90) | 31 | 133.05 (22.03) |
| *4* | 6 | 108.92 (11.50) | 5 | 152.1 (7.12) |

Table 19: Average score (standard deviation in parentheses) as a function of the number of dimensions in the all-pay auctions.

The table suggests that increasing the number of considered dimensions generally improves average performance. We found partial support for that in All-pay 3 and strong support in All-pay 4, using a ranksum Mann–Whitney test to compare the scores of strategies with different numbers of dimensions.[7] For subjects who mentioned at least one dimension, we studied the effect of particular dimensions on the scores. In a linear regression explaining the score in All-pay 3, the coefficients for the dummy variables for the dimensions D1A, D2L, and D3 were positive and significant. In All-pay 4, the coefficient for D2L, D2H, and D3 were significant and positive. In both games, the use of D4 was significant and negative, suggesting that saving a portion of the endowment is not the right tool, given the experimental behavior of the other players; see Table S6 in the Appendix. Although consideration of particular aspects of a strategy could lead to a lower strategy score, overall we found that the use of more dimensions is beneficial, as in the Blotto games.

### 5.4 Suggested and final strategies in all-pay multi-object auctions

In this section, the dimensions are analyzed solely on the basis of the suggested strategies and the final strategies. In the suggested strategies, 74% of the subjects used all 60 points in All-pay 3, and 72% did so in All-pay 4. In each game, 9 subjects suggested not bidding at all and the average spending was about 50 points. These findings are consistent with the proportion of D4 in the messages and confirm that this dimension was much more common in the all-pay auctions than in the first-price auctions. In the final strategies, the use of all 60 points went down to 63% in All-pay 3 and 68% in All-pay 4.

Looking at dimension D1, Table 20 presents the proportion of strategies that correspond to each number of reinforced fronts. The numbers are consistent with the finding that the vast majority of subjects considered D1. In All-pay 3 the majority decided to have one reinforced front, and in All-pay 4 the majority focused on one or two fronts.

[7] In All-pay 3, we find that the score of strategies that correspond to four mentioned dimensions is significantly higher than that of strategies with one dimension ($p<0.05$), and also marginally significantly higher than strategies with two dimensions (0.06) and three dimensions ($p=0.09$). In All-pay 4, the score of strategies that correspond to one dimension is significantly lower than that of strategies with two ($p<0.01$), three ($p<0.001$), and four dimensions ($p<0.01$). The score of strategies that correspond to two dimensions is significantly lower than that of strategies with four dimensions ($p<0.05$), and the score of strategies with three dimensions is marginally significantly lower than that of strategies with four dimensions ($p=0.071$).

| *High-bids in* | **All-pay 3 (n=123)** | | **All-pay 4 (n=123)** | |
|---|---|---|---|---|
| | **suggested** | **final** | **suggested** | **final** |
| *0 auctions* | 16% | 20% | 15% | 14% |
| *1 auction* | 59% | 59% | 41% | 36% |
| *2 auctions* | 24% | 21% | 36% | 43% |
| *3 auctions* | | | 8% | 7% |

Table 20: Number of high bids in the all-pay auctions. High bid is defined as a bid higher than B/3 or B/4, respectively, where B is the used endowment.

Table 21 summarizes the use of various unit digits in the strategies, which corresponds to an aspect of D2L and D2H (but does not capture these dimensions entirely). The table provides information on the individual level and suggests that 41%–47% of the subjects did not consider the unit digit aspect of their bids and chose all their bids to be multiples of tens or zero in their suggested strategies. We distinguish between high-bid auctions and neglected auctions. Aggregating over all subjects and all auctions, we find that the unit digit 0 is used more frequently in the neglected fronts. In All-pay 3, 53% of the bids in reinforced auctions (n=206) and 61% in neglected auctions (n=163) have the unit digit 0. In All-pay 4, 43% of the bids in high-bid auctions (n=247) and 65% in neglected auctions (n=245) have the unit digit 0. Thus, the above pattern in the all-pay auctions is similar to that observed in the first-price auctions and the Blotto games.

In All-pay 4, the unit digit patterns suggest the higher sophistication of the final strategies compared to the suggested strategies, while in All-pay 3 no such tendency is observed.

| *Strategies* | All-pay 3 (n=123) | | All-pay 4 (n=123) | |
|---|---|---|---|---|
| | suggested | final | suggested | final |
| *All bids have the unit digit 0* | 47% | 47% | 41% | 36% |
| *Some bids have the unit digit 1,2, or 3* | 33% | 33% | 30% | 42% |
| *The rest of the strategies* | 20% | 20% | 28% | 22% |

Table 21: Type of bids in terms of unit digit in strategies in the all-pay auctions.

As for dimension D3, Figures 8 and 9 show the average bid in the different auctions and suggest that the first auction is less frequently reinforced in both games, as is the low-stakes Auction D in All-pay 4. The pattern of focusing on the intermediate auctions is similar to that observed in the previous games. In All-pay 3, the final strategies assign fewer resources to fronts A and C, compared to the suggested strategies. In All-pay 4, smaller bids are found in fronts A and B, compared to the suggested strategies.

