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Inhomogeneity effects on the magnetoresistance and the ghost critical field above T_c in thin mixture films of In–Ge

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Abstract. We have investigated the effects of inhomogeneity on the magnetoresistance (MR) of thin mixture films of In–Ge. By performing these measurements near the superconducting transition temperature, we could identify the *ghost critical field*, H^*_{c2}. We have used the data on MR and H^*_{c2} to calculate quantities such as the coefficient of diffusion and the inelastic scattering time. A homogeneous–inhomogeneous transition is observed when the relevant length scale (the inelastic diffusion length or the superconducting coherence length) becomes of the order of the inhomogeneity scale in the sample.

1. Introduction

Electronic conduction in two-dimensional (2D) disordered systems where Anderson localisation may be relevant (Anderson 1958) has been studied extensively both theoretically and experimentally in recent years (Lee 1982, Fukuyama 1982). Advances were achieved in the understanding of the effects of inelastic scattering at finite temperature (Lee 1982, Fukuyama 1982, Imry and Strongin 1981), and spin–flip and spin–orbit interactions (Maekawa and Fukuyama 1981, Deutscher and Fukuyama 1982).

The effects of superconducting fluctuations on the magnetoresistance (MR) were first studied by Larkin (1980). The influence of localisation on the transition temperature, T_c, was studied further (Imry and Strongin 1981, Maekawa and Fukuyama 1981).

In comparing experiment and theory it is usually assumed that the samples are homogeneous on the relevant length scale. Localisation effects in inhomogeneous systems, defined as systems where the inelastic diffusion length is smaller than the characteristic length imposed by the inhomogeneity, have not been given much attention, although such effects may be important near the metal–insulator transition.

In this paper, we report on the study of the MR of co-evaporated thin films of In–Ge at temperatures somewhat above the superconducting transition temperature and at various concentrations. Far from the metal–insulator (MI) transition the films behave as homogeneous 2D systems in the presence of strong spin–orbit coupling and significant superconducting fluctuations. However, close to the MI transition the apparent inelastic

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scattering time is anomalously short. We interpret this effect as being due to the inhomogeneity of the system on the length scale of the inelastic diffusion length.

By fitting the MR data to the theoretical expression for localisation in 2D, it is possible to identify the field (the ghost critical field) $H_{c_2}^*$ necessary to suppress superconducting fluctuations. Near the MI transition we observe an anomalous behaviour of $H_{c_2}^*$ due to inhomogeneity on the scale of the effective superconducting coherence length.

The paper is organised as follows. In § 2 we discuss the characteristic inhomogeneity scale in relation to the sample preparation and morphology. The MR measurements and the method of fitting them to the theoretical formulae are described in § 3. Inhomogeneity effects are discussed in § 4 and a discussion of the results obtained in the inhomogeneous limit is given in § 5. Section 6 is devoted to a brief discussion.

2. Sample preparation and characterisation

Films were prepared by co-evaporation of In and Ge from two electron beam guns at an average rate of 30 Å s^{-1} onto a microscope slide held at room temperature, to a combined thickness of about 500 Å. The pressure during evaporation was about 5×10^{-6} Torr. From each evaporation eight samples were produced with a range of metal concentrations, due to different distances of the strips to the sources. The average concentration of In was about 65%. To analyse the microscopic structure, the same evaporation procedure was performed onto electron microscope (TEM) grids and onto SiO substrates. The structures of the In–Ge mixture on TEM substrates is identical to that on SiO; however, the resolution of the micrographs is much worse for the latter.

Thus it is plausible that the structure of our actual samples is the same as on the grids, as shown in the bright-field micrograph (figure 1). It is known that In is highly mobile on

