RAPID INFLATION - DELIBERATE POLICY OR MISCALCULATION?

ALEX CUKIERNAN*
Tel-Aviv University

1. INTRODUCTION

High inflations, particularly when they reach the hyperinflation stage, seem like bizarre events. Inflation imposes costs on society and existing evidence suggests that in most, if not all hyperinflations, steady state real revenues from monetary expansion could have been increased by decreasing the rate of inflation. This raises doubts about the rationality of governments that inflate at high rates or, at the very least, about their understanding of the way the economy operates. A basic question is why these governments inflate at high and accelerating rates only to ultimately stabilize by changing the monetary regime. Answering this and related questions requires better understanding of the motives and constraints facing the inflating governments.

Allan Meltzer has long been committed to a line of investigation that tries to bridge the gap between economics and politics in order to produce a better understanding of how economic policy is actually made. This paper follows this tradition by trying to uncover some of the motives, institutional constraints, and historical conditions that may induce government to follow policies whose ultimate consequence is very rapid or even hyperinflation. The central theme is that, although policy errors may be part of the story, a substantial part of hyperinflationary policies can be understood in terms of deliberate choices made by rational governments.

*Tel-Aviv University. I benefited from useful discussions with Alberto Alesina, Karl Brunner, Zvi Eckstein, Robert Flood, Jacob Frenkel, Peter Garber, Zvi Hercowitz, Carl Helfferich, Finn Kydland, Allan Meltzer, Manfred Neumann, Pierre Siklos, Guido Tabellini, and Juergen von Hagen. The usual disclaimer applies. The Horowitz Institute project on Central Banking provided partial financial assistance.
The recent macro-policy-oriented literature has identified several motives that tempt governments to inflate. Among the most prominent are the employment and the revenue motives. This paper focuses exclusively on the latter motive which probably was the major driving force during the classic post World War I hyperinflations as well as during the recent Bolivian hyperinflation.¹

Section 2 opens the paper by presenting evidence on the behavior of revenues from seigniorage in Germany during the well-known post World War I hyperinflation. A puzzling feature is that those revenues tended to increase, on average, as the rate of inflation increased contrary to what is implied by available estimates of the demand for money in Germany during the hyperinflation. This result is robust to alternative ways of evaluating seigniorage. A related regularity is that actual real money balances move substantially more smoothly than those that are predicted by making use of any of the available estimates of money demand under perfect foresight.²

Section 3 presents indicators for the behavior of inflationary expectations during the German hyperinflation. These indicators are drawn from the forward exchange market and from the domestic credit market. Both indicators suggest that expectations seriously underestimated actual developments particularly in 1923 when the already high rate of monetary expansion climbed to unprecedented heights.³

Section 4 presents a framework in which government's willingness to trade higher inflation for seigniorage revenues undergoes stochastic changes over time and in which government has more accurate information about the persistence of those changes than the public. Because of this information structure the public learns gradually but optimally about changes in government's inflationary intentions. Government cares about both the present and the future but may have a positive time preference. Due to the slow adjustment of expectations current seigniorage revenues can be increased by raising monetary growth even if such policy eventually decreases seigniorage. Governments that become more desirous of immediate seigniorage revenues may find it rational to follow such a policy. This result provides a resolution of the seeming paradox that most high inflation governments ultimately operated their economies in a range of expected inflation in which a decrease in inflation would have increased steady state revenues from seigniorage.⁴ It also resolves the puzzle raised by the positive correlation between monetary expansion and seigniorage revenues during much of the German hyperinflation and provides an explanation for the large and somewhat persistent downward biases in expectations within an optimal forecasting framework. Last but not least the discussion in Section 4 is consistent with the view that even during hyperinflationary episodes the governments in charge were following deliberate but rational (given their objectives and constraints) inflationary policies. The tremendous inflationary acceleration which occurred in Germany in 1923 is rationalized in terms of an increase in government's need for immediate seigniorage revenues following the Ruhr's invasion.

Section 5 provides an explanation for the fact that hyperinflations are allowed to accelerate for a while only to be finally stabilized. The main idea is that due to expectational and other lags, an acceleration in the rate of monetary expansion can temporarly increase seigniorage even if it ultimately decreases it. Policymakers with strong time preference for seigniorage may find it profitable to engage in such a policy as long as expectations have not adjusted. But once this adjustment takes place and seigniorage revenues drop below a certain threshold, the old regime is no longer useful to the policymaker who then finds it advantageous to introduce a monetary reform. This provides a qualitative analysis of the factors that determine the timing of stabilization.⁵

¹However, during the recent high (but not hyper) inflations in Argentina, Brazil, and Israel the employment motive was relatively more important although the revenue motive played a role as well. An analysis of the interaction between those two motives during the recent Israeli inflation appears in Cukierman (1988). The Argentinian and Brazilian experiences are described in Dornbusch and Fischer (1986) and Helpman and Levchenko (1986), and the recent Bolivian experience in Sachs (1987) and Morales (1988). A third motive for inflationary policies from which I abstract is reduction of deficits in the balance of payments (Cukierman and Lieberman (1987)). A general historical survey of conditions under which high inflations have appeared is in Capie (1986).

²Such estimates have been provided by Cagan (1956), Sargent (1977), and Christiano (1987).

³This statement is based on data until September 1925 since data on forward rates and interest rates is not available thereafter.

⁴Sargent (1977), for example, expresses the widely held view that this observation reflects an "...apparent; irrational behavior by the creators of money." (page 60).

⁵As far as I know all existing literature on stabilization takes its timing as exogenous. Examples are Flood and Gerber (1980a, 1980b), Sargent (1986), Drazen and Helpman (1986), and Benhabib and Eckerstein (1987).
Europe in the immediate post World War I period was the nursing ground for some of the classic hyperinflations. Against this background Sections 6 and 7 provide answers to two questions: First, why was deficit financing used? Second, given the size of deficits, why was the fraction financed by money creation so large in some of the countries? The answer given to the first question is that many individuals in all of the countries that emerged from the war (expecting to have substantially higher future incomes) wanted to borrow resources against their future higher labor incomes but could not do so because of financial constraints. Being unable to borrow through private capital markets, they managed to partially fulfill this desire via the political system by inducing governments to create deficits. This factor also provides a partial answer to the second question.

But for the countries that lost the war, and in particular for Germany, the large foreign exchange denominated reparations provided an additional powerful incentive to resort to inflationary finance. In the German case a large fraction of reparations was conditional on Germany's ability to pay. A smoothly functioning fiscal system would have been taken as a signal that Germany was able to pay more and would have triggered an increase in reparation demands on the part of the allies. By contrast heavy reliance on inflationary finance, by creating the opposite impression, would have contained the demands of the allies, as was actually the case at the end of the hyperinflation. In addition, because of the stipulation that reparations had to be paid in foreign exchange, it was more likely that the fraction expropriated for reparations out of conventional taxes would be larger than the same fraction from seigniorage revenues. Under those circumstances most German voters preferred the inflation tax to regular current or future taxes independently of the size of their wealth. Those ideas are described verbally in Section 6 and demonstrated within the framework of a decisive voter model of the type pioneered by Meltzer and Richard (1981) in Section 7. Concluding remarks follow.

2. THE PUZZLING BEHAVIOR OF REVENUES FROM SEIGNIORAGE DURING THE GERMAN HYPERINFLATION

The post World War I German hyperinflation is one of the best known and relatively well-documented hyperinflation. As opposed to some recent high inflations in Argentina, Brazil and Israel, the German government financed well over fifty percent of its expenditures by means of seigniorage during the hyperinflation years. It is therefore well accepted that the revenue motive was the major force behind the inflationary policies followed by the German authorities. But given this premise and estimates of the elasticity of money demand with respect to expected inflation, most of the rates of inflation prevailing in Germany, at least since mid-1922, imply irrational behavior on the part of government. The reason is that, to the extent that the public did not systematically underestimate actual inflation, estimates of money demand imply that inflation was in a range in which a decrease in it would have increased government's revenue from seigniorage.

