

A note on the relation between the real rate and the central bank's output target under perfect and imperfect information

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Preliminary

1 Introduction

A widely observed phenomenon is that real rates of interest tend to be high during periods of disinflation and Israel is no exception to this empirical regularity. Data on real rates during disinflation supporting this assertion for the OECD countries appears, inter alia, in Table 2.10 of Liviatan (2003). This note utilizes a New Keynesian framework to explore whether the increase in real rates can be explained by a decrease in the output target of the CB from a level above potential output to the potential level.

The first section shows that, when the public is fully informed about this change (perfect credibility) the reduction in the output target of the CB cannot explain the increase in the real rate. The second section shows that when the public does not fully recognize the reduction in the output target (imperfect credibility) the real rate **does** increase, on average, above its natural level. Another possible explanation for high real rates during disinflations relies on the liquidity effect. The note concludes with brief comments on the relation between those two

*This note was triggered by discussions with Nissan Liviatan.

mechanisms and argues that they are likely to be complementary. In particular their interaction appears to provide a potential explanation for the persistence of above natural real rates in the wake of reduced output targets on the part of the CB.

2 Invariance of real rates to the CB output target under full information

One aspect of a high effective degree of conservativeness or independence is that the output target of the CB is close to potential output and possibly even identical to it. Broad intuition suggests that there should be a negative relation between the real rate set by the CB and the magnitude of the (non negative) difference between the CB output target and potential output. However it turns out that, under full information this intuition is false. Admittedly, for given inflationary expectations, an increase in the output target induces the CB to reduce the real rate. However this change also raises inflationary expectations, and this induces the Bank to raise the real rate in a compensating manner. This benchmark result is illustrated here within a New Keynesian framework due to Clarida, Gali and Gertler (1999) (CGG). The behavior of the economy is described by

$$x_t = -\varphi(i_t - E_t\pi_{t+1}) + E_tx_{t+1} + g_t \quad (1)$$

$$\pi_t = \lambda x_t + \beta E_t\pi_{t+1} + u_t \quad (2)$$

Here x_t and π_t are the output gap and inflation, E_tx_{t+1} and $E_t\pi_{t+1}$ are the expected values of those variables conditioned on the information available in period t , i_t is the short term nominal rate of interest, g_t is a demand shock, u_t is a cost shock and φ , λ and β are positive coefficients. The stochastic behavior of the two shocks is stationary but persistent, and is given by

$$\begin{aligned} g_t &= \mu g_{t-1} + \hat{g}_t & 0 < \mu < 1 \\ u_t &= \rho u_{t-1} + \hat{u}_t & 0 < \rho < 1. \end{aligned} \quad (3)$$

Here \hat{g}_t and \hat{u}_t are the innovations to the cyclical components of demand and of costs respectively, and μ and ρ characterize the persistence of those shocks. The first equation states that the output gap is negatively related to the ex ante real rate of interest and positively related to the expected future output gap. The latter appears in the output gap equation to reflect the notion that, since individuals smooth consumption, expectations of higher consumption next period (associated with higher expected output) leads them to demand more current consumption, which raises current output. In this framework the CB affects the output gap and inflation through the choice of the short term nominal rate of interest which, given expected inflation, determines the short term real rate. Note that the CB affects inflation through the output gap.

As in stylized models of sticky staggered prices pioneered by Calvo (1983), current inflation depends on future expected inflation. In this type of model only a fraction of firms has the opportunity to adjust its price each period and, due to costs of price adjustment, each firm adjusts its price at discrete intervals. Hence when it is given the chance to adjust its price the firm adjusts it by more the higher is expected future inflation. This interpretation implies that β is a discount factor. The positive dependence of inflation on the output gap is based on the view that this gap is a measure of excess demand and is shared by both forward, as well as backward, looking models of the economy in which output is demand determined.

One of the dimensions of effective conservativeness or independence is that the non negative discrepancy between desired and potential output is small or even zero. For simplicity and without loss of generality I will consider two extreme types of central bankers. One that incurs losses whenever the output gap deviates from a target that is k percent above potential and another type whose output target is equal to potential output. Obviously, the second CB is more conservative than the first in the sense that it is satisfied with a lower output target. We may therefore refer to it as a relatively "level conservative" CB to distinguish it from Rogoff's (1985) conception of conservativeness which refers to the relative importance attributed to the stabilization of inflation and of output. Svensson has renamed the latter type of conservativeness as "weight conservativeness". Whenever the term "conservativeness" alone appears in this note it should be understood as referring to the first type of conservativeness.