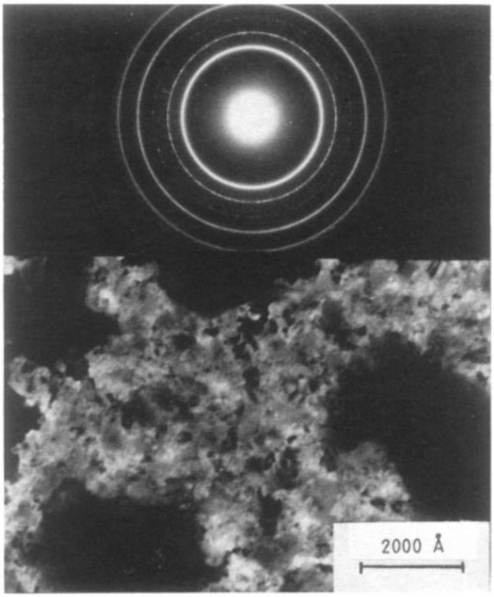

Figure 1. A TEM micrograph and diffraction pattern for In–Ge (the sharp circles are for Ge and the dotted ones for In).

a glass substrate and this was indeed observed during the TEM observation when the grid was heated by the electron beam. However, at room temperature no change in the location or in the size of the In crystallites was detected.

The diffraction pattern indicates that the In and the Ge are both crystalline and since the In rings are made of distinguishable bright dots the In has considerable texture. Analysing carefully the dark-field micrographs (not shown) we came to the conclusion that the mixture is composed of large ($\simeq$2000 Å) rounded clusters of pure In up to 1 μm apart.

These In lumps are connected by a random mixture of small In and Ge crystallites of size $\simeq$70 Å on the average. There exist, therefore, two inhomogeneity length scales in these mixtures: a very large one ($\sim$1 μm) and a smaller one that characterises the small-grain random matrix that connects the In lumps. The characteristic length for this matrix is the percolation length (Deutscher *et al* 1982) $\xi_p = a|p - p_c|^{-\nu}$, where a is the typical grain size and $p - p_c$ the distance from the percolation threshold measured in vol% of In. We estimate that in our samples $p - p_c$ varies from about 20% (sample F) to a few per cent (sample A). We shall assume in the following that the transport properties of the films are dominated by the fine-structure matrix. The relevant inhomogeneity scale should be ξ_p, which varies from a few 100 Å for sample F to several 1000 Å for sample A.

3. Magnetoresistance measurements

The MR measurements were performed using a high-sensitivity multimeter ($\Delta R/R$ could be measured down to 10^{-6}) with a four-terminal method. The positive MR was measured for applied magnetic fields varying from $\simeq$150 G to 13 kG at 4.2 K. The superconducting transition temperature was also measured. Characteristics of the six successive strips are given in table 1 (in the order they were on the slide there is one strip missing between samples A and B).

Table 1. Sample characteristics.

Sample	A	B	C	D	E	F
R ($\Omega_\square$)	131	17.4	13.2	10.8	9.0	6.3
R(300 K)/R(4.2 K)	0.77	1.3	1.37	1.50	1.51	1.74
T_c (K)	<1.4	2.75	3.15	3.2	3.27	3.3
$g(T/T_c)$†	<0.85	2.36	3.47	3.6	3.9	4.14
$\beta - \alpha$	2.2	4.05	4.45	4.5	4.65	4.95
$\beta_{th} + \frac{1}{2}$‡	<1.33	3.81	6.1	6.4	7.0	7.6

† From Bergman (1982): $g(T/T_c) = -1/\ln(T/T_c)$ where T_c was obtained by measurement.
‡ Calculated using results from Bergman (1982) and the value of $g(T/T_c)$.

A typical MR plot is shown in figure 2, where three regions can be distinguished. At very low fields a H^2 dependence is clearly seen. At intermediate fields one observes a tendency towards a dependence on ln H, as seen in figure 3. Finally, a slower variation is observed at very high fields particularly for samples with a low $R_\square$. The fact that the MR is positive and only weakly anisotropic is indicative of strong spin–orbit coupling, since the samples are essentially two-dimensional as discussed below. However, the

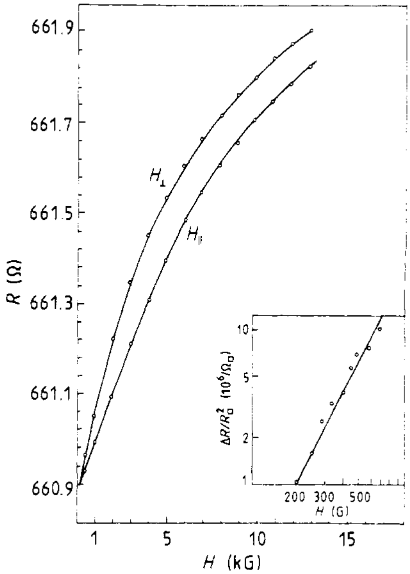

Figure 2. A typical MR measurement (sample C). The inset shows the low-field behaviour of H^2. The full curves are guides to the eye.