Under perfect foresight Cagan's (1956) original estimate of the semi-elasticity of money demand (a) which is 5.46 implies that steady state revenues from seigniorage decrease as inflation goes up beyond 18.3% per month. Sargent (1977) points out some econometric problems in Cagan's estimate and reestimates α, using both Cagan's as well as Barro's (1970) data under the assumption of rational expectations. The lowest point estimate obtained by Sargent is 2.344. This would imply that at any rate of inflation above 42.7% per month, further inflationary acceleration decreases government's steady state revenues from seigniorage. However, Sargent is not able to pinpoint α very precisely. Christiano (1987) points out that the estimation procedure is sensitive to the shock restrictions imposed on the money supply process and manages to obtain more precise

---

6 This is analogous to the role of bequest constraints in creating deficits through the political system (Cukierman and Meltzer (1987)).

7 Graham (1967), pp. 40-41. By contrast during the high Israeli inflation seigniorage usually was less than 3 percent of total government expenditures (Cukierman (1988)).

8 A brief discussion of other motives for inflationary policies appears in the concluding section.

9 This statement is based on the well-known result (Bailey (1956) and Cagan (1956)) that in the absence of growth, with perfect foresight, and a Cagan-type money demand function of the form \( \frac{1}{1+\pi} \) where \( \pi \) is inflation, steady state seigniorage is maximized when \( \pi \) is equal to \( 1/\alpha \).
estimates of \( \alpha \) by imposing several alternative money supply rules. The lowest significantly different than zero estimate of \( \alpha \) he obtains is 1.76. This estimate implies that the critical point beyond which steady state seigniorage decreases with inflation is 56.8%. A quick look at column (1) of Table 1 establishes that inflation was most of the time above even this large rate from July 1922 until the stabilization at the end of 1923.\(^{10}\)

Does this mean that the German government was acting irrationally in terms of its own objectives? A natural point of departure for an investigation of the rationality of a government that persistently speeds up the rate of nominal growth in order to get more seigniorage is to examine whether seigniorage increased or decreased as inflation accelerated. In general seigniorage in month \( t \) can be calculated as

\[
\frac{M_{t+1} - M_t}{\bar{p}_t} \quad (1)
\]

where \( M_t \) are nominal money balances at the beginning of month \( t \) and \( \bar{p}_t \) is the average price level over the month. Such seigniorage figures (in percentages of November 1920 real money balances) are reported in column (3) of Table 1. It is apparent that in general, as the magnitude of inflation increased, seigniorage increased. Particularly striking is the large increase in seigniorage during the last several months of the hyperinflation when the monthly rate of inflation was well above one thousand percent.\(^{11}\)

Since this finding is so blatantly in conflict with conventional wisdom about the relationship between inflation and seigniorage, it is advisable to examine the possibility that it is due to a measurement problem. In particular it should be noted that the expression in equation (1) is a discrete approximation of the precise expression for monthly seigniorage which is the integral over the month of momentary rates of seigniorage. While the difference between those two measures is usually small at low or moderate rates of inflation, it may become quite important at high rates. In order to evaluate the correct measure of seigniorage, rates of money growth and of inflation at each instant are required. Such data is unavailable. But it is possible to approximate this ideal measure by computing it analytically and by assuming that, within each month, the instantaneous rates of inflation and of monetary expansion are distributed uniformly. More precisely let \( \mu_t^i \) and \( \tau_t^i \) be the instantaneous rates of monetary growth and of inflation in month \( t \). Then

\[
M_{t+\tau} = M_t e^{\mu_t^i \tau_t^i} ; \quad P_{t+\tau} = P_t e^{\tau_t^i \tau_t^i} \quad 0 \leq \tau \leq 1
\]

where \( M_t \) and \( P_t \) are nominal money balances and the price level at the beginning of month \( t \). Instantaneous seigniorage revenues are

\[
\frac{M_{t+\tau} - M_t}{P_{t+\tau}} = m_t^i e^{(\mu_t^i - \tau_t^i) \tau} \quad 0 \leq \tau \leq 1
\]

where \( m_t^i = M_t/P_t \). Integrating equation (3) over the month

\[
\int_0^{\tau_t^i} \frac{m_t^i}{P_{t+\tau}} d\tau = m_t^i e^{(\mu_t^i - \tau_t^i) \tau_t^i}
\]

The relationships between monthly and instantaneous rates of change are given by

\[
\frac{\text{inflation rate}}{\text{monetary growth rate}} = \ln(1 + \tau_t^i)
\]

where \( \mu_t^i \) and \( \tau_t^i \) are the rate of monetary expansion and the rate of inflation from the beginning to the end of month \( t \). By using equations (4), (5) and data on \( \mu_t^i \) and \( \tau_t^i \), it is possible to obtain estimates of seigniorage that are not subject to biases caused by the use of an average

\(^{10}\)Studies of money demand in Israel summarized by Offenbacher (1985) also imply that during the last two years preceding the 1985 stabilization, inflation was in the elastic range of the money demand function.

\(^{11}\)Dental and Eckstein (1987) report a similar trend.
<table>
<thead>
<tr>
<th>Period</th>
<th>(1) Prices</th>
<th>(2) Money</th>
<th>(3) Calculated Discretely</th>
<th>(4) Calculated Continuously</th>
<th>(5) Actual</th>
<th>(6) Predicted</th>
<th>(7) Real Money Balances in (Percentages of 1/10 balances)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1920-12</td>
<td>-0.024</td>
<td>0.071</td>
<td>7.33</td>
<td>7.86</td>
<td>109.66</td>
<td>119.88</td>
<td>1.09</td>
</tr>
<tr>
<td>1921-1</td>
<td>-0.011</td>
<td>-0.032</td>
<td>-3.45</td>
<td>-3.46</td>
<td>107.84</td>
<td>117.27</td>
<td>1.09</td>
</tr>
<tr>
<td>2</td>
<td>-0.036</td>
<td>0.012</td>
<td>1.33</td>
<td>1.37</td>
<td>112.62</td>
<td>122.44</td>
<td>1.09</td>
</tr>
<tr>
<td>3</td>
<td>-0.019</td>
<td>0.030</td>
<td>3.37</td>
<td>3.58</td>
<td>118.20</td>
<td>118.85</td>
<td>1.01</td>
</tr>
<tr>
<td>4</td>
<td>-0.012</td>
<td>0.021</td>
<td>2.43</td>
<td>2.58</td>
<td>122.09</td>
<td>117.37</td>
<td>0.96</td>
</tr>
<tr>
<td>5</td>
<td>0.015</td>
<td>0.014</td>
<td>1.73</td>
<td>1.70</td>
<td>121.97</td>
<td>112.02</td>
<td>0.92</td>
</tr>
<tr>
<td>6</td>
<td>0.045</td>
<td>0.049</td>
<td>5.78</td>
<td>5.86</td>
<td>122.43</td>
<td>106.26</td>
<td>1.07</td>
</tr>
<tr>
<td>7</td>
<td>0.184</td>
<td>0.028</td>
<td>3.29</td>
<td>2.73</td>
<td>106.29</td>
<td>83.19</td>
<td>0.78</td>
</tr>
<tr>
<td>8</td>
<td>0.203</td>
<td>0.055</td>
<td>3.17</td>
<td>2.92</td>
<td>91.43</td>
<td>80.44</td>
<td>0.88</td>
</tr>
<tr>
<td>9</td>
<td>0.132</td>
<td>0.079</td>
<td>6.93</td>
<td>6.47</td>
<td>87.17</td>
<td>91.22</td>
<td>1.05</td>
</tr>
<tr>
<td>10</td>
<td>0.285</td>
<td>0.060</td>
<td>4.75</td>
<td>3.81</td>
<td>71.91</td>
<td>69.65</td>
<td>0.96</td>
</tr>
<tr>
<td>11</td>
<td>0.191</td>
<td>0.103</td>
<td>6.26</td>
<td>6.26</td>
<td>66.58</td>
<td>82.14</td>
<td>1.23</td>
</tr>
<tr>
<td>12</td>
<td>0.036</td>
<td>0.126</td>
<td>8.27</td>
<td>8.95</td>
<td>72.35</td>
<td>107.88</td>
<td>1.49</td>
</tr>
</tbody>
</table>

Notes to Table 1:

Inflation and money growth are rates of change from the beginning to the end of each month in decimal points. They have been calculated using data on the mid-month Wholesale Price index and on end-of-the-month quantity of notes in circulation from pp. 526-530 of Young (1935). Monthly mid-month to mid-month rates of inflation were converted to rates of inflation over each month by taking (geometric) averages of adjacent mid-month to mid-month inflation rates.