Both CB types examined here dislike deviations of inflation and of output from their respective targets and differ from each other only in their output targets. The objective of

either type is to minimize

$$\frac{1}{2}E_0 \sum_{t=0}^{\infty} \delta^t [\alpha(x_t - k)^2 + \pi_t^2] \quad (4)$$

where δ is a discount factor and α is the relative importance attributed by both types to the stabilization of output. Here $k > 0$ for the more liberal (or more dependent) CB and $k = 0$ for the more conservative (or more independent) CB. We shall proceed by solving this problem for any k and then compare the levels of the real rate chosen by each type.

Under discretion a CB of "type k " chooses in each period the nominal interest rate, i_t , and a contingent path for future interest rates so as to minimize the expression in equation (4) subject to the structure of the economy in equations (1), (2) and (3) and taking expectations as given. The Euler equations necessary for minimization of this problem are given by

$$x_t - k = -\frac{\lambda}{\alpha}\pi_t, \quad t = 0, 1, 2, \dots \quad (5)$$

This condition states that, in each period the marginal cost of deviating from the output target is equated to the marginal cost of deviating from the inflation target (which is zero). Note that this condition depends on the effective level of independence (or conservativeness) of the CB as characterized by the size of k . In particular a positive k implies that the CB is willing to tolerate a positive deviation of inflation from its target even when output is at its potential level. Substituting equation (5) into the inflation equation (2) one can express current inflation in terms of next period's expected inflation and of the current cost shock u_t . Iterating this equation forward, imposing rational expectations, and using equation (3) repetitively yields

$$\pi_t^k = \alpha q u_t + \frac{\alpha}{\lambda} k \quad (6)$$

where

$$q \equiv \frac{1}{\lambda^2 + \alpha(1 - \beta\rho)}. \quad (7)$$

and the superscript " k " attached to inflation and to other variables in the sequel highlights the fact that the equilibrium values of those variables may depend on k . Combining equation (6)

with equation (5) it is possible to also express the optimal level of x_t in terms of u_t . This yields

$$x_t^k = -\lambda q u_t. \quad (8)$$

Substituting this expression into the output gap equation, (1), and rearranging, it is possible to backup the optimal level of the real rate of interest

$$r_t^k \equiv i_t - E_t \pi_{t+1} = \frac{(1-\rho)\lambda}{\alpha\varphi\rho} E_t \pi_{t+1}^k + \frac{1}{\varphi} g_t - \frac{(1-\rho)}{\varphi\rho} k. \quad (9)$$

For a relatively independent (or conservative) CB, $k = 0$ and equations (6) and (9) reduce to

$$\pi_t = \alpha q u_t \quad (10)$$

$$r_t \equiv i_t - E_t \pi_{t+1} = \frac{(1-\rho)\lambda}{\alpha\varphi\rho} E_t \pi_{t+1} + \frac{1}{\varphi} g_t. \quad (11)$$

respectively while equation (8) remains the same. Here r_t denotes the real rate chosen by a relatively independent CB whose output target is equal to the potential level and $E_t \pi_{t+1}$ denotes the level of inflationary expectations when the public is fully aware of the fact that the CB targets potential output. Equations (6) and (10) imply that

$$E_t \pi_{t+1}^k = \alpha q \rho u_t + \frac{\lambda}{\alpha} k = E_t \pi_{t+1} + \frac{\alpha}{\lambda} k. \quad (12)$$

Using equation (12) in equations (9) and (11) **both** of those expressions reduce to

$$r_t^k = r_t \equiv i_t - E_t \pi_{t+1} = \frac{(1-\rho)\lambda}{\varphi} q u_t + \frac{1}{\varphi} g_t. \quad (13)$$

Hence, when the public is fully informed about the output target of the CB, the path of the real rate is independent of this target. This invariance is a consequence of two opposing effects that exactly offset each other. Taking the case $k = 0$ as a benchmark and raising it to a positive value triggers two opposing effects on the real rate. For given inflationary expectations the real rate goes down as the CB tries to push output above potential (equation (9)). But the increase in k also raises inflationary expectations by $\frac{\alpha}{\lambda} k$ inducing the CB to raise the interest rate back

by an equivalent amount (equations (12) and (9)). The upshot is that, when the public is fully informed about the output target of the CB, the real rate of interest is at its natural Wicksellian level on average. This can be seen formally by noting, from equation (13), that the expected value of r_t^k is zero.

3 The effect of imperfect credibility on the real rate

I turn now to the case in which there is a discrepancy between the true output target of the CB and the public's beliefs about this target. Situations like that are common during periods of inflation stabilization characterized by a decrease in the output target of the CB. In cases of this type the public that has been accustomed to the high output target does not adjust its perception of the bank's output target downward for some time. I refer to such conditions as imperfect credibility. This section shows that, during periods of imperfect credibility, the real rate of interest is above its natural level.