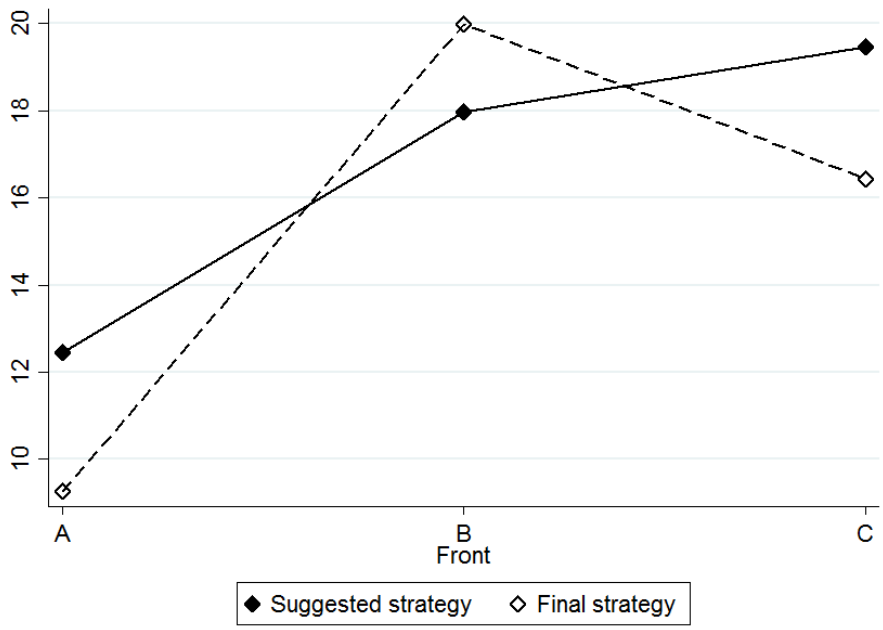


Figure 8: Average bids in each auction in All-pay 3.

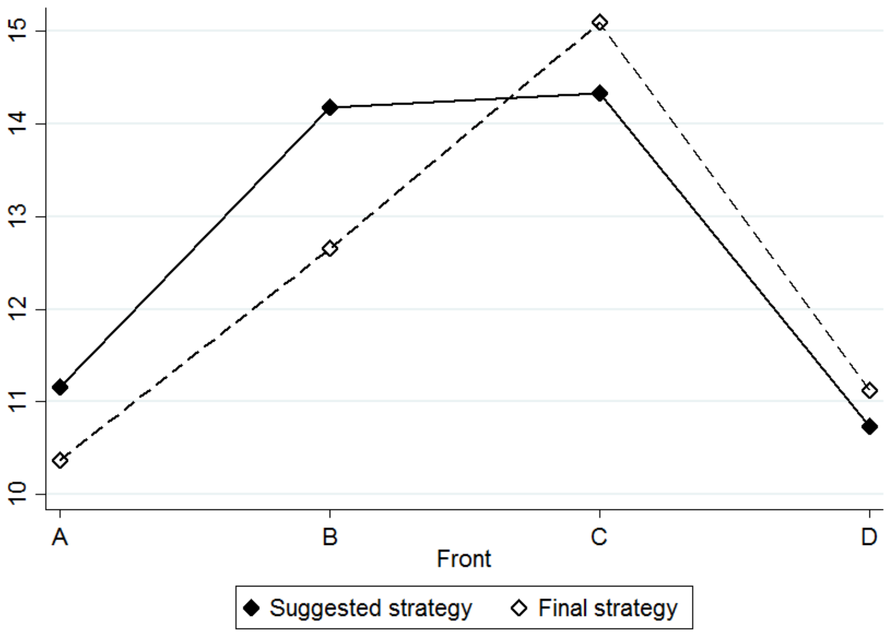


Figure 9: Average bids in each auction in All-pay 4.

## 6. Linking the reasoning process in the six allocation games

In all the games studied here, we find evidence of multi-dimensional reasoning as players think in terms of characteristics of strategies and often deliberate over multiple dimensions at the same time. In this section, we want to understand the extent to which the dimensions considered in the three types of games resemble each other. We use the message classification to explore the connection between the reasoning in the different games in two analyses. First, we provide a between-subject analysis of the reasoning in the various games on an aggregate level. This analysis addresses the fundamental question of whether different resource allocation games trigger similar multi-dimensional reasoning. Second, we measure the within-subject correlation of the reasoning between pairs of games that a subject played. As in the previous sections, the analysis is based only on the classified messages.

### 6.1 The connection between the games on the aggregate level

In order to examine whether the three different types of games trigger similar reasoning, we consider only the reasoning in the first game played by the subject. This eliminates any possible influence that the reasoning in one game might have on the reasoning in another. Thus, we focus here on the reasoning in the three games: Blotto 6, Auction 3, and All-pay 3. Note that in each of these games, the different fronts have equal values. Figure 10 presents the distribution of the number of dimensions per message for the three games and Table 22 reports the proportion of subjects' messages reflecting the corresponding dimensions.