Monthly real revenues from inflation are expressed in percentages of end-of-November 1920 real money balances. The inflation tax in column (3) is calculated by dividing the change in notes in circulation over the month by the average level of the Wholesale Price index during the month. The inflation tax in column (4) is obtained by expressing the monthly inflation tax as the integral, over the month, of momentary inflation taxes under the assumption that monetary expansion and inflation are uniformly distributed over the month. Further details appear in the text.

Actual and predicted real money balances refer to the end of each month and are expressed in percentages of end-of-November 1920 real money balances. Real money balances at the end of each month are obtained from those at the end of the previous month by using the rates of money growth and inflation over the month. Predicted real balances (in terms of 1/10 balances) are calculated by using Christiano's (1987) recent estimate of the semi-elasticity of money demand (a) which is 1.76 and the assumption of perfect foresight.
seigniorage did not decrease as high inflation penetrated the hyper-inflation range and continued to accelerate. A related regularity is that monetary expansion and seigniorage are positively correlated on a month by month basis during most of the hyperinflation (compare columns (2) and (4) of Table 1).

In order to obtain more information about the cause of the divergence between actual seigniorage and the amount predicted by theory and available estimates of $\alpha$, it is instructive to compare actual real money balances at the end of each period with those predicted under perfect foresight by conventional estimates of money demand for Germany during the hyper-inflation. Since the tax rate (which is the rate of monetary expansion) is the same for both actual and predicted seigniorage, any differences between those two magnitudes must originate from differences between the actual and predicted behavior of real money balances. Actual and predicted real money balances, measured in terms of end-of-November 1920 real balances are presented in columns (5) and (6) of Table 1. Predicted balances are computed using the low estimate of 1.76 for $\alpha$ from Christiano (1987). The ratio between predicted and actual real money balances is presented in column (7). A striking regularity that emerges from comparison of this ratio with inflation is that it tends to go down when inflation accelerates and to go up when inflation decelerates. This feature holds practically throughout the entire three years but is particularly striking in 1921 and in 1923. From the beginning to the second half of 1921 inflation goes from negative to positive values. Concurrently the ratio between predicted and actual balances goes down. In 1923 this negative correlation is even more strongly in evidence. As inflation plunges into the negative range in March 1923, the ratio between predicted and actual balances shoots up. Then as inflation rapidly moves up into the high cliffs of the second half of 1923, the ratio goes down abruptly and becomes zero from July 1923 until the end of the hyperinflation.

The upshot is that actual real balances adjust to changes in the rate of inflation more slowly than predicted balances do. This feature becomes even stronger if higher estimates of $\alpha$ are used instead of the low 1.76 estimate. Actual balances generally move in the same direction that predicted balances do but more slowly. This phenomenon is consistent with

<table>
<thead>
<tr>
<th>Year</th>
<th>Seigniorage</th>
<th>Inflation</th>
<th>Monetary Expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1921</td>
<td>42.77</td>
<td>142</td>
<td>66</td>
</tr>
<tr>
<td>1922</td>
<td>50.27</td>
<td>4226</td>
<td>1030</td>
</tr>
<tr>
<td>1923a</td>
<td>277.80</td>
<td>$85.54 \times 10^9$</td>
<td>$38.75 \times 10^7$</td>
</tr>
<tr>
<td>1923b</td>
<td>71.06</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^{a}\)Seigniorage is in percentages of November 1920 real money balances. It is calculated from column (4) of Table 1.

\(^{b}\)Seigniorage during the first ten months of 1923 at a yearly rate.

monthly price level. Such estimates, again in percentages of real balances at the end of November 1920, are presented in column (4) of Table 1.\(^2\) Seigniorage still goes up as the rate of inflation accelerates from negative rates in early 1921 to over ten thousand percent per month at the end of 1923. The total amount of yearly seigniorage from column (4) of Table 1 is presented along with the yearly rates of inflation and of monetary expansion in Table 2. The increase in seigniorage is particularly large in 1923. Since most of this increase is due to a tremendous increase in the last month of the hyperinflation, total seigniorage in 1923 is also presented only for the first ten months of 1923 at a yearly rate. For this alternative calculation seigniorage in 1923 is still larger than seigniorage in 1922 which in turn is larger than total seigniorage in 1921.

I conclude that contrary to what is implied by most empirical estimates of money demand functions during the German hyperinflation,\(^{12}\) Note that for low rates of change the two measures of seigniorage are similar. But they are sometimes quite different at high rates of inflation (compare columns (3) and (4)).

\(^{13}\) Evidence presented in Graham (1967) pp. 40-41 is consistent with the view that seigniorage was substantially higher in 1923 than in 1922.
each of the following two distinct but not necessarily mutually exclusive hypotheses. One is that the long-run adjustment of money demand to a change in expected inflation is larger than the short-run adjustment because the institutions of monetary exchange adjust slowly. The other is that, due to an inability to separate persistent from transitory inflationary developments, inflationary expectations lag behind actual inflationary developments even if the public uses all the information at its disposal in an optimal manner. The following section presents evidence from the foreign exchange and from the capital markets on the behavior of inflationary expectations during the German hyperinflation.

3. EVIDENCE ON THE BEHAVIOR OF INFLATIONARY EXPECTATIONS DURING THE GERMAN HYPERINFLATION

There is no direct model free measure of inflationary expectations during the German hyperinflation. But it is possible to get an idea about the behavior of those expectations by examining data on the one month forward rate of exchange between the Mark and the Pound Sterling during the hyperinflation. The advantage of this data, besides being model free, is that it is based on market data which reflects the beliefs of market participants about the future course of the exchange rate. Although exchange rate expectations need not perfectly coincide with inflationary expectations, the correlation between them is likely to be very high. Frenkel (1977, p. 656) reports that the correlation among various price indices and the exchange rate exceeded 0.99 during the hyperinflation and concludes that it seems reasonable to identify the expectations on the exchange rate with the expectations on the price level. Even if we do not subscribe to such a strong relation between those expectations, it is highly likely that one can learn about the qualitative features of the relationship between actual and expected inflation from the parallel relationship between the actual and the previously expected rate of depreciation of the exchange rate. In particular, lags in the adjustment of expected depreciation behind actual depreciation will be taken as being indicative of similar lags in the adjustment of expected inflation to actual inflationary developments.

The first two columns of Table 3 present data on the expected and the actual rates of depreciation of the Mark with respect to the Pound Sterling. At each moment the one month forward percentage discount is taken as a measure of the rate of depreciation expected to occur over the next month. The corresponding actual depreciation over the same forecast span is displayed in column 2. Thus, according to the table, a 0.8% appreciation of the Mark was expected to occur at the beginning of January 1921 until the beginning of February 1921. The actual appreciation was 6.9%. An examination of the relationship between actual and expected depreciation in Table 3 reveals that, generally, the fluctuations in the actual rate of change in the exchange rate were substantially larger than the fluctuations in its expected counterpart. Individuals underpredicted large depreciations as well as large appreciations. Particularly striking is the fact that as the rate of depreciation accelerated from 1921 to 1923, expected depreciation lagged behind quite substantially. Until May of 1922 the one month forward Mark consistently sold at a small premium with respect to the pound sterling in spite of the fact that between May of 1921 and May of 1922, the actual average rate of depreciation was 18.7% per month. As the rate of depreciation accelerated through the rest of 1922 and 1923, expected depreciation also increased gradually but, at least for as long as there is data on forward rates, it trailed far behind actual developments. The lag in the adjustment of exchange rate expectations became particularly dramatic during July-September 1923 when the Mark price of sterling rose by leaps and bounds. The data is therefore consistent with the view that there were substantial lags in the adjustment of exchange rate expectations. Since movements in the exchange rate and in local price indices were highly correlated, it is likely that inflationary expectations similarly lagged behind inflationary developments.