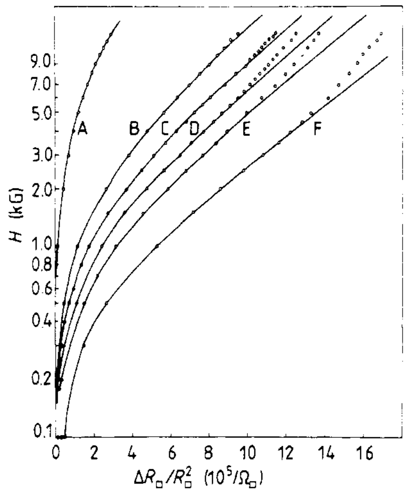

Figure 3. Semi-log plot of the perpendicular-field MR for all the samples. The full curves were fitted. H_{c2}^* is the field where the curve stops fitting the data.

slopes, d $(\Delta R/R_0^2)/\mathrm{d}\ln H$ are too large by almost one order of magnitude to be explained by the spin–orbit interaction only.

To interpret our results, we thus consider Larkin's formula for the MR in the presence of superconducting fluctuations:

$$\Delta R_\square/R_\square^2 = -(e^2/2\pi^2\hbar)(\alpha - \beta(T))Y(H/H_0). \tag{1}$$

where $Y(x)$ is the digamma function entering the usual formula for the MR (Lee 1982, Fukuyama 1982). H_0 is the crossover field where the Landau radius, $L_L(H)$ is of the order of the inelastic scattering length, L_ε (L_ε is related to the inelastic mean time, τ_ε, by the relation $L_\varepsilon^2 = D\tau_\varepsilon$). Superconducting fluctuations result in the change of α into $(\alpha - \beta(T/T_c))$, where β is a function of T/T_c only which diverges when its argument approaches unity.

When neglecting spin–orbit coupling, $\alpha = 1$, while in the limit of strong spin–orbit scattering, which we have argued could be the case for our samples, $\alpha = -\frac{1}{2}$ (Maekawa and Fukuyama 1981, Deutscher and Fukuyama 1982).

We have fitted the data to (1) using a least-squares fitting method and for each sample have extracted the parameters $(\beta - \alpha)$ and H_0, summarised in table 1. Figure 3 shows the experimental points and the fitted curve. At high fields a considerable deviation from (1) was observed in fitting the data. Therefore, the fitting procedure was started using *only low-field data points*, and including higher-field points as long as the resulting H_0 and $(\beta - \alpha)$ remained constant. Thus, for each sample the fit was stopped at a different field value: H_{c2}^*. We interpret H_{c2}^* as the field where β becomes *dependent on field*, i.e. the field that suppresses the amplitude of the superconducting fluctuations.

The good fit to (1) obtained in the low- and intermediate-field regions (see e.g. figure 4) is a confirmation that our strong-spin–orbit assumption is correct. Otherwise, we

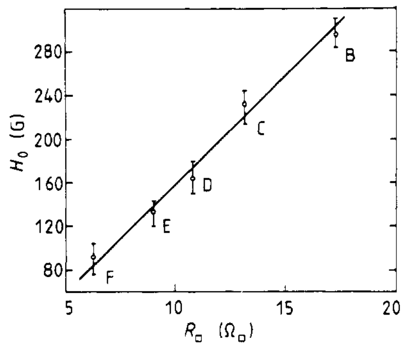

Figure 4. H_0 extracted from the fit against $R_\square$.