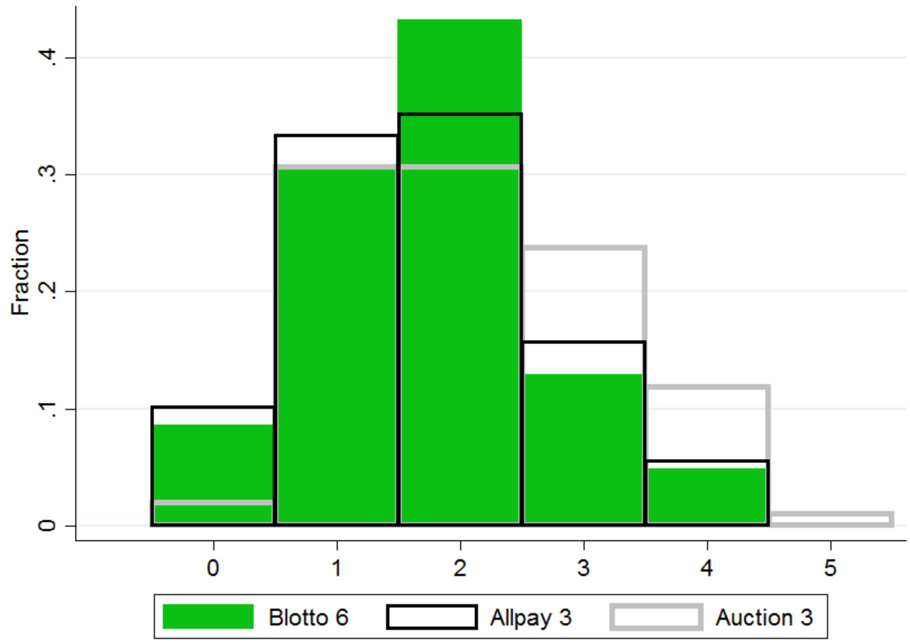


Figure 10: Number of dimensions in classified messages in the first game played.

Figure 10 suggests that the distributions of the number of considered dimensions differ between the games, although the proportions are similar. The median number of considered dimensions is 2 in all three games and the average is 1.84 in Blotto 6, 2.31 in Auction 3, and 1.87 in All-pay 3. A ranksum Mann–Whitney test indicates that the numbers of dimensions in Blotto 6 and All-pay 3 are not significantly different ($p=0.87$), whereas the numbers of dimensions in Auction 3 are significantly larger than their counterparts in both Blotto 6 ($p<0.01$) and All-pay 3 ($p<0.05$).

| | Blotto 6 (n=98) | Auction 3 (n=58) | All-pay 3 (n=52) | Blotto 6 vs. Auction 3 | Blotto 6 vs. All-pay 3 | Auction 3 vs. All-pay 3 |
|---|---|---|---|---|---|---|
| *D1* | 87% | 67% | 77% | ** | | |
| *D1A* | 7% | 12% | 2% | | | * |
| *D2L* | 24% | 22% | 12% | | | |
| *D2H* | 22% | 60% | 23% | *** | | *** |
| *D3* | 43% | 66% | 56% | ** | | |
| *D4* | - | 3% | 17% | | | * |

Table 22: Frequency of dimensions in classified messages in the first game played. Statistical significance of a proportion test is indicated for each pair of games and each dimension.
* indicates p<0.05, ** indicates p<0.01, *** indicates p<0.001.

Table 22 reveals that the frequency of usage of the various dimensions follows a similar pattern in all three games: D1 is the most popular dimension, D3 is the second most popular, D2H and D2L follow, and D1A is rarely considered. Overall, the frequencies of those dimensions are similar in all three games. However, as indicated in the table, there are some statistical differences between the games in terms of the proportions of usage of particular dimensions. Two qualitative differences stand out. First, only in All-pay 3 does a considerable group of players consider D4, which is natural since it is the only game in which players can benefit from an unused budget. Second, in Auction 3 the consideration of D2H occurs with a frequency that is almost 40 percentage points higher than in the other games. This finding is naturally reflected in the higher average number of dimensions per message in the game. Thus, in the Blotto games and the all-pay auctions a decision to concentrate only on a number of fronts does not necessarily trigger thinking about the particular assignment, whereas in Auction 3, the type of assignment to an auction with a high bid is frequently discussed.[8]

[8] Recall that in the Blotto games, D2H included only arguments about the unit digit in the assignment to reinforced fronts whereas in the two types of auctions, D2H had a wider definition and included both arguments on the rough magnitude of the bid and arguments on the unit digit.

To summarize, the aggregate data presented in the above tables indicate that in the first resource allocation game they play, people think of the same dimensions in the different games. In other words, these three games trigger a similar type of reasoning. Yet, each different game has some unique aspects and a slightly different focus due to its particular nature and specific parameters. In the next subsection, we inquire whether an individual player tends to reason in a similar manner in different allocation games.

### 6.2 The connection between the games on the individual level

Does a participant who thinks about a large number of dimensions in one game tend to think of many dimensions in another? When comparing games, both within and across blocks, we find a positive correlation between the numbers of considered dimensions in each pair of games (0.19-0.58). Of the two kinds of correlations, the within-block correlation tends to be stronger; see Table S4 in the Appendix.

We further investigate whether players who think of a particular dimension in one game tend also to think of that dimension in another game. We start by looking at the dimensions considered in a pair of games of the same type. Table 23 presents the within-block correlations in the various dimensions for the three types of games. Overall, the use of particular dimensions is highly correlated within the Blotto games and within the all-pay auctions, whereas the two first-price auctions, except for their shared dimension D2L, seem to trigger the consideration of different dimensions. This is in line with the aggregate-level findings in Auction 4 of a more frequent use of D3 and D1 and a less frequent use of D2H compared to Auction 3. These differences thus reflect a smaller individual persistence in the use of dimensions in the multi-object auctions.

| ***Dimension/Block*** | **Blotto (n=197)** | **Auction (n=93)** | **All-pay (n=98)** |
|---|---|---|---|
| *D1* | 0.327*** | 0.161 | 0.256* |
| *D1A* | 0.187** | 0.06 | -0.015 |
| *D2L* | 0.531*** | 0.254* | 0.573*** |
| *D2H* | 0.543*** | 0.161 | 0.218* |
| *D3* | 0.512*** | 0.053 | 0.261** |
| *D4* | NA | -0.051 | 0.334*** |

Table 23: Within-block correlation in the usage of particular dimensions.

* indicates $p<0.05$, ** indicates $p<0.01$, *** indicates $p<0.001$.