This conclusion is confirmed by data on the daily money rate. This rate is consistently below the actual rate of inflation as can be seen from a comparison of columns (3) and (4) in Table 3. The daily rate fluctuates

---

14 Most empirical studies of the demand for money during the hyperinflation, like those of Cagan (1956), Sargent (1977), and Christiano (1987), test a joint hypothesis that includes both the form of money demand and the process of expectations formation. An exception is Frenkel (1977).

15 This data passes simple tests of market efficiency (Frenkel (1977)).

16 This data terminates in September 1923 and is available again only from November 1924 and on.
### TABLE 3

Market indicators of expected inflation during the German hyperinflation (Decimals)

<table>
<thead>
<tr>
<th>Year</th>
<th>Expected Monthly Depreciation of the Mark</th>
<th>Actual Monthly Depreciation of the Mark</th>
<th>Daily Money Rate (Monthly)</th>
<th>Actual Monthly Rate of Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1921</td>
<td>-0.008</td>
<td>-0.069</td>
<td>0.003</td>
<td>-0.011</td>
</tr>
<tr>
<td></td>
<td>-0.005</td>
<td>0.000</td>
<td>0.003</td>
<td>-0.036</td>
</tr>
<tr>
<td></td>
<td>-0.008</td>
<td>0.019</td>
<td>0.003</td>
<td>-0.019</td>
</tr>
<tr>
<td></td>
<td>-0.008</td>
<td>0.049</td>
<td>0.003</td>
<td>-0.012</td>
</tr>
<tr>
<td></td>
<td>-0.006</td>
<td>-0.046</td>
<td>0.003</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>-0.005</td>
<td>0.116</td>
<td>0.003</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>-0.005</td>
<td>0.042</td>
<td>0.003</td>
<td>0.184</td>
</tr>
<tr>
<td></td>
<td>-0.005</td>
<td>0.112</td>
<td>0.003</td>
<td>0.203</td>
</tr>
<tr>
<td></td>
<td>-0.005</td>
<td>0.352</td>
<td>0.004</td>
<td>0.132</td>
</tr>
<tr>
<td></td>
<td>-0.003</td>
<td>0.060</td>
<td>0.003</td>
<td>0.285</td>
</tr>
<tr>
<td></td>
<td>-0.003</td>
<td>-0.069</td>
<td>0.003</td>
<td>0.191</td>
</tr>
<tr>
<td></td>
<td>-0.002</td>
<td>-0.109</td>
<td>0.003</td>
<td>0.056</td>
</tr>
<tr>
<td>1922</td>
<td>-0.004</td>
<td>0.109</td>
<td>0.004</td>
<td>0.024</td>
</tr>
<tr>
<td></td>
<td>-0.002</td>
<td>0.245</td>
<td>0.003</td>
<td>0.216</td>
</tr>
<tr>
<td></td>
<td>-0.001</td>
<td>0.197</td>
<td>0.004</td>
<td>0.225</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>-0.034</td>
<td>0.004</td>
<td>0.091</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.731</td>
<td>0.003</td>
<td>0.052</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.725</td>
<td>0.004</td>
<td>0.254</td>
</tr>
<tr>
<td></td>
<td>0.001</td>
<td>0.798</td>
<td>0.004</td>
<td>0.653</td>
</tr>
<tr>
<td></td>
<td>0.005</td>
<td>0.720</td>
<td>0.005</td>
<td>0.681</td>
</tr>
<tr>
<td></td>
<td>0.026</td>
<td>0.561</td>
<td>0.006</td>
<td>0.717</td>
</tr>
<tr>
<td></td>
<td>0.052</td>
<td>2.164</td>
<td>0.006</td>
<td>1.003</td>
</tr>
<tr>
<td></td>
<td>0.154</td>
<td>0.353</td>
<td>0.007</td>
<td>0.614</td>
</tr>
<tr>
<td></td>
<td>0.111</td>
<td>0.089</td>
<td>0.007</td>
<td>0.555</td>
</tr>
<tr>
<td>1923</td>
<td>0.076</td>
<td>2.097</td>
<td>0.009</td>
<td>0.997</td>
</tr>
<tr>
<td></td>
<td>0.182</td>
<td>-0.358</td>
<td>0.019</td>
<td>0.325</td>
</tr>
<tr>
<td></td>
<td>0.160</td>
<td>-0.071</td>
<td>0.009</td>
<td>-0.059</td>
</tr>
<tr>
<td></td>
<td>0.062</td>
<td>0.804</td>
<td>0.010</td>
<td>0.292</td>
</tr>
<tr>
<td></td>
<td>0.106</td>
<td>1.202</td>
<td>0.015</td>
<td>0.928</td>
</tr>
<tr>
<td></td>
<td>0.128</td>
<td>1.357</td>
<td>0.015</td>
<td>2.026</td>
</tr>
<tr>
<td></td>
<td>0.391</td>
<td>10.413</td>
<td>0.056</td>
<td>5.979</td>
</tr>
<tr>
<td></td>
<td>0.962</td>
<td>22.513</td>
<td>1.118</td>
<td>16.895</td>
</tr>
<tr>
<td></td>
<td>0.444</td>
<td>24.847</td>
<td>1.267</td>
<td>85.692</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.280</td>
<td>175.075</td>
<td></td>
</tr>
</tbody>
</table>

**Notes to Table 3:**

Expected and actual rates of depreciation of the Mark are calculated using data on one month forward rates and spot rates, respectively. The raw data is from Eicheng (1937) pp. 450-455 and it refers to the exchange rate of the Mark to the Pound Sterling. The expected rate of depreciation over the next month is provided by the relative forward discount of the Mark, and actual depreciation is calculated over a matching time span. The dates for which forwards and spot quotations are used are located at the beginning of each month. The precise date varies between the first and the eighth of each month. Until November 1921 one month forward contracts terminate at the end of each calendar month. Hence, the time span covered by the contract may be somewhat shorter than one month if the quotation is taken after the first of the month. From December 1921 forward contracts are for a full one month forward period.

The figures for the daily money rate are from Table 23 of Hoffgerich (1966). They have been converted into decimals per month to allow quick comparison with the actual inflation figures.

The rates of inflation figures are from column 1 of Table 1.
in a relatively narrow range (between 0.3 and 0.4 of a percent per month) until mid-1922 and starts an almost steady climb thereafter, reaching a maximum of 128 percent per month in October of 1923. However, the gap between actual inflation and the daily rate tends to widen as inflation accelerates in 1923, indicating that the extent of inflation underestimation rose with inflation. This data is therefore consistent with the view that there were substantial lags in the adjustment of inflationary expectations and that the degree of underestimation became particularly high during the last phases of hyperinflation, when the rate of inflation reached unprecedented heights.

Casual historical evidence lends further support to the view that the extent of inflation had been seriously underestimated at the time. Holtfrerich (1986, p. 191) notes that foreigners continued to buy paper marks in the form of bank notes and mark deposits into the year 1922. This behavior reflected the belief, held roughly until the beginning of 1922, that the prewar parity of the Mark was going to be reestablished by the Reichsbank. Economic policy debates at the time left the impression that such a course of action was a real possibility. The publication in June 1922 of the recommendation by the J.P. Morgan committee to postpone long-term lending to Germany and the murder of Rathenau shattered this belief (Holtfrerich op. cit.). The forward premium became a discount and the nominal daily market rate started to increase, too. But the actual depreciation of the Mark and actual inflation increased much more as can be seen in Table 3.

