Let Game Theory Be?

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In the Passover 2009 supplement of Calcalist and on the newspaper's website, three strategic games were presented. An analysis of the results reveals that the readers' decisions are similar to those made in these games throughout the world – but are not consistent with game theory.

You are driving to Jerusalem on Highway 1. As you approach the Ben Shemen interchange, you notice unusually heavy traffic and consider exiting to Road 443. You only have a few seconds to decide whether to stay on the highway or turn off to the alternative route. On a regular day, nearly all the drivers travel on Highway 1, so it seems to you that it would be best to take Road 443 instead. It occurs to you that the drivers ahead of you may have noticed the heavy traffic and exited to Road 443, creating a traffic jam there. If that's the case, then it would be better for you to stay on Highway 1. But you suspect that the other drivers are no less clever than you and that they likely underwent a similar reasoning process and stayed on Highway 1. So, perhaps it is best after all to take Road 443? You are quite confused.

The decision you face is a strategic one – the best action for you to take depends on the decisions made by the other participants. Game theory analyzes the ways people think in such situations. In the Passover supplement of *Calcalist*, we offered readers a peek into the world of game theory via three games that entail considerations similar to those that arose when driving to Jerusalem. We'll see below that game theory does not provide helpful advice about how to play the three games – and certainly cannot tell you what to do at the Ben Shemen interchange.

GAME 1 – Easy Money

Imagine that you and another *Calcalist* reader are participating in the following game:
Each of you must request a sum of money between 91 to 100 shekels. Each player will receive the sum he requests. In addition, if one of the players requests exactly one shekel less than the other player requests, he will receive an additional 100 shekels. Of course, each player must make a decision without knowing what the other player has decided.

What sum of money will you request?

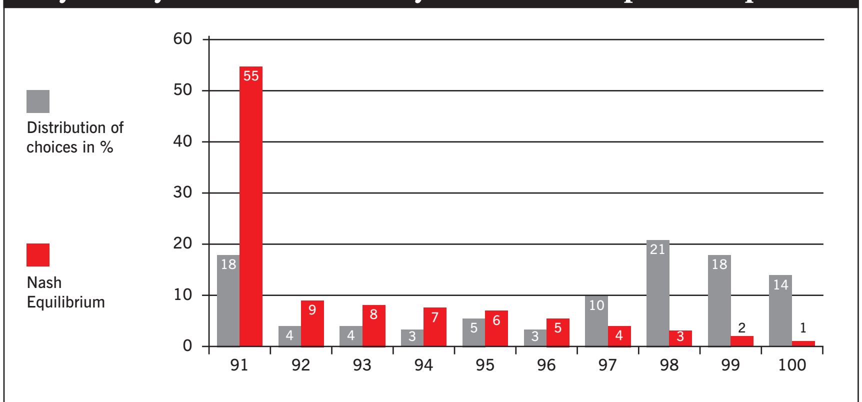
In the Easy Money game, your goal is to request one shekel less than your competitor. If you are correct, you will win a bonus of 100 shekels in addition to the sum you requested. Of course, you do not know what your competitor is about to do. The first thought that comes to mind is to ask for the highest sum – 100 shekels. But if most of the other readers also request 100, you should ask for 99. While you receive only 99 shekels for sure, there is a good chance that you will be playing against a reader who demanded 100 – and you will then win the bonus. At this stage, it will occur to you that the other readers might go through a similar reasoning process and will request 99. And, therefore, you should request 98. And perhaps you'll apply again the same kind of reasoning and request 97?

We call such a chain of argumentation "iterative reasoning". If you continue, you will reach 91. But then you will conclude that if everyone requests 91, it is better for you to request 100 after all – though you will not receive the bonus, you are certain to receive the maximum sum. You have returned to the starting point. At what step in the chain do people stop?

The classic game theoretical analysis of the game is not based on the assumption that the participants engage in iterative reasoning. The analysis uses the concept called "Nash Equilibrium" (which we will not explain here) that predicts a distribution of choices described in the graph below. The graph also displays the distribution of choices made by 1,928 *Calcalist* readers who participated in the game. As the graph shows, the projected distribution is very different than the distribution of choices made by the participants in the game. According to the Nash Equilibrium, 55% of the participants are expected to request 91 shekels and only 1% are supposed to request 100 shekels. Among the readers of *Calcalist*, only 18% requested 91 shekels and 14% asked for 100 shekels.

Who excelled? The choice of 97 (by 10% of the readers) was the best because the most popular strategy was 98. But the 21% of the readers who requested 98 were only slightly inferior as 18% of the participants requested 99.

Easy Money – How Much Money Did the Participants Request?



Based on these results, as well as other studies, it appears that most people stop at one of the first three steps of iterative reasoning or choose the starting point of the chain. Thus, the data from the field – and not the game theory analysis – are what teach us about the way people think.

And how much did they receive?

Request	91	92	93	94	95	96	97	98	99	100
Average Payment	95	96	96	99	98	106	118	116	113	100

GAME 2 – Tennis Tournament

You are a tennis coach and are about to send your team to a tournament. The other readers of *Calcalist* are the coaches of the other teams in the tournament. There are four tennis players on each team: one at level A+ (the highest level), one at level A, one at level B+, and one at level B (the lowest level).

Each coach must assign his players to one of four positions: 1, 2, 3, or 4.

After all the coaches assign their players, the tournament will begin, with each team competing against each of the other teams. A game between two teams will include four matches. A tennis player assigned to a particular position on his team's list will play against the player assigned to the corresponding position on the opposing team's list.

In any match between two tennis players of different levels, the higher-ranked player will win and his team will receive a point. In a match between two players of the same level, the game will end in a draw and each team will receive half a point.