Consistent modeling of imperfect credibility requires some adjustment in the informational assumptions made in CGG. In particular, if the public has full current information about all economic variables and shocks (as CGG assume) it can immediately back up the current value of k from the interest rate, thus eliminating imperfect credibility. To focus on situations of imperfect credibility I assume that, except for i_t that is observed contemporaneously, the public observes all variables and shocks with a one period lag while the CB still observes them contemporaneously. This represents a minimal deviation from the original framework and prevents the public from immediately inferring the output target of the CB from the observation on the current interest rate. It also moves the CGG framework a bit towards realism.

Suppose that, after a sustained period during which it had an output target above potential ($k > 0$) the output target of the CB is set equal to potential output ($k = 0$). However the public's perception of the target, denoted k^e , remains positive. Since the actual value of k is 0, the CB behaves so as to satisfy condition (5) above with $k = 0$.

$$x_t = -\frac{\lambda}{\alpha}\pi_t, \quad t = 0, 1, 2, \dots \quad (14)$$

Combining this condition with the Phillips relation in equation (2) and rearranging yields

$$\pi_t = \frac{\alpha}{\alpha + \lambda^2} (\beta E_t \pi_{t+1} + u_t). \quad (15)$$

Thus, as in CGG current inflation depends on the rate of inflation expected for next period. But, unlike in CGG, the public believes that the CB will behave in period $t + 1$ so as to satisfy the condition

$$x_{t+1}^e - k^e = -\frac{\lambda}{\alpha} \pi_{t+1}^e, \quad t = 0, 1, 2, \dots \quad (16)$$

where the superscript "e" denotes the values of the respective variables as currently perceived by the public. CGG show (see their equation (4.4)) that when the CB behaves according to equation (16) actual inflation is given by

$$\pi_{t+1} = \alpha q u_{t+1} + \frac{\alpha}{\lambda} k. \quad (17)$$

Since, except for the fact that they do not know k and do not possess full current information, expectations are formed rationally, a similar relation characterizes the public's current perceptions.¹ Taking expected values of both sides of equation (17) conditional on the information available to the public in period t

$$\pi_{t+1}^e \equiv E_t \pi_{t+1} = \alpha q \rho u_{t+1}^e + \frac{\alpha}{\lambda} k^e. \quad (18)$$

Substituting equation (18) into equation (15), noting (from equation (3)) that $u_{t+1}^e = \rho^2 u_{t-1}$, and rearranging

$$\pi_t = \alpha q \rho u_{t-1} + \frac{\alpha}{\alpha + \lambda^2} \hat{u}_t + \frac{\alpha^2 \beta}{\lambda(\alpha + \lambda^2)} k^e. \quad (19)$$

This equation describes the behavior of the actual rate of inflation when the public's belief is given by $k^e > 0$ in spite of the fact that the CB is targeting potential output. Substituting

¹Note that equation (17) does not characterize the **actual** behavior of the CB (which is given by equation (14) led by one period). Equation (17) is used only to characterize the public's beliefs about the relation between next period's inflation and next period's cost shock, given that the public believes that k is positive.

equation (19) into the first order condition of the CB (equation (14)) and rearranging

$$x_t = -\lambda q \rho u_{t-1} - \frac{\lambda}{\alpha + \lambda^2} \hat{u}_t - \frac{\alpha \beta}{\alpha + \lambda^2} k^e. \quad (20)$$

This equation describes the behavior of the output gap when the public's belief is given by $k^e > 0$ in spite of the fact that the CB is targeting potential output. Finally, to find the behavior of the real rate of interest we substitute equation (20) into equation (1) and rearrange.

$$r_t = \frac{1}{\varphi} \left[\lambda q \rho u_{t-1} + E_t x_{t+1} + \frac{\lambda}{\alpha + \lambda^2} \hat{u}_t + g_t + \frac{\alpha \beta}{\alpha + \lambda^2} k^e \right]. \quad (21)$$

Equation (8) above provides an equilibrium expression for the output gap for all values of k . Since, except for the fact that they do not know k and do not possess full current information expectations are formed rationally, a similar relation characterizes the public's current perceptions. That is

$$x_t^e = -\lambda q u_t^e. \quad (22)$$

Equations (22) and (3) imply that

$$E_t x_{t+1} = -\lambda q \rho^2 u_{t-1} \quad (23)$$

Substituting equation (23) into equation (21) we obtain

$$r_t = \frac{1}{\varphi} \left[\lambda q \rho (1 - \rho) u_{t-1} + \frac{\lambda}{\alpha + \lambda^2} \hat{u}_t + g_t + \frac{\alpha \beta}{\alpha + \lambda^2} k^e \right] \quad (24)$$

from which it follows that the expected value of the real rate of interest is given by