In conclusion the evidence that is available from both the foreign exchange market and the capital market supports the view that inflationary expectations lagged behind actual inflationary developments, particularly when inflation accelerated abruptly. The next section presents a framework in which such lags are compatible with rational expectations and uses their presence to resolve the puzzle raised by the behavior of seigniorage during the hyperinflation.

4. A RESOLUTION OF THE PUZZLE

The evidence of the previous sections raises several issues. First, it calls for a reconciliation of the fact that seigniorage generally increased as inflation and monetary expansion rose during the German hyperinflation, with the commonly expressed view that lowering inflation would have increased seigniorage. Second, it raises the possibility that German policymakers were not necessarily acting irrationally. Finally, it raises a question about the extent to which the systematic underprediction of inflation and deflation (particularly in 1923) reported in Section 3 is consistent with a rational formation of expectation.

This section presents a framework that is consistent with the view that, even in 1923 when inflation accelerated to unprecedented heights, policymakers were acting rationally given their objectives and constraints. This framework is also consistent with the evidence in Section 2, according to which seigniorage generally increased with inflation. Furthermore, it implies that the large and sustained downward biases in the public's prediction of inflation, particularly in 1923, were not inconsistent with the view that expectations were being formed optimally.

The two main elements of the framework are first that government cares about the entire future profile of seigniorage revenues and inflation with, possibly, strict preference for nearer outcomes. Second, government's preference for seigniorage versus inflation avoidance shifts stochastically through time and government or the central bank has more timely information about those shifts than the public. In particular government has better information about the degree of persistence of changes in the relative importance it attributes to price stability and seigniorage revenues. The public learns about those shifts in objectives by observing past policies chosen by government. But since those objectives are affected both by persistent and by transitory shocks, optimal learning implies that persistent changes in relative objectives are detected by the public only gradually. This enables policymakers to temporarily increase seigniorage revenues by increasing the rate of monetary expansion even if this policy eventually drives expected inflation above the point at which the elasticity of money demand is larger than unity. Governments that become more desirous of seigniorage revenues use this temporary trade-off to increase seigniorage in...
the short run at the expense of higher inflation and even lower seigniorage in the long run. Since the trade-off is temporary, policymakers with strong time preference are more likely to follow policies that will eventually increase inflation and decrease seigniorage.

The German experience, particularly in 1923, conformed to this pattern. In particular the analytical framework that follows implies that:

a. Monetary expansion and seigniorage revenues may be positively related even at very high rates of inflation as is the case in Table 1. b. A government with a strong desire for immediate seigniorage does not necessarily act irrationally when it increases current seigniorage at the cost of higher inflation and lower future seigniorage. c. When large persistent increases occur in government's desire for seigniorage, expectations may be systematically biased downward for a while as is the case in Table 3. This is due to the process of gradual but optimal learning.

Before plunging into the specifics of the model used to illustrate those ideas, it is important to remark that sluggish adjustments of the institutions that are designed to reduce the need for holding money can also create a temporary trade-off between current and future seigniorage. In particular it may be rational for a government with a sufficiently strong time preference to inflate beyond the unit elasticity point of the long-run demand for money even when the public has perfect foresight. Part of the events in Germany, particularly during the early phases of the hyperinflation, were probably due to this delayed adjustment of institutions. I have chosen to ignore possible differences between the long and the short-run demand for money because of this element and to focus instead on lags in the adjustment of expectations. This choice was
dictated first by the evidence of Section 3 which suggests that lags in expectations were quite important during the German hyperinflation and second in order to maintain the analytical framework within manageable proportions.

A. A Model Of Seigniorage, Governmental Objectives, And The Public's Beliefs

Let $M_t$ be nominal money balances in period $t$. Seigniorage obtained by government because of monetary expansion between period $t$ and period $t+1$ is

$$g_t = \frac{M_{t+1} - M_t}{P_t} - \frac{M_{t+1} - M_t}{P_t} = \frac{\nu_{t+1} M_t}{P_t}$$ (7)

where $\nu_{t+1}$ is the rate of monetary expansion between period $t$ and period $t+1$ and $M_t$ are real money balances in period $t$. The decision-making process of government is such that the decision about $\nu_{t+1}$ is made in period $t$. The specification in (7) embodies the assumption that government spends the addition to money balances between period $t$ and period $t+1$ at period $t$'s prices.

Demand for real money balances in period $t$ assumes the familiar form due to Cagan

$$D^e = \alpha^{\ast} t+1$$ (8)

where

$$\alpha^{\ast} t+1 = E[\nu_{t+1}]$$ (9)

is the rate of inflation expected by the public, as of $t$, to occur between $t$ and $t+1$. $I_t$ denotes the information set available to the public in period $t$ and $D$ and $\alpha$ are positive constants. In order to focus on the effects of the policymaker's informational advantage, I abstract from possible lags in the adjustment of money-economizing institutions by assuming that there is no difference between the long- and the short-run demand for money as implicitly by equation (8). The price level in each period is determined from

---

18 A previous version of the paper investigated the polar case in which there is perfect foresight but there are differences between the short- and the long-run adjustments of money demand to changes in expected inflation. (Evidence that such differences may be non-negligible at least for mild inflations is provided by Cukierman, Lennan, and Papada (1985) for major EEC countries and by Piterman (1985) for Argentina and Israel.) This framework, too, implies that governments with high time preference may choose to inflate at rates above the rate at which the elasticity of long-run money demand is unity.

Differences between the short- and the long-run demand for money arise because a sustained increase in expected inflation triggers institutional adjustments designed to further reduce the need to hold and to use the costly money (Brunner and Meltzer (1971)). Use of alternative means of payments, limited barter arrangements, changes in the frequency of wage payments, and more efficient banking are costly institutional changes. Hence they are introduced gradually only after the public has become convinced that the higher inflation is a persistent phenomenon.

19 It implies that the increment to nominal money balances raises prices only after it has been spent on the market.
the money-market equilibrium condition
\[ M_t = \frac{\mathbf{p}^*_t}{\mathbf{p}_t} = \mathbf{D} e^{-\alpha x_{t+1}}. \] (10)

Equation (10) and its lagged counterpart imply
\[ 1 + x_t = (1 + p_t) e^{x_{t+1} - x_t}. \] (11)

Taking natural logarithms of (11) and using the approximation
\[ \ln(1 + x) = x \]
and
\[ x_t = \ln(1 + x_t) = \mu x_t + \alpha (x_{t+1} - x_t). \] (12)

No policymaker, including the one who presided over the post World War I German hyperinflation, likes inflation per se. But governments are willing to tolerate some monetary expansion in order to obtain seigniorage. An essential feature of hyperinflation is that the desire for seigniorage in comparison to inflation aversion fluctuates in a not totally predictable manner over time and that policymakers become informed of such shifts more quickly than the public at large. The policymaker may not be fully informed about his future objectives, but he is in a better position than the public to evaluate those objectives. Furthermore, in choosing current monetary expansion government takes account of the future as well as of the present. But different governments may discount the future to a different degree. Those characteristics are modeled by postulating that the policymaker's decision strategy is

\[ \text{Max}_{\nu_i, i = 1, 2, \ldots} E_{t_0} \sum_{t=0}^{\infty} \beta^t [x_t g_t - \nu_{t+1}^2]. \] (13)

\[ x_t = A + p_t + \epsilon_t, \quad A > 0, \] (14)

\[ p_t = \rho p_{t-1} + \nu_t, \quad 0 < \rho < 1, \] (15)

where \( v \) and \( \epsilon \) are serially and mutually uncorrelated normal variates with zero means and variances \( \sigma^2_v \) and \( \sigma^2_{\epsilon} \), respectively. \( \beta \) is the government's subjective discount factor and \( x_t \) measures government's desire for seigniorage in comparison to its aversion to inflation.\(^{21}\) \( E_{t_0} \) is a conditional expected value operator that is conditioned on the information available to government in period 0.