Next, we explore the connection of the dimensions associated with the first game of each of the two blocks. For some players, these games are Blotto 6 and All-pay 3 and for the others they are Blotto 6 and Auction 3 in the two possible orders. Again, for all subjects, in each of the two games the fronts' values are identical. Among the 191 subjects whose messages were classified, we found a correlation in the use of D3 (Spearman coefficient=0.17, p=0.019) and D2L (Spearman coefficient=0.23, p=0.001). No correlation was found in the usage of other dimensions. Taking into account that D1A is rarely used and D1 is very frequent in all games, the results suggest that the reasoning of a player between different games is linked through the tendency to consider similar dimensions.

We further measure the correlation in the usage of dimensions between specific games that are not in the same block. Table 24 summarizes the findings and indicates that between Blotto games and all-pay auctions, the use of mainly D2L but also of D2H are correlated, whereas between Blotto games and the first-price-auctions, the use of mainly D2H but also of D1 and D3 are correlated (see Table S5 in the Appendix for more details). We are aware that we are looking at more than 40 tests and therefore some of them should turn out to be significant just by chance. Note, however, that the number of significant correlations is higher than what would be expected by pure chance.

| | ***Auction 3*** | ***Auction 4*** | ***All-pay 3*** | ***All-pay 4*** |
|---|---|---|---|---|
| ***Blotto 6*** | $^{+}$ D2H, D3 | D1, D2H | D2L | D2L, D2H |
| ***Blotto 7*** | $^{+}$ D2H, $^{+}$D3 | D1, D2H | D2L | D2L, D2H, $^{+}$D3 |

Table 24: Summary of the between-block correlation in the usage of particular dimensions. $^{+}$ indicates a marginally significant correlation (p is in the range 0.05–0.07).

The interpretation of the results in this subsection is that, while different people may focus on different dimensions, a person's reasoning in two allocation games tends to show similarities. It is intuitive that a person who believes that the order of assignments matters in the Blotto games also believes that it matters in the auctions and that the group of subjects who figure out that many people neglect fronts in the Blotto games will suspect that a parallel tendency exists in the first-price and all-pay auctions as well. On the other hand, it is possible that a particular game makes a person think of an aspect that was not considered in another game. For example, a person may be more inclined to consider the identity of reinforced auctions in games with asymmetric values of items than in other games.

## 7. Conclusion

This study was designed to investigate the reasoning process in competitive resource allocation games and to explore whether there are components of the process that are common to different games in this class. We experimentally studied three types of allocation games: Blotto games, first-price multi-object auctions with budget constraints, and all-pay multi-object auctions with a limited endowment. The analysis of the written communication between team members in the experiment indicates that in all the games studied, players classify the strategies in a number of dimensions and perform their strategic deliberation in the space of those dimensions. Moreover, the main dimensions are common to the three types of games.

The subjects' written messages suggest that almost all of them reasoned in terms of characteristics of strategies rather than in terms of strategies per se. While a group of subjects appeared to consider only one dimension, 57%–85% mentioned two or more dimensions in their message, depending on the particular game. Overall, the most frequently considered dimensions were how many fronts to focus on (D1) and the identity of those fronts (D3). In games with asymmetric values of items, D3 was naturally more frequent. The less frequently considered dimensions D2L and D2H, which focus on the details of the assignment to a front, distinguished their users as more sophisticated and are often significant determinants of good performance. In some of the games, a consideration of D3 was associated with a high score as well. One of the most prominent dimensional decision rules was L1, i.e., a response to a belief that others choose instinctively within the dimension, e.g., by allocating in multiples of ten. Similarly prominent was R, i.e., a collection of reasonable arguments that are neither instinctive nor explicitly belief-based, e.g., focusing on the majority of fronts under the assumption that it is impossible to win all the fronts and that winning less than half is not enough.

In Section 6, we found that a player tends to consider similar dimensions in different interactions and that multi-dimensional sophistication is persistent across games. We also showed that in the aggregate some patterns of multi-dimensional reasoning are found in different competitive resource allocation games and do not depend on the details of the interaction. Thus, for example, we imagine that both cyber attackers with limited resources and bidders in multi-object auctions focus on some fronts while nonetheless allocating a small amount of resources to other, almost neglected fronts.

The focus of the paper was on individual reasoning and in particular on understanding the relevance of multi-dimensional reasoning via written messages. The suggested strategies that accompanied the written messages reflected the reasoning identified in the messages. For example, the proportion of people who used unit digits 1–3 in their assignments is generally similar to the proportion of messages in which this aspect was mentioned. Although final decisions were not accompanied by a written message, they turn out to be informative about multi-dimensional reasoning as they are qualitatively similar to the suggested strategies in terms of the number of reinforcements, the use of unit digits, and the location of assignments. However, the final strategies seem to be slightly more sophisticated. In the Blotto games, we found fewer reinforcements of 0, 1, and 2 fronts and in the first-price auctions we found fewer reinforcements of 0 and 1 front. We further found a lower frequency of the unit digit 0 and a higher frequency of the digits 1, 2, and 3 in almost all games. Overall, we found more neglect of the first front, which can be beneficial if one allocates a small assignment to it. We conclude that multi-dimensional reasoning is not diminished by communication.

The above finding makes us conjecture that in a repeated interaction of such games, players organize the information on other players' past behavior in the space of dimensions, form dimensional beliefs, and respond to those beliefs as they did in this study. Throughout the interaction, beliefs may become more precise and behavior within dimensions may converge to a stable one. Thus, the evidence that players consider similar dimensions in different games and that the patterns of multi-dimensional reasoning are recurrent encourages a discussion of an equilibrium concept that is based on such reasoning. Arad and Rubinstein (2017) offer one way to model an equilibrium with multi-dimensional reasoning. Future studies shall provide experimental evidence on the perception of dimensions and on the nature of multi-dimensional reasoning in repeated interactions to inform such models.