$$E r_t = \frac{\alpha \beta}{\varphi (\alpha + \lambda^2)} k^e \quad (25)$$

which is positive as long as k^e is positive. Thus, as long as the public believes that the CB targets an output level that is above potential the CB is led to choose an interest rate that is above the Wicksellian natural rate. The positive value of k^e leads to a systematically positive expected inflation and this induces the CB to maintain a real rate that is above the natural

rate. The corresponding average values of inflation and of the output gap are, from equations (19) and (20), are

$$E\pi_t = \frac{\alpha^2\beta}{\lambda(\alpha + \lambda^2)}k^e. \quad (26)$$

$$Ex_t = -\frac{\alpha\beta}{\alpha + \lambda^2}k^e. \quad (27)$$

Thus, due to imperfect credibility inflation is above target and output is below its potential level so that the economy is in a recession. There is a certain analogy between the last result and a result from the earlier literature on the inflation bias within flexible price frameworks in the presence of imperfect information about the ability to commit. In particular, the frameworks of Barro (1986) and Cukierman and Liviatan (1991) imply that, in the presence of uncertainty about whether a preannounced inflation target represents a commitment on the part of policymakers or whether it is just cheap talk, a truly committed policymaker produces a recession.² However, it was not possible to make definite statements about the effect of imperfect credibility on the real rate in those frameworks because they did not explicitly feature the interest rate.

3.1 A possible extension

One could argue that the contemporaneous observation of the interest rate from equation (24) can be used by the public to sharpen its perception of the value of k . One way to extend the model in order to account for such a possibility is to reformulate k as a stochastic variable and to recognize that, from equation (24), an observation on the interest rate amounts to an observation on a linear combination of k and of the innovations, \hat{u}_t and \hat{g}_t to costs and aggregate demand. In such an extended framework k^e will be equal to the statistical prediction of k from a regression of k on this linear combination of stochastic variables.

It is not too difficult to see that, although such an extension will change some of the model's details, it will not change the basic conclusion that, a decrease in k from a positive value to zero, will be followed by a period of above natural real rates of interest. The argument supporting this claim follows. Since the coefficient of the regressor in the optimal prediction

²See also Cukierman (2000).

equation is between zero and one (as is the case, for example, in Lucas (1973)) **only part** of any given change in k is perceived by the public. As a consequence, following an observation on the contemporaneous interest rate by the public, k^e will decrease below k but will still remain positive, implying through an extended version of equation (25) that the real rate will still be, on average, above its natural value. The precise drafting of such a model is left for future work.

4 Concluding reflections

Besides Israel during the second half of the nineties, the phenomenon of high real rates has been quite frequent during periods of inflation stabilization. During Volcker's disinflation in the early eighties the US experienced some of the highest real rates it had over the twentieth century and many other OECD countries had relatively high real rates during their disinflations.

Another, not mutually exclusive, explanation for high real rates during periods of disinflation is that the restrictive monetary policies enacted during such times lead, by lowering the rate of growth of the money supply in the presence of sticky prices and wages, to a decrease in the real money supply. This leads to an excess demand for money causing an increase in the real rate of interest. This is the well known liquidity effect that accompanies restrictive monetary policies. It is likely that in the Israeli case during the second half of the nineties both mechanisms operated. An interesting challenge for future work is to get some feel about the relative contribution of imperfect credibility and of the liquidity effect to the increase in real rates during periods of disinflation.

In a broader sense a combination of both mechanisms is likely to do a better job in explaining the rather long time that Israeli interest rates had to be maintained above their natural level during disinflation. The reason is that the liquidity effect operates only as long as the prices and wages that have been set prior to stabilization have not been adjusted. A reasonable upper bound for adjustment of most nominal magnitudes seems to be about two years. Most prices are adjusted at higher frequencies and nominal wage contracts rarely extend beyond two years.

However, in Israel the period of high real rates that started in 1994 extended substantially beyond the two year bound. A possible explanation is that, due to imperfect credibility, when

prices and wages were adjusted, they were reset at levels that were too high in view of the subsequent policies of the CB. This triggered a second round of restrictive liquidity effects on the real rate on top of the direct upward impact of imperfect credibility of the type analyzed in the previous section. In fact the economy may go through a number of such (hopefully convergent) rounds before credibility is finally established allowing the CB to ultimately reduce the average level of the real rate to its natural level.

Note that the longer is the time period over which the real rate is maintained above its natural level following a disinflation attempt, the more likely it is that the CB will not persist with the disinflation. This is due to political economy reasons. The longer the period of unusually high real rates the stronger the pressures on the CB to deal with this problem by liquidity injections. This mechanism may extend the period of imperfect credibility and its vicious externality with the liquidity effect. Thus, through this route, the interaction between imperfect credibility and the liquidity effect provides a mechanism that amplifies the persistence of above natural real rates following the CB decision to reduce its output target.

5 References

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