The higher is \( x_t \) the more willing are policymakers to trade higher monetary expansion for more seigniorage. Equations (14) and (15) state that, in addition to the nonstochastic term, \( A \), the shift parameter \( x_t \) is affected by a persistent stochastic component \( p_t \), (whose degree of persistence is measured by \( \rho \)) and by a transitory shock \( \epsilon_t \).

The policymaker knows the current and past values of \( p_t \) and \( \epsilon_t \) and therefore also of \( x_t \). The public does not have direct knowledge about the realizations of those variables, but it can make inferences about them from its knowledge of past rates of monetary expansion. Hence the policymaker has better information than the public not only about the state of its current preferences but about their persistence as well. This information advantage can be used, by him, to temporarily increase current seigniorage at the expense of future seigniorage when this is advantageous.

The information set, \( I_t \), of the public includes, in addition to the deterministic and stochastic structure of the model, observations about rates of monetary expansion up to and including period \( t \). Using this information, the public forms expectations about future monetary growth which are used in turn to form rational forecasts of inflation as in equation (9). Linearity of government's decision rule is essential for the calculation of those expectations. Such linearity obtains if seigniorage, \( g_t \), is linear in its arguments. It is shown in part 1 of the Appendix that a linear approximation of seigniorage is

\[ g_t = G_0 + G_1 \nu_{t+1} - G_2 \nu_{t+1} \] (16)

where
\[ G_1 = D e^{-\alpha x_m}, \quad G_2 = \alpha \mu, \quad G_0 = \mu \theta. \] (17)

and \( \mu \) is some mean steady state rate of inflation. This approximation neglects the nonlinearities in the seigniorage function \( g_t \). But it has the

\footnote{For simplicity the rate of monetary expansion is used as a proxy for the costs of inflation to government.}
the virtue of making it possible to investigate explicitly the effects of the policymaker's informational advantage on monetary expansion and seigniorage in a world of rational expectations.\textsuperscript{22}

Using (16) in (13) the policymaker's decision problem may be rewritten as

$$\begin{align*}
\max_{\{v_i\}, i=1,2,\ldots} & \quad E_0 \sum_{i=0}^{\infty} \beta^i x_i \left[ (G_0 + G_1 + \mu_{i+1} - 2\mu_{i+1}^*) - \frac{\mu_{i+1}^*}{2} \right] \\
& \text{where, due to the rationality of expectations and the persistence in governmental objectives, } \mu_{i+1}^* \text{ depends on actual rates of monetary expansion prior to period } i+1.
\end{align*}$$

(13a)

The policymaker knows the process by which the public forms the expectation $\mu_{i+1}^*$. This process must be consistent with the actual strategy followed by government. The government's strategy is derived, in turn, by solving the maximization problem in equation (13a), taking the process for the formation of $\mu_{i+1}^*$ as given. In other words, $\mu_{i+1}^*$ is a rational expectation of next period's inflation formed by using the public's knowledge about the policymaker's strategy in conjunction with all the relevant information available.

In order to derive the equilibrium solution of the model, I proceed in two steps. First I postulate the public's beliefs about the behavior of money growth in the economy. Then I show that when government maximizes (13a), given those beliefs, the strategy that emerges validates the public's beliefs.

The public believes that the rate of monetary expansion is given by

$$\mu_{i+1} = B_0 A + B_1 + \psi_i$$

(18)

where $\psi_i$ is a normal variate with zero mean and variance $\sigma_\psi^2$, and $B_0$ and $B$ are known constants which depend on the underlying parameters of government's objective function. It is shown in part 2 of the Appendix that those beliefs about $\mu$ in conjunction with the public's knowledge of equation (12) imply

$$\begin{align*}
x_{i+1}^* &= B_0 A + \theta^* \sum_{s=0}^{\infty} (\mu_{i-s} - \beta B_0 A) \\
\theta^* &= \frac{\alpha - \lambda}{1 + \alpha (1 - \rho)}, \\
\lambda &= \frac{1}{2} \left( \frac{1 + \rho}{\alpha} + \rho \right) - \frac{1}{4} \left( \frac{1 + \rho}{\alpha} + \rho \right)^2 - 1, \\
\rho &= \frac{\sigma_\psi^2}{\sigma_\psi^2},
\end{align*}$$

(20a, 20b)

where $\lambda$ is a number between $0$ and $\rho$. Using (17) and (19) in (13a), the policymaker's decision problem may be rewritten

$$\begin{align*}
\max_{\{v_i\}, i=1,2,\ldots} & \quad E_0 \sum_{i=0}^{\infty} \beta^i x_i \left[ (G_0 + G_1 + \mu_{i+1} - 2\mu_{i+1}^*) - \frac{\mu_{i+1}^*}{2} \right] \\
& - G_1 \mu_{i+1}^* \sum_{s=0}^{\infty} \lambda^s (\mu_{i-s} - \frac{\mu_{i+1}^*}{2})
\end{align*}$$

(22)

The policymaker chooses the actual value of $\mu_i$ and a contingency plan for $\mu_{i+1}$, $i \geq 2$. Recognizing that in each period in the future the policymaker faces a problem that has the same structure as period zero's problem, the stochastic Euler equations necessary for an internal maximum of (22) are\textsuperscript{23}

$$G_1 [x_{i+1}^* - \beta \mu_{i+1}^* - \mu_{i+1}] - \mu_{i+1} = 0 \text{ for } i = 1, 2, \ldots.$$ 

(23)

Although the policymaker knows $x_i$ in period $i$ (and the public does not), he is uncertain about values of $x$ beyond period $i$. Based on the information available to him in period $i$, he computes a conditional expected value for $x_{i+j}$, $j \geq 1$. In view of (14) and (15)

\textsuperscript{22}Since the approximation is used only in the solution of government's decision problem, it can be interpreted as describing the behavior of a government that, due to computational constraints, operates subject to bounded rationality.

\textsuperscript{23}Sargent (1979, Chapter 14). The problem in (22) is formally equivalent to that on page 1 of Glickman and Nefzger (1966b). An argument similar to that in footnote 14 of that article implies that the transversality condition is satisfied for any $\beta < 1$. This condition is sufficient for an internal maximum.
\[ E_{Gi} x_i + j = p^j x_i + (1-p^j)A. \]  \hfill (24)

Substituting (24) into (23), summing up the resulting infinite geometric progressions and rearranging

\[ \nu_{i+1} = D \left[ (1 - \frac{\rho \bar{\sigma}_m (\rho - \lambda)}{1 - \rho \bar{\sigma}_m (1 - \rho)} A + (1 - \frac{\rho \bar{\sigma}_m (\rho - \lambda)}{1 - \rho \bar{\sigma}_m (1 - \rho)}) (p_i + \epsilon_i) \right]. \]  \hfill (25)

Rationality of expectations implies that the coefficients of \( A \) and of \( p_i \) should be the same across equations (18) and (25), respectively, so

\[ B_0 = 1 - \frac{\rho \bar{\sigma}_m (\rho - \lambda) (B)}{1 - \rho \bar{\sigma}_m (1 - \rho)} D \]  \hfill (26a)

\[ B = 1 - \frac{\rho \bar{\sigma}_m (\rho - \lambda) (B)}{1 - \rho \bar{\sigma}_m (1 - \rho)} D. \]  \hfill (26b)

The dependence of \( \lambda \) on \( B \) through equation (21) is stressed by writing \( \lambda \) as a function of \( B \). It is shown in part 3 of the Appendix that subject to plausible restrictions

\[ 1 \geq B_0, \quad B > 0 \]  \hfill (27)

and that equations (26) imply a unique solution for \( B_0 \) and \( B \). Rationality of expectations also implies (by comparing (18) and (25)) that

\[ \psi_i = B_e \epsilon_i, \quad \sigma^2_p = B^2 \sigma^2_e. \]  \hfill (28)

This completes the demonstration of the fact that the public's beliefs are rational and that equation (25) (or (18)) describes the behavior of money growth chosen by government. In view of (18) and (28), equation (19) may be rewritten

\[ x_{i+1} = B_0 A + \sum_{s=0}^{\infty} \lambda^s (p_i + \epsilon_i) \]  \hfill (29)

which implies that there are persistent effects of past values of government's relative desire for seigniorage and the inflation tax on current inflationary expectations as is the case, for example, with adaptive expectations. This sluggishness is caused by the public's inability to separate persistent from transitory changes in government's desire to use seigniorage as a source of revenue for financing the budget. It induces temporary but persistent deviations between actual and expected inflation. This enables government in turn, to increase current seigniorage even when inflation is in the range of money demand in which the elasticity of this demand is larger than unity.

