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LETTER TO THE EDITOR

Conductivity measurement on percolation fractal

Alexander Palevski and Guy Deutscher
Department of Physics and Astronomy, Tel-Aviv University, Tel-Aviv, Israel

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Abstract. DC conductivity measurements on percolating networks were performed on scales smaller than the correlation length ξ_p. The scale dependence follows a power law from which we deduce $\mu/\nu=0.95$, where μ and ν are respectively the conductivity and connectivity indices.

In recent years there has been much interest in the transport properties of percolation systems near the percolation threshold, since percolation is one of the mechanisms for the description of metal–insulator transitions (Deutscher *et al* 1983). In particular, random walks on percolation fractals (Mandelbrot 1982) and related to it the fracton dimension (Alexander and Orbach 1982) were studied in detail using different methods of computation (Ben Avraham and Havlin 1982, Pandey and Stauffer 1983, Angles d'Auriac *et al* 1983) and good agreement was found between the random walk exponent and the dynamical scaling prediction (Gefen *et al* 1983). The anomalous diffusion on fractals was studied by computing the variation of the average end-to-end distance of a random walk with the number of steps (Ben Avraham and Havlin 1982, Pandey and Stauffer 1983). The fracton dimensionality was measured by computing the average number of distinct visited sites (Angles d'Auriac *et al* 1983). However, the dynamical properties of fractals have never been studied in real systems. This letter reports on the first conductivity measurements in a real 2D percolation system on scales L where the anomalous diffusion regime is observed. The ratio μ/ν was evaluated and the crossover from $L>\xi_p$ to $L<\xi_p$ was observed.

According to percolation theory, the (DC) conductivity of the metal–insulator mixture σ and the probability of belonging to the infinite cluster P_∞ approach zero as $\sigma\propto(p-p_c)^\mu$ and $P_\infty\propto(p-p_c)^\beta$ respectively, when the concentration p of the conducting constituent approaches its critical value p from above ($p>p_c$). Since the percolation correlation length diverges as $\xi_p\propto|p-p_c|^{-\nu}$, the variation of the DC conductivity can be expressed in terms of ξ_p as $\sigma\propto\xi_p^{-\mu/\nu}$. Since on self-similar scales any physical quantity should be independent of ξ_p, one can apply finite-size scaling considerations to deduce the scale dependences of physical quantities in powers of L (Deutscher *et al* 1983) for $L\ll\xi_p$. One thus finds $\sigma\propto L^{-\mu/\nu}$ and for the diffusion coefficient $D\propto L^{-\theta}$, where θ relates to other exponents through the relation (Gefen *et al* 1983): $\theta=(\mu-\beta)/\nu$. While ν is known exactly, $\nu=\frac{4}{3}$ (Nienhuis 1982), numerical values of μ obtained from theoretical (including computer) studies (Skal and Shklovski 1974, de Gennes 1976, Straley 1978) as well as from real experiments (Last and Thouless 1971, Abeles *et al* 1975, Coutts 1976, Deutscher *et al* 1978) have ranged from 0.9 to 1.4. Recent theoretical values in 2D are 1.274 (Alexander and Orbach 1982), 1.3 (Zabolitzky

1984) and 1.31 (Adler 1984) while a recent experiment gives 1.25 ± 0.05 (Palevski *et al* 1984).

It has previously been reported that structures of Pb on Ge (Kapitulnik and Deutscher 1982) and Au thin films (Voss *et al* 1982) exhibit percolation characteristics. However, one realises that a direct measurement of the conductivity on scales $L < \xi_p$ on metal–insulator thin mixture films is extremely difficult, since it implies fabrication of at most 100 Å wide microcontacts separated by distances varying from 1 μm down to a few 100 Å. We have found an alternative solution by reproducing the original percolation network on much larger scales. This has been achieved by using a TEM (transmission electron microscope) picture of the thin film taken at a magnification of 40 000 in combination with photolithographic techniques. The TEM picture was used as a mask to perform lift-off photolithography on Cu or Al films with typical thicknesses of 100 Å. The samples produced have a geometrical shape identical to that of the original Pb on Ge film but on a much larger scale as shown in figure 1.

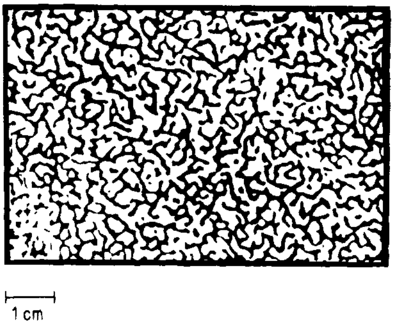

Figure 1. Sample produced using original micrograph as a mask for photolithography.