The team's score at the end of the tournament will be the sum of the points the team earned playing against all of the other teams. The objective of each coach is for his team to accumulate the highest score among the teams.

How will you assign the four players on your team?

In the Tennis Tournament, the natural starting point of the chain of reasoning is to assign the players in a list according to the ranking of their abilities (starting with the best player in the first position on the list, and ending with the worst player in the last position). The first iterative step is to elevate the worst player to the first position in the list and to assign the other three players to the next three positions on the list according to their abilities (see the table below). With this assignment, if you play against a team that assigns all four players according to their relative abilities, three of your four players will be better than the players they compete against and you will win by a score of 3 to 1.

There are 24 possible strategies in this game, but 39% of the *Calcalist* readers chose one of two strategies: 18% assigned the players according to their relative abilities, from the best to the worst, and 21% chose the "the first step strategy". All other strategies were far less popular. The winning strategy in the tournament – which was chosen by only 3% of the participants – is the "the second step strategy". This strategy assigns the strongest player to the third position on the list and defeats the "first step strategy" by a score of 3 to 1.

Tennis Tournament – How Were the Tennis Players Assigned?			
Position	Starting point	1st iterative step	2nd iterative step
1	A+	B	B+
2	A	A+	B
3	B+	A	A+
4	B	B+	A
Achievement in tournament	Last place	Second place	First place
% of choices	18%	21%	3%

GAME 3 – War of the Generals

You are the commander of an army in a war being waged on six battlefields.
You are contending against the commander of the enemy's army. Each of you has 120 troops under his command.

It is nighttime, and each of you has to decide how to deploy his troops on the six battlefields. At dawn, the forces on each battlefield will engage in battle. The army that allocates a larger number of troops to a particular battlefield will win that battlefield.

In the event that both armies assign the same number of troops to a particular battlefield, neither of the armies will record a victory on that battlefield. The number of points you win in your war against the enemy's army is the number of battlefields on which you are victorious.

You will play with the deployment you choose against each of the *Calcalist* readers who participate in the game. Your final score will be the total points you accumulate in the wars against all of the other participants.

How will you deploy your 120 troops on the six battlefields?

In the War of the Generals, the first and obvious game strategy that comes to mind is "6 x 20" – deploying 20 troops on each of the six battlefields. Since this strategy immediately suggests itself, it serves as a natural starting point for the iterative process. However, contrary to the two previous games, the next steps in the iterative process are not so clear. There are many strategies that defeat the "6 x 20" on five of the six battlefields – for example, all of the strategies that allocate the troops evenly over five battlefields. These strategies achieve the highest possible score against the "6 x 20" and, therefore, each of them can be regarded as a "first step strategy".

In this game, there are about 250 million (!) strategies, and about 1,000 of them were selected by *Calcalist* readers. The "6 x 20" was the most popular strategy, chosen by 11% of the participants. The six strategies of the "5 x 24" type were chosen by 4%. Another popular class of strategies (12%) included the 15 strategies that deploy 30 troops on four battlefields.

The three winning strategies are described in the box below. The winning general chose a deployment of 2-31-31-31-23-2. And his explanation – like the explanations of the other winners – indicates that he used arguments of an iterative nature: "First, I decided that I would 'surrender' two battlefields, but not so easily. I thought that other people would decide to allocate only a few troops to some of the battlefields and perhaps would not deploy any troops on others. Then, I could win an 'abandoned' battlefield at the inexpensive cost of one troop. Ultimately, I decided to deploy two troops on the weak battlefields in order to overpower those who thought like me and deployed a single troop on the weak battlefields. It seemed logical to me that the weak battlefields would be the furthest ones from the center. I was left with 116 troops to allocate to four battlefields, which is an average of 29 troops per battlefield. I decided to reinforce three of the four remaining battlefields with two troops rather than one – that is, to deploy 31 troops in order to defeat those who deployed the remaining troops equally. In this way, I would also defeat those who deployed 30 troops on each of the central battlefields."

The results among *Calcalist* readers are very similar to those collected among 4,600 students in game theory courses in 25 countries, from China to Mexico. Amazingly, the exact same strategy was also the winner in the students' tournament! Incidentally, game theory itself does not help us in understanding the game. In fact, we do not even know what the Nash Equilibrium looks like in this complex game.

The Victorious Generals and the Deployment of Their Forces	
1	**Sagi Zamir** Student of Software and Electrical Engineering at Tel Aviv University **2-31-31-31-23-2**
2	**Shlomo Philip** Accountant at the Finance Ministry **2-32-31-31-22-2**
3	**Arik Kahane** Project manager at Matrix in Ra'anana **2-23-31-31-31-2**
4	**Lior Shahaf** Graduate student in Physics at Ben-Gurion University **2-23-31-31-31-2**

The lesson?

In the three games, iterative reasoning occurs naturally. It seems that the participants who used iterative reasoning in Easy Money and in the Tennis Tournament exercised similar kind of reasoning in the War of the Generals and received there much higher scores than the other players. However, the overwhelming majority of the participants did not use a consistent pattern of reasoning in all three games. It is evident that they sometimes relied on iterative reasoning and sometimes on other types of strategic thinking.

Classic game theory does not take into consideration the fact that people employ various patterns of reasoning, some iterative and some not, and it does not succeed in predicting their behavior in such games. An interesting finding: 16% of the participants noted that they had taken a course in game theory; their behavior in all three games was nearly identical to that of the other participants.

Everyone talks about game theory and expects it to help in understanding strategic situations in life. Game theory is undeniably a fascinating field regardless of its applicability. But even a simple survey of the type in which *Calcalist* readers participated provides us with a better grasp of the way in which people think in the games of life. And in regard to the drivers who encounter traffic jams on the way to Jerusalem, neither game theory nor the survey findings can offer any salvation.