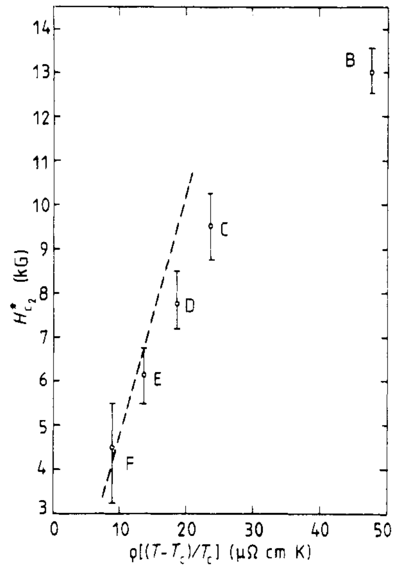

Figure 5. $H^*_{c_2}$, the critical field where the deviation from the fitted curve starts, versus the sample resistivity. The broken line is drawn with the slope that intersects the pure In parameters (T_c = 3.4 K and $H^*_{c_2} \simeq 300$ G).

should find different behaviour in the low-field and strong-field regions (Deutscher and Fukuyama 1982).

The linear dependence of the H_0 on $R_\square$ (figure 5) is as expected for homogeneous samples if τ_ε is independent of D. According to the theory of Abrahams *et al* (1979), one should have:

$$H_0 = \hbar c/4eD\tau_\varepsilon = \phi_0/4\pi D\tau_\varepsilon \tag{2}$$

where ϕ_0 is the flux quantum.

The values of L calculated from H_0 vary from $\simeq 1200$ Å for sample F to 400 Å for sample A. This justifies the assumption that our films are essentially two-dimensional.

4. Inhomogeneity and superconducting fluctuations

This linear dependence of H_0 on $R_\square$ for samples F through to B is indicative that these are homogeneous *on the scale of* L_ε. This agrees with our assumption about the relevant homogeneity scale, since for these samples $\xi_p < L_\varepsilon$. The deviation observed for sample A shows that it is no longer in the homogeneous limit. In this case the relation $L^2 = D\tau_\varepsilon$ is no longer valid because, on the time scale τ_ε, D has become dependent on time (Gefen *et al* (1983) and references therein). Qualitatively, the diffusion length remains finite at short times near the MI transition, which should indeed lead to a saturation of $H_0 \propto L_\varepsilon^{-2}$. The value of L_ε obtained for sample A for H_0 = 840G is 400 Å, which must then be considered as an effective inhomogeneity length scale in our samples. This is not unreasonable, since it is smaller than ξ_p mentioned in § 2.

We interpret the field $H^*_{c_2}$ as the characteristic field that suppresses the superconducting fluctuations. Since the characteristic spatial extension of these fluctuations is $\xi(T)$ (see e.g. Tinkham 1975), we expect $H^*_{c_2} \simeq \phi_0/2\pi\xi^2(T)$ or

$$H^*_{c_2} \simeq \phi_0[(T - T_c)/T_c]v_F/2\pi(0.855)^2\xi_0 D \qquad (3)$$

where D is the (2D) diffusion constant, and is related to the conductivity via the Einstein relation $\sigma = e^2N(0)D$. Here we have taken into account the normalisation factor $2^{1/2}$ for $\xi(T)$ above and below T_c.

We therefore expect $H^*_{c_2}$ to be linear in $\rho(T - T_c)/T_c$ *as long as* $\xi(T)$ *is larger than the characteristic inhomogeneity scale.* This is indeed almost verified for the low-R samples (see figure 5).

Hence, from the standpoint of superconductivity, we find that our samples are essentially in the crossover region from homogeneous to strongly inhomogeneous. It is remarkable that this conclusion agrees quantitatively with that reached by the study of the upper critical field H_{c_2} in In–Ge films *below* T_c (Deutscher *et al* 1982).

The second parameter extracted from the data is the variation of $(\beta - \alpha)$ with T_c. Comparing it with the theoretical $(\beta + \frac{1}{2})$ value (taken from Larkin (1980) and assuming $\alpha = -\frac{1}{2}$) the agreement seems in general to be poor, except in the case of sample B. We propose that this inconsistency is due to the complicated structure of our samples.