B. Resolution of the Seigniorage Puzzle in Terms of the Model

A striking feature of seigniorage during all phases of the German hyperinflation is that it usually increases when the rate of monetary growth increases (columns 2, 3, and 4 of Table 1). This is puzzling when (as was the case during the second part of the hyperinflation) the rate of inflation is in the range of money demand in which the elasticity of this demand is larger than one. The puzzle disappears when the informational advantage of policymakers is recognized explicitly as done in the model of the previous subsection. As can be seen from equation (16), seigniorage increases from one period to the next if the rate of monetary expansion increases by enough in comparison to the increase in expectations. Since the change in expectations depends on a weighted average of differences between past rates of monetary expansion (equation (19)), seigniorage always increases when current monetary growth is sufficiently larger than past rates of monetary expansion.

Essentially, the policymaker always possesses the ability to increase current seigniorage because inflationary expectations, being based on information up to the previous period, are temporarily given. This ability is independent of the elasticity of money demand since expectations are fixed for the period. It is related to Calvo's (1978) result that under perfect foresight optimal monetary policy is bound to be time inconsistent. However, unlike in Calvo's model, the fact that the currently chosen policy affects future expectations checks the temptation of the policymaker to inflate without bounds.24

The factors that determine the correlation between current seigniorage and current monetary expansion can be seen more clearly by substituting

---

24Another important difference is that the decision rule of the policymaker here is time consistent.
(17) and (18) into (16) and by rewriting period i's seigniorage as

\[ s_i = K + G_s \{ \mu_{i+1} - \frac{\alpha_{s-1}}{1 - \rho} \} \sum_{s=0}^{\infty} \mu_{i+s} \cdot \frac{1 - e^{-\lambda s}}{\lambda^s} \frac{1}{1 - \lambda} \].

Equation (30) implies that seigniorage increases over time when the rate of monetary expansion increases in comparison to past rates of monetary expansion and decreases when the current rate decreases in comparison to past rates. This implication is qualitatively consistent with the behavior of money growth and seigniorage during the German hyperinflation and therefore with the view (from equation (25)) that the urge of the German government to obtain seigniorage progressed over time and reached an acute dimension in 1923.26

An additional, not exclusive interpretation of the German government's policy, particularly during the last phases of the hyperinflation, is in terms of an increase in the rate of time preference. It is useful, before fully developing this point, to step back and relate the formal analysis below to the last stages of the German hyperinflation in 1922 and 1923. In January of 1923 French and Belgian troops occupied the Ruhr in an attempt to make Germany pay the high reparations that were decided upon in May 1921. The German government reacted by decreeing a state of passive resistance that required higher governmental expenditures and therefore larger amounts of seigniorage. In terms of the model this can be thought of as a situation in which, due to the conflict, the rate of time preference for current seigniorage increased (i.e. went down). Accordingly I turn next to an investigation of the effect of a change in \( s \) on the rate of monetary expansion.

Inspection of equations (26) reveals that both \( B_0 \) and \( B \) are decreasing functions of \( s \). Equation (25) implies, therefore, that the lower is \( s \), the larger is the average monetary expansion \( DB/A \). Moreover, for positive realizations of \( p_i + \epsilon_i \), lower values of \( s \) reinforce this average effect by increasing the effect of \( p_i + \epsilon_i \) on the rate of monetary expansion chosen by the policymaker. Hence the acceleration of inflation in 1923 is consistent with the view that, following the Ruhr's invasion, the urge of the government for seigniorage intensified dramatically and so did the urgency of this urge. In terms of the model, 1923 can be characterized by particularly large realizations of \( x_i \) as well as by a downward shift in the structural parameter \( \alpha \).

It is also interesting to note that, other things the same, the larger is the short-run instability in the policymaker's objectives as measured by \( \sigma_2 \), the larger the average rate of monetary expansion picked by him.27 The intuitive reason is that in the presence of larger short-run instability in government's desire for seigniorage, it takes longer for the public to detect changes in governmental objectives. As a consequence the negative effect of a current monetary acceleration on the policymaker's ability to collect seigniorage in the future occurs later, making it more profitable for him to obtain more current seigniorage by picking a higher rate of monetary expansion.28

C. A RATIONAL EXPECTATIONS EXPLANATION FOR THE LAGGING FORWARD PREMIUM AND FOR THE RELATIVE SLUGGISHNESS OF REAL BALANCES

The consistent underestimation of the Mark's depreciation implied by the behavior of the forward premium in 1923 can be understood within the above framework, as the natural consequence of a large persistent increase in \( x_i \). The model implies that such an increase will cause money growth to rise. But due to the public's inability to perfectly separate persistent from transitory changes in the policymaker's relative preference for seigniorage, the consequent rise in \( u \) and in inflation may be underestimated for a while. This can be illustrated by using equations (12), (10), and (25) to calculate unexpected inflation.

---

26In terms of the model this implies large positive realizations of \( p_i \) and perhaps also of \( \epsilon_i \) in 1923.

27This can be seen by noting that \( DB/A \) is an increasing function of \( \lambda \), since the sign of the partial derivative of \( B_0 \), with respect to \( \lambda \), is the same as that of \( 1 - p \) which is positive and by noting that \( \lambda \) is an increasing function of \( \sigma_2 \). The last fact is demonstrated in Cukierman and Heitzer (1986b).

28In terms of Grossman and Van Huyck (1986), terminology, the "punishment" for high current monetary expansion comes later and is therefore discounted more heavily. But unlike in Grossman and Van Huyck the loss in the policymaker's "reputation" due to a higher current monetary expansion is a byproduct of the public's attempt to optimally learn about the changing pattern of the policymaker's objectives rather than to exogenous deterrence mechanisms.
Equation (31) suggests that when a large persistent shock occurs in government's preferences, it induces a systematic divergence between actual and expected inflation for several periods. In particular when there is a large positive innovation, $v$, to the persistent component of the policymaker's preferences in period $i-1$, $P_{i-1}$ is large in comparison to previous values of $p$. As a result $x_i - x_i^\star$ becomes positive implying an understimation of inflation. The higher value of $p$ raises subsequent inflationary expectations but only gradually. Formally this is reflected in equation (31) through the fact that in the expressions for $x_{i+j-1} - x_{i+j}$, $j \geq 1$, the larger values of $p$ from period $i-1$ on are weighted along with the lower, preperiod $i-1$ values of $p$ pulling $x_{i+j}$ down and creating a downward bias in expectations for a while after the occurrence of the large persistent shock. Those implications are fully consistent with the evidence of Table 3 according to which during 1923 the forward premium systematically underpredicted the subsequent devaluation of the Mark. They also provide an explanation for the sluggish adjustment of interest rates.

Equations (12) and (25) suggest that when the value of $p$ in period $i-1$ rises, so does the rate of inflation in period $i$. Equation (31) implies that a rise in $p_{i-1}$ also causes an increase in the size of unexpected inflation. The upshot is that the model implies a positive correlation between inflation and unexpected inflation, providing an explanation for the fact (Table 3) that the gap between inflation and interest rates widened as inflation increased.