After all finite clusters were eliminated, the resistance R of squares ($L \times L$) was measured as a function of L. Contact lines of length L were made with silver paint whose resistance never exceeded ~1% of the film's resistance. The cluster inside the measured square was isolated by disconnecting the two other sides of the square from the rest of the sample. The centres of the squares were always located on the infinite cluster.

$R(L)$ is shown in figure 2. It is seen clearly that below a certain scale, which we claim is ξ_p, the resistance shows huge fluctuations which survive until the length becomes of the order of the unit grain size a (the channel's thickness). There are range of length scales over which the resistance is infinite, i.e. there is no channel connecting the opposite sides of the square. Hence the resistance in this experiment is not a statistically averageable quantity. Instead we averaged the conductance as an empirical choice, and the result of averaging over ten different central points of the same sample is given in figure 3. The curve is quite smooth, but since the crossover region is very broad, it is impossible to deduce the value of μ/ν from this measurement.

The same sample was studied using a different electrode geometry. One contact was placed on a point located on the infinite cluster and the second was placed on a circle of radius L centred on that point. This method has several advantages; (a) there is no need to eliminate finite clusters; (b) it is not necessary to isolate the inner cluster; (c) each measurement gives a smooth curve $R = f(L)$ without averaging over many

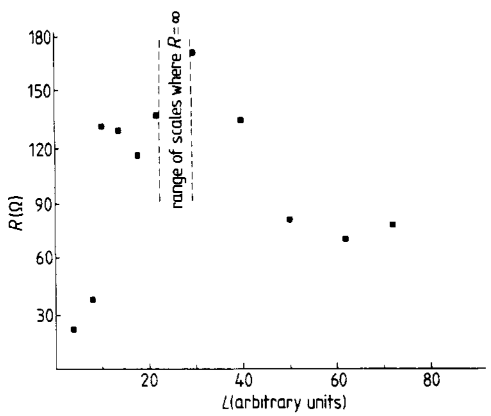

Figure 2. A typical result of a $R(L)$ dependence measured using method of squares ($L\times L$) for one central point.

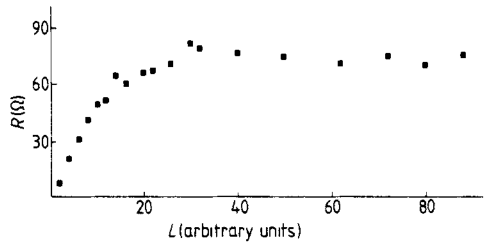

Figure 3. $R(L)$ curve for the sample with ξ_p smaller than the size of the substrate (after averaging over ten central points).

central points. However, the data points of this measurement (not shown) also exhibited a broad crossover thus not allowing a determination of the critical exponent.

Another sample with ξ_p larger than the size of the sample was studied using the latter method. In figure 4, we show the results of this measurement. The slope in the log–log insert gives $\mu/\nu = 0.95 \pm 0.05$. This value agrees well with the latest results for μ/ν. The Alexander–Orbach (AO) rule yields $\mu/\nu = 0.948$, while the latest value for μ/ν (Adler 1984) yields $\mu/\nu = 0.985$. The accuracy of our measurement is not sufficient to favour one of the theoretical values but we hope that using this method for large scales and better statistics the value of μ/ν can be measured very accurately.

Finally, we hope that the method for producing samples described in this letter can be useful for further study of critical phenomena. This method can be combined with computer simulations. It is easy to produce pictures of the percolation network with well known values of the concentration p close to p_c. Converting them into metal–insulator samples, as we did here, one can perform conductivity, magnetoresistance, Hall effect, heat flow (diffusion) etc measurements on both homogeneous and non-homogeneous scales.

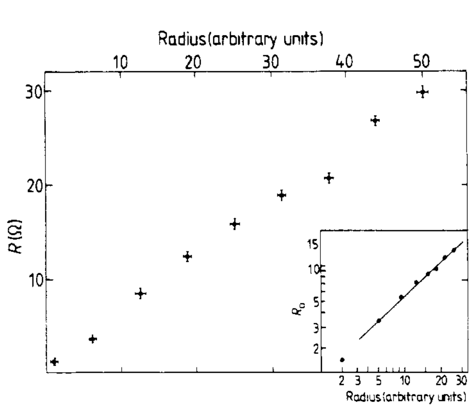

Figure 4. Variation of $R(L)$ for a sample with ξ_p larger than the size of the substrate (measured using the circle method). The insert shows slope $\mu/\nu = 0.95 \pm 0.05$.

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