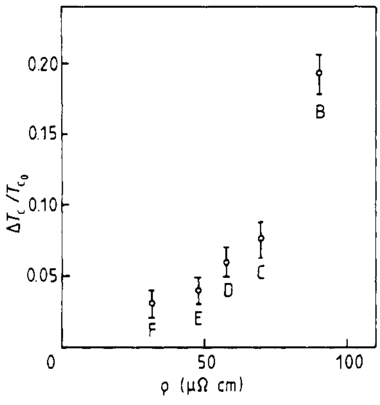

Figure 6. The variation of the superconducting transition temperature with the normal-state resistivity.

In figure 6 we show the variation of T_c with the sample resistivity. It is seen that for the dirtier samples the change is large (samples A and B) compared with that for the cleaner samples (E and F). The lowering of T_c could be attributed to weak links and Josephson couplings (Entin-Wohlman *et al* 1981) in the extreme inhomogeneous samples (A and B), but for samples E and F it may be due to electron localisation (Maekawa and Fukuyama 1983) which gives $\Delta T_c/T_c \propto \rho$. A quantitative comparison requires a precise knowledge of the interactions and microscopic parameters and therefore could not be performed.

5. Discussion of the homogeneous limit

We can combine (2) and (3) to calculate τ_ε from the values of H_0 and $H_{c_2}^*$ in the homogeneous limit:

$$\tau_\varepsilon = [\xi_0(0.855)^2/2v_F]/H_{c_2}/H_0[(T - T_c)/T_c]. \tag{4}$$

Assuming sample F to be in the homogeneous limit (both on the scale of L_ε and on that of $\xi(T)$), we can calculate τ_ε (see table 2) using the bulk In values for ξ_0 and v_F ($\xi_0 = 4.4 \times 10^{-5}$ cm, $v_F = 1.8 \times 10^8$ cm s^{-1}). We then obtain $\tau_\varepsilon = 1.7 \times 10^{-11}$ s. (Repeating the same procedure for samples E through to B gives a systematic error on τ_ε due to the inhomogeneity of the samples on the scale $\xi(T)$. This time can now be used to calculate the *effective* value of D through equation (3), for samples F through to B, which are 'homogeneous' on the scale L_ε, assuming τ_ε to be independent of D (Bergmann 1982).

Table 2. Sample microscopic properties.

Sample	A	B	C	D	E	F
$\tau_\varepsilon(10^{-11}$ s)†		0.82	1.16	1.43	1.50	1.70
D(cm^2 s^{-1})‡		6.8	6.0	7.0	7.9	9.7
ρ (μΩ cm)§	203.5	68.3	55.3	38.5	32.2	22.3
Measured ρ (μΩ)	705	90.8	70.0	58.1	47.7	31.8

† Calculated from the ratio H_{c2}^*/H_0.
‡ Using τ_ε, calculated from H_0.
§ Calculated using Einstein relation and D and using $\tau_\varepsilon = 1.6 \times 10^{-11}$ for all samples.

We find this effective value to be roughly a factor of 1.5 larger than what was calculated from the measured value of R and the Einstein relation using the *bulk* value for $N(0)$. In other words, it appears that localisation effects in the macroscopically homogeneous limit are the same as in the microscopically homogeneous limit, provided a properly normalised (reduced) density of states is taken into account.

This is confirmed when we compare the values of $(\beta - \alpha)$ obtained from our fit to those calculated from the theory of Larkin (table 1). Values obtained from the fitted curves use the measured values of $R_\square$, and are smaller than Larkin's values. Agreement with theory would require an effective $R_\square$ smaller than the measured one by the same factor, 1.5. Again it seems that the coefficient of diffusion is the fundamental scaling parameter, rather than the measured value of $R_\square$.

An interpretation of the values of $(\beta - \alpha)$ obtained for high-resistivity samples must await a more complete understanding of the properties of inhomogeneous systems.

6. Conclusion

In conclusion we have shown that sample inhomogeneity can affect physical properties such as electron localisation and the superconducting transition. The basic result is that as long as the characteristic length imposed by homogeneity is small compared with the relevant physical length (the inelastic diffusion length or the superconducting coherence length in our case) the material under investigation is essentially homogeneous with the appropriate correction of the density of states.

Our detailed investigation now calls for much more precise measurements of the influence of inhomogeneity on other electronic processes, such as the electron–electron interaction.

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