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## Appendix A: Supplementary tables

| | D1 (n=367) | D1A (n=32) | D2L (n=102) | D2H (n=82) | D3 (n=163) |
|---|---|---|---|---|---|
| *L0* | 2% | 0% | 2% | 1% | 20% |
| *L1* | 27% | 34% | 57% | 78% | 44% |
| *L2* | 1% | 0% | 5% | 4% | 0% |
| *R* | 64% | 50% | 22% | 14% | 3% |
| *N* | 6% | 16% | 15% | 2% | 33% |

Table S1: Decision rules in the Blotto games. Messages from the two Blotto games are pooled together.

| | D1 (n=145) | D1A (n=24) | D2L (n=42) | D2H (n=109) | D3 (n=144) | D4 (n=9) |
|---|---|---|---|---|---|---|
| *L0* | 6% | 8% | 7% | 3% | 6% | 0% |
| *L1* | 48% | 38% | 17% | 19% | 66% | 22% |
| *L2* | 1% | 0% | 7% | 4% | 8% | 0% |
| *R* | 28% | 17% | 50% | 62% | 4% | 33% |
| *N* | 18% | 38% | 19% | 13% | 17% | 44% |

Table S2: Decision rules in the first-price auctions. Messages from the two auctions are pooled together.

| | D1 (n=130) | D1A (n=3) | D2L (n=62) | D2H (n=37) | D3 (n=110) | D4 (n=59) |
|---|---|---|---|---|---|---|
| *L0* | 13% | 0% | 2% | 16% | 22% | 10% |
| *L1* | 12% | 33% | 84% | 51% | 50% | 2% |
| *L2* | 0% | 0% | 3% | 3% | 10% | 0% |
| *R* | 58% | 33% | 5% | 19% | 3% | 71% |
| *N* | 17% | 33% | 6% | 11% | 15% | 17% |

Table S3: Decision rules in the all-pay auctions. Messages from the two auctions are pooled together.

| *Pairs of games* | **Spearman correlation** |
|---|---|
| *Blotto 6 & Blotto 7 (n=197)* | 0.579** |
| *Auction 3 & Auction 4 (n=93)* | 0.483** |
| *All-pay 3 & All-pay 4 (n=98)* | 0.397** |
| *Blotto6 & Auction 3 (n=92)* | 0.494** |
| *Blotto7 & Auction 3 (n=93)* | 0.442** |
| *Blotto 6 & Auction 4 (n=91)* | 0.399** |
| *Blotto 7 & Auction 4 (n=93)* | 0.431** |
| *Blotto 6 & All-pay 3 (n=99)* | 0.192* |
| *Blotto7 & All-pay 3 (n=99)* | 0.336** |
| *Blotto 6 & All-pay 4 (n=98)* | 0.248** |
| *Blotto 7 & All-pay 4 (n=98)* | 0.385** |

Table S4: Correlation between the number of dimensions in messages in different games.

* indicates p<0.05, ** indicates p<0.01.

| *Pairs of games: Dimension* | **Spearman correlation** |
|---|---|
| *Blotto 6 & Auction 3 (n=92): D2H* | 0.194 (p=0.064) |
| *Blotto 6 & Auction 3 (n=92): D3* | 0.276** |
| *Blotto 7 & Auction 3 (n=93): D2H* | 0.199 (p=0.056) |
| *Blotto 7 & Auction 3 (n=93): D3* | 0.192 (p=0.065) |
| *Blotto 6 & Auction 4 (n=91): D1* | 0.343*** |
| *Blotto 6 & Auction 4 (n=91): D2H* | 0.253* |
| *Blotto 7 & Auction 4 (n=93): D1* | 0.233* |
| *Blotto 7 & Auction 4 (n=93): D2H* | 0.238* |
| *Blotto 6 & All-pay 3 (n=99): D2L* | 0.359*** |
| *Blotto 7 & All-pay 3 (n=99): D2L* | 0.426*** |
| *Blotto 6 & All-pay 4 (n=98): D2L* | 0.346*** |
| *Blotto 6 & All-pay 4 (n=98): D2H* | 0.275*** |
| *Blotto 7 & All-pay 4 (n=98): D2L* | 0.347*** |
| *Blotto 7 & All-pay 4 (n=98): D2H* | 0.253** |
| *Blotto 7 & All-pay 4 (n=98): D3* | 0.184 (p=0.07) |

Table S5: Between-block correlation in the usage of particular dimensions.

* indicates p<0.05, ** indicates p<0.01, *** indicates p<0.001.

| | Blotto 6 | Blotto 7 | Auction 3 | Auction 4 | All-pay 3 | All-pay 4 |
|---|---|---|---|---|---|---|
| *D1* | -0.211 | -0.344* | 3.230 | -6.026 | 4.142 | 5.739 |
| | (0.175) | (0.169) | (2.149) | (4.203) | (2.803) | (4.669) |
| *D1A* | 0.194 | -0.269 | 1.852 | -0.212 | 19.17* | 7.614 |
| | (0.165) | (0.145) | (3.133) | (4.975) | (8.475) | (21.82) |
| *D2L* | 0.377*** | 0.378*** | -7.457** | 4.469 | 12.93*** | 22.20*** |
| | (0.0901) | (0.0872) | (2.555) | (3.789) | (2.919) | (4.710) |
| *D2H* | 0.195* | 0.0635 | -0.0202 | -0.160 | 4.032 | 12.57* |
| | (0.0915) | (0.101) | (2.179) | (3.137) | (2.980) | (6.206) |
| *D3* | 0.216** | -0.0194 | 1.344 | 3.891 | 5.642* | 11.19* |
| | (0.0777) | (0.0791) | (2.131) | (6.057) | (2.473) | (4.995) |
| *D4* | | | | | -9.079** | -11.28* |
| | | | | | (2.722) | (5.274) |
| *Constant* | 2.647*** | 3.456*** | 41.14*** | 63.80*** | 93.76*** | 107.3*** |
| | (0.180) | (0.178) | (2.648) | (6.718) | (3.019) | (6.115) |
| *N* | 193 | 198 | 97 | 98 | 97 | 99 |