D. The Model And The Facts - A Final Overview

In conclusion the data on seigniorage, money growth, and expectations during the German hyperinflation in conjunction with the analysis of this section provide a possible explanation for the inflationary acceleration of 1923 as the consequence of a desperate but deliberate attempt on the part of a policymaker with an extremely strong and rising desire for immediate seigniorage to satisfy this urge. The positive correlation between monetary growth and seigniorage suggests that German policymakers were able to temporarily satisfy this urge. The substantial lags in the adjustment of inflationary expectations explain why they were able to achieve this aim even at rates of inflation for which the elasticity of money demand was substantially above unity. Finally, the model suggests that they chose to use their short run information advantage in spite of its obvious ultimate inflationary consequences because of their strong preference for current seigniorage.

The lag in the adjustment of expectations also provides an explanation for the sluggish behavior of actual real balances in comparison to those that are predicted, under perfect foresight, by Christiano's (1987) estimate of $\alpha$. But as pointed out at the beginning of this section, part of the sluggishness in real balances, particularly during the final phases of the hyperinflation, may additionally be due to slow adjustment of money-saving institutions. Both types of lags induce policymakers with strong time preference to inflate at high rates.

Obviously part of the gigantic acceleration of inflation in 1923 might have been due to miscalculations as well. For example, it is likely that German policymakers underestimated the extent to which conventional tax revenues would fall as a result of the inflationary acceleration of 1923. Since some of the increased desire for seigniorage in 1923 was a consequence of this inflation-induced decrease in ordinary tax receipts, part of the inflation in this year may be due to a prior underestimation of this effect. But there is little doubt that a substantial part of the tremendous inflationary acceleration of 1923 was a direct result of the increased hunger of the German government for immediate seigniorage triggered by the policy of passive resistance to the Ruhr's occupation.

The framework of this section also sheds a different light on the

---

29 This divergence cannot be used, at the time, by individuals to improve their forecasts since expectations are rational. Further details about why this is the case appear in Section 5 of Brunner, Meltzer, and Meltzer (1982) and in Adam and Meltzer (1982).

30 This statement implicitly relies on a weak version of purchasing power parity (PPP). Frenkel (1977) presents evidence that PPP held during the German hyperinflation and that exchange rate expectations are therefore a good proxy for price level expectations.

31 Under perfect foresight the unit elasticity point would have been crossed around mid-1922. But the data on expectations from the forward market suggests that it was crossed only about a year later.

32 This is the so-called "Oliver-Tanzi" effect familiarized by the more recent South American and Israeli inflations.
the apparent irrationality of the Israeli government that inflated at more than 100% per year during the early eighties in spite of the fact that total revenues from inflation were negative (Fischer (1984)). This was due to the fact that the Israeli government had large balances of low interest, unindexed loans, when inflation accelerated at the end of the seventies (Sokoler (1987)). Due to the existence of those balances inflation decreased the future real redemption value of government loans but did not affect current cash flows since most of those loans were long term. The negativity of the total inflation tax was due to the fact that this loss on capital account was larger than the amount of current resources obtained through current expansion of the base. The discussion in this section suggests that such behavior is compatible with maximization of the present value of seigniorage revenues when government has strong time preference.33

5. WHY IS HYPERINFLATION ALLOWED TO DEVELOP ONLY TO BE STABILIZED LATER ON?

The framework of the previous section also sheds light on why inflation is allowed to accelerate for a while during hyperinflation and is finally stabilized. Such behavior can be understood as the consequence of a string of persistent increases in $\xi$ in equation (13a) during the accelerating phases of the hyperinflation. Those increases induce government to increase monetary expansion and therefore inflation (equations (25) and (12)) in order to temporarily increase seigniorage. This effect induces a higher inflation the larger the time preference of the policymaker (the lower $\alpha$). After a while expectations adjust upward and reach the high inflation range, too. Provided $\xi$ rises sufficiently or if $\alpha$ is small enough or both, inflation rises above the point of unit elasticity. Due to the persistent-transitory confusion, expectations do not necessarily follow actual inflation into this range for a while. During this period, higher monetary expansion serves the policymaker's purposes since it increases his revenues from seigniorage.

But once expectations rise into the range in which the elasticity of real money demand with respect to expectations is larger than unity, the higher rate of monetary expansion unambiguously decreases the value of the policymaker's objectives. The reason is that, in comparison to the period just prior to the string of increases in $\xi$, monetary expansion is higher and revenues from seigniorage are lower. Moreover, this situation is expected (by government) to persist under the old regime, making the maximized value of the objective function in (13a) rather low. If this value decreases below a certain threshold that is determined by the cost of monetary reform, there is a switch in regime. More precisely, when the expected difference between the value of government's objectives with and without a monetary reform becomes larger than the cost of such a reform, the policymaker decides to stabilize inflation. From the point of view of the policymaker the main target of a stabilization is to decrease inflationary expectation swiftly. After those expectations have adjusted to a highly inflationary environment, the policymaker may experience difficulties even if he becomes more conservative in his attitude to seigniorage (x went down) in the meantime. The reason is that inflationary expectations are now slow to adjust downward, maintaining the maximized value of his objectives in (13a) at a low level. Under such circumstances a credible stabilization improves the policymaker's objectives by bringing expectations down swiftly.

Paradoxically the likelihood that high inflation will be terminated by a change in regime is larger the higher is the range of inflation attained. With higher inflation, inflationary expectations reach a higher range more quickly, making the maximized value of government's objective function under the old regime lower. This may help explain why hyper-inflations usually end with very credible stabilizations, whereas South American type inflations, in which yearly inflation fluctuates between one hundred and one thousand percent, are not always stabilized with the same level of determination.

6. WHY DO GOVERNMENTS RESORT TO INFLATIONARY FINANCE?

To this point the analysis has taken the fact that government requires as much revenue as possible from the inflation tax as given exogenously. This section provides an answer to the obvious underlying positive question: Why or under what circumstances is inflationary finance used? This question can be divided into two subquestions; the first asks under

33 This is not meant to imply that the Israeli governments of the time did not miscalculate, nor that they inflated only, or even mainly, because of revenue considerations. See also Parkin (1984).
what circumstances government is likely to resort to deficit financing, and
the second asks why this deficit is financed by money rather than by bonds.
Analysis of these questions requires some positive hypotheses about
government behavior. Here I take the view that, in a democracy,
governmental decisions reflect to a large extent the wishes of the decisive
voter. The question then becomes: Why do the majority of voters prefer
the costs associated with a large inflation tax to those resulting from
direct taxation, bond financing or a smaller total government budget.

A notable feature of most classical hyperinflations is that they
occurred following wars after a part of the infrastructure of those
countries was destroyed. As a consequence the social rates of return to
rebuilding of the infrastructure and to public outlays for this purpose
were high. The average individual emerged from the war with little wealth
and with an expectation that as the process of reconstruction proceeds at a
fast pace, his income is going to rise substantially. He therefore wanted
to borrow resources from his future higher income in order to finance current
consumption. Individuals at the top of the wealth distribution could
satisfy this desire by running down some of their wealth. However, the bulk
of the population who had little or no wealth desired to borrow against its
future labor income. But markets in which individuals could borrow against
their future (expected) higher labor incomes were largely unavailable. As a
consequence a large part of the population in countries like Germany,!
Hungary, Austria, and even France found itself constrained to hold current
consumption below the level they would have chosen, had more perfect
capital markets been available. Clearly, given the level of government
expenditures, those individuals had a preference for bond rather than for
tax financing of the budget because such financing enabled them to achieve
a superior intertemporal allocation of consumption. Some of them might even
have desired a lower government budget in order to alleviate their current
tax burden even more. But this preference was probably checked for many
individuals by the high private