Table S6: Coefficients (standard errors in parentheses) in a linear regression where the dependent variable is the score in the game and the explanatory variables are dummies for the various dimensions.
* indicates p<0.05, ** indicates p<0.01, *** indicates p<0.001.

## Appendix B: Instructions for the experiment

The following instructions were provided to subjects in the experiment and were read out loud:

**Welcome to the experiment**

You are about to participate in an experiment on making decisions in a group. Please pay close attention to the instructions.

During the course of the experiment, you will be able to earn a considerable sum of money. Your decisions and those of the other participants in this room are the factors that will determine the size of the sum (a detailed explanation is provided below). In each game, you will be able to earn points that can be converted to shekels at the end of the experiment at the conversion rate of 5 points = 1 shekel. **In addition to your winnings in the various games, each participant who completes the experiment will receive a sum of 35 shekels**. You will receive the total sum of money you earn immediately upon conclusion of the experiment, personally and in cash.

You are not allowed to talk to other participants or look at their computer screen during the experiment.

If you have any questions, please raise your hand and one of the staff members will be glad to respond.

You will play four games in the experiment. The experimenter will read the instructions for each game out loud, and then all of the participants will play the game on the computer.

At the beginning of each game, each participant will be randomly teamed with another participant from this room – each such pair will play together as one group. The team's earnings will be divided equally between the two participants.

In each game, the computer will randomly pair the participants into teams.

How is the "team decision" determined in the game?

In each of the games, each member of the team will be asked to enter a "final decision." After the computer receives the final decision of both team members, it will randomly select one of the two possibilities. That is, there is a 50% probability that the computer will select your final decision as the one to represent the team, and a 50% probability that it will choose the final decision of the other team member. However, you can influence the final decision of your teammate in the following way: Before entering your final decision, you can send (only) **one** written message to the other member of the team. **The message should include your proposed course of action, together with a detailed**

**explanation of why you chose this option**. This message is your only way to influence your teammate's decision – use it wisely and explain your decision in a clear and persuasive way.

This message will appear on your teammate's computer screen before he or she makes a final decision. In the same way, before you make your final decision, a message will appear on your computer that includes your partner's proposal, together with an explanation of why he or she believes it is the correct course of action. As noted above, the computer will randomly select one of the two final decisions your team submits. (If one of the participants succeeds in persuading his or her teammate, then the two final decisions will be identical and the fact that the computer selects only one of them is insignificant.) With the final decision that the computer selects to represent your team, you will compete against the other teams participating in the game.

The experiment will be conducted as follows:

<u>Proposals and messages stage</u>:

1. Game 1: You will be asked to submit a proposed action + a message to your teammate
2. Game 2: You will be asked to submit a proposed action + a message to your new teammate
3. Game 3: You will be asked to submit a proposed action + a message to your new teammate
4. Game 4: You will be asked to submit a proposed action + a message to your new teammate

<u>Final decision stage</u>:

1. Game 1: You will be asked to make a final decision
2. Game 2: You will be asked to make a final decision
3. Game 3: You will be asked to make a final decision
4. Game 4: You will be asked to make a final decision

The results of the games will be reported only at the end of the experiment.

If you have questions at this stage, please raise your hand.

**We'll start with the proposals stage and will read the instructions for the first game**

* Numbers of games in practice corresponded to the order of games played in the session.

**Instructions for Game 1**

The computer assigns a teammate to each participant in the experiment.

In this game, your team will compete in a tournament against **all of the other teams** in the experiment.

You will play the role of a general commanding an army in time of war; the other teams in the experiment are commanders of enemy armies. Each team has **120 soldiers** it must deploy on **6 separate fronts**.

In each game against another team, you win on the fronts where you deploy more soldiers than the other team. If both teams deploy the same number of soldiers to a particular front, both lose on that front. You must decide how to deploy your soldiers without knowing the deployments selected by the other teams.

You play automatically with your selected deployment vis-à-vis each of the teams in the experiment (you cannot select different deployments against different teams).

**Your team's final score in the game will be the total number of victories you win against all of the teams.**

The winner in the tournament will be the team that receives the highest score among the teams.

In the case of a tie, the computer will randomly select the winner.

The prize for the winning team is 150 points.

- **Now send your teammate your proposed course of action and your message**
- **Remember that this is not your final decision**
- **Remember that in this game you were paired with a new teammate**

**Instructions for Game 2**

The computer now reassigns a teammate to each participant in the experiment.

In this game too, your team will compete in a tournament against **all of the other teams** in the experiment.

The game is similar to the previous game, but now each team has **210 soldiers** to deploy to **7 separate fronts**.

You play automatically with your selected deployment vis-à-vis each of the teams in the experiment (you cannot select different deployments against different teams).

**Your team's final score in the game will be the total number of victories you win against all of the teams.**

The winner in the tournament will be the team that receives the highest score among the teams.

The prize for the winning team is 150 points.

- **Now send your teammate your proposed course of action and your message**
- **Remember that this is not your final decision**
- **Remember that in this game you were paired with a new teammate**

**Instructions for Game 3**

The computer assigns teams in the experiment.

In this game, your team will compete against **two other teams** only. Those teams will be randomly selected by the computer.

The game entails three different "auctions" (A, B, and C) in which your team can choose to participate. The team is entitled to submit a bid in each auction. The bid is the number of points it is willing to pay in the event that it wins the auctions. The winning team in each auction is the one that submits the highest bid from among the competing teams.

Each team must simultaneously decide which bid to submit in each of the auctions – of course, without knowing what the other teams are planning.

- If your team wins Auction A, it will receive a prize of **100 points**
- If your team wins Auction B, it will receive a prize of **100 points**
- If your team wins Auction C, it will receive a prize of **100 points**

As noted, each auction is separate. The way to win is to submit the highest bid among the three competing teams. If there is a tie in a particular auction (if two teams submit the same highest bid), the computer will randomly select one of them as the winner.

You are entitled to participate in each of the three auctions, as long as the total of your bids does not exceed **120 points**.

**Please note! You will be required to pay the bid you submitted only if you win the auctions.**

Illustration of the game**:**

Let's assume a particular team proposes x points in Auctions A, y points in Auctions B and z points in Auctions C. The team is happy to learn that it won in Auctions A and in Auctions B. In this case, it receives 200 points in prizes and pays x+y points (the sum of points it bid in Auctions A and B).

If the team wins only in Auctions B, it receives a prize of 100 points and pays y points.

- **Now send your teammate your proposed course of action and your message**
- **Remember that this is not your final decision**
- **Remember that in this game you were paired with a new teammate**

**Instructions for Game 4**

The computer now reassigns a teammate to each participant in the experiment.

In this game too, your team will compete against **two other teams** only. Those teams will be randomly selected by the computer.

This game is similar to the previous game, but there are four different auctions here:

- If your team wins Auction A, it will receive a prize of **90 points**
- If your team wins Auction B, it will receive a prize of **90 points**
- If your team wins Auction C, it will receive a prize of **90 points**
- If your team wins Auction D, it will receive a prize of **110 points**

You are entitled to participate in each of the four auctions, as long as the total of your bids does not exceed **120 points**.

- **Now send your teammate your proposed course of action and your message**
- **Remember that this is not your final decision**
- **Remember that in this game you were paired with a new teammate**

**Instructions for Game 5**

The computer now reassigns a teammate to each participant in the experiment.

In this game, your team will compete against **two other teams** only. Those teams will be randomly selected by the computer.

The game entails three different "auctions" (A, B, and C) in which your team can choose to participate. The team is entitled to submit a bid in each auction. The winning team in each auction is the one that submits the highest bid from among the competing teams.

Each team receives **60 points** it can use for bidding in the auctions. **In this game, each bid in each auction must be paid, even if the team does not win the auctions**. Unused points will remain in the possession of the team at the end of the game, and will be added to the team's winnings.

Each team must simultaneously decide which bid to submit in each of the auctions – of course, without knowing what the other teams are planning.

- If your team wins Auctions A, it will receive a prize of **90 points**
- If your team wins Auctions B, it will receive a prize of **90 points**
- If your team wins Auctions C, it will receive a prize of **90 points**

As noted, each auction is separate. The way to win is to submit the highest bid among the three competing teams. If there is a tie in a particular auction (if two teams submit the same highest bid), the computer will randomly select one of them as the winner.

**Please note! You will be required to pay the total number of points you bid in a particular auction, even if you lose the auctions.**

Illustration of the game**:**

Let's assume a particular team proposes x points in Auctions A, y points in Auctions B, and z points in Auctions C. The team is happy to learn that it won in Auctions A and in Auctions B. In this case, it receives 180 points in prizes and pays x+y+z points (the sum of points it bid in all of the auctions). That is, in this case, the team receives 60 plus 180 minus (x+y+z).

If the team only wins in Auctions B, it receives a prize of 90 points and pays x+y+z points. That is, the sum the team receives at the end of the game is 60 plus 90 minus (x+y+z).

- **Now send your teammate your proposed course of action and your message**
- **Remember that this is not your final decision**
- **Remember that in this game you were paired with a new teammate**

**<u>Instructions for Game 6</u>**

The computer now reassigns a teammate to each participant in the experiment.

In this game, your team will compete against **two other teams** only. Those teams will be randomly selected by the computer.

This game is similar to the previous game, but there are <u>four</u> different auctions here:

- If your team wins Auctions A, it will receive a prize of **90 points**
- If your team wins Auctions B, it will receive a prize of **90 points**
- If your team wins Auctions C, it will receive a prize of **90 points**
- If your team wins Auctions D, it will receive a prize of **80 points**

Again, each team receives **60 points** it can use for bidding in the auctions. **In this game, each bid in each auction must be paid, even if the team does not win the auctions**. Unused points will remain in the possession of the team at the end of the game, and will be added to the team's winnings.

- **Now send your teammate your proposed course of action and your message**
- **Remember that this is not your final decision**
- **Remember that in this game you were paired with a new teammate**

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**Last page:**

Now we'll move on to the stage of final decisions in the four games

- You can proceed to Game 2 only after you make a final decision in Game 1, and so on.
- Before you make the decision, please read the reminder about the rules of game and the proposal + message you received from your teammate in this game

## Appendix C: Classification instructions

*For each type of game, two research assistants were assigned. Each research assistant received specific instructions for either the two Blotto games or the two multi-object first-price auctions or the two all-pay multi-object auctions.*

**1. General classification instructions**

Each of you is asked to classify the message of every subject from the Blotto 6 (or Auction 3 or All-pay 3) game and then classify the message of every subject from the Blotto 7 (or Auction 4 or All-pay 4) game. It is important that you do not link between subject's choices in these two games and hence each game has its own classification sheet in the attached Excel file.

After completing the classification, please send me the file.

Then, I will ask you two to meet and examine incompatibilities in your classifications. In your meeting, you will try to convince each other of the accuracy of your classification and may agree on one of the classifications or keep both if you still disagree. Incompatibilities are legitimate and may be common due to the vagueness of the messages.

The main part of the classification is to understand whether the subject deliberated in terms of dimensions of strategies. If the subject did, you are asked to mention only the dimensions that were explicitly specified in the subject's message. You may use the subject’s suggested strategy (the vector of assignments) to better understand the subject’s message, but classify it based only on considerations that were explicitly mentioned in the message.

There may be subjects who did not think in terms of dimensions. For example, a subject may have chosen a strategy in response to a concrete belief (vector of assignments) on competitors’ strategies or a distribution of such. Another type of non-dimensional reasoning might be a “random allocation.” In such cases, you should note in the classification sheet “not multi-dimensional reasoning.” If you are not sure, please state it on the sheet.

### 2.1 Specific classification instructions for the Blotto games

**<u>The following are potential dimensions of a strategy (a vector of 6 or 7 components)</u>:**

**D1** How many fronts should be "neglected" (or how many should be reinforced).

**D1A** Asymmetry in reinforced fronts (e.g., "one should be stronger than the other").

**D2L** What to allocate to "neglected" fronts: the unit digit in neglected fronts.

**D2H** The specific assignment in reinforced fronts: the unit digit in such fronts.

**D3** Which fronts should be neglected / reinforced or the "order" of assignments.

**<u>Decision rules within the above dimensions could be, for example</u>:**

**L0** Intuitive explanation.

**L1** A response to the belief that the competitor chooses intuitively.

**L2** A response to the belief that the competitor is sophisticated and uses decision rule L1.

**R** A reasonable explanation that is not a response to a certain belief on the opponent's behavior (e.g., aiming at winning the majority of fronts).

**S** Safety (win for certain a minimal number of fronts).

* After classification was completed, S (which is a sub-category of R) was merged with R.

**2.2 Specific classification instructions for the first-price and all-pay multi-object auctions**

**<u>The following are potential dimensions of a strategy (a vector of 3 or 4 components)</u>:**

**D1** How many auctions should be “neglected” (or how many auctions are reinforced).

**D1A** Asymmetry in “reinforced” auctions (e.g., “one bid should be larger than the other").

**D2L** What to allocate to “neglected” auctions.

**D2H** Values in “reinforced” (high-bid) auctions (considerations of the specific bid or an approximate value of the bid, e.g., the value may stem from the auction prize value, or from beliefs on the competitor's bid).

**D3** Which auctions should be neglected / reinforced ("identity" of auctions or "order" of bids).

**D4** Budget usage (how much of the budget to use for bids and how much to save).

**<u>Decision rules within the above dimensions could be, for example</u>:**

**L0** Intuitive explanation.

**L1** A response to the belief that the competitor chooses intuitively.

**L2** A response to the belief that the competitor is sophisticated and uses decision rule L1.

**R** A reasonable explanation that is not a response to a certain belief on the opponent's behavior (e.g., aiming at winning the majority of auctions).

**S** Safety (win for certain a minimal number of auctions).

**M** Bid value that ensures a minimal reasonable gain in case of winning.

**P** Bid value that is proportional to the auction's prize value.

* After classification was completed, S, M, and P (which are sub-categories of R) were merged with R.

**3. Additional classification instructions given to all research assistants**

- If there is no message or a short message without any strategic content, please note **N** (no explanation).
- If there is clear dimensional thinking (e.g., "We need to think of how many auctions/fronts we should reinforce") but there is no explicit decision rule, classify the decision rule in the certain dimension as **N**.
- If there are two decision rules in one dimension, please indicate both of them.
- If you are not sure whether a dimension or a decision rule was mentioned in the text, you can note that with a question mark.

**Additional comments:**

1. An argument about the status of the player and not his strategy ("I am an engineer" or "I studied game theory") should be noted and quoted briefly.
2. An argument that reveals hesitation should be noted and quoted.
3. An argument that reveals confidence should be noted and quoted.
4. Inconsistency between the suggested strategy and the accompanying message should be noted.
5. A misunderstanding of the game by the subject should be noted.
6. Unusual arguments should be noted.
7. If the message includes rows with "0," it means that the last sentence prior to the 0 was erased. Please refer to the whole text but mention in your file that the subject erased text.

If there are dimensions/arguments that are not specified in the instructions, you may add them. Before adding any new dimension/argument, you should inform me. Adding a rule will be approved if it appears multiple times in the messages of different subjects.

Attached please find an Excel file to fill in your classification. Please e-mail me if there is any question.

**4. Oral Clarifications and Amendments**

At the beginning and throughout the classification process, the following clarifications and amendments were made to the instructions:

- Excel classification sheet was introduced and explained to the research assistants.
- The difference between dimension 1' and dimension 3 was clarified (D1A refers only to asymmetry between fronts/auctions without mention of order).
- Goals were added to the auction games classification (subjects used "Goals" as a major part of their strategy in the auction games. An example of such a goal: winning a minimal number of auctions).
- A decision rule called “Le” (Leftovers) was added: allocation of the budget residues to an auction/front. After classification was completed, it was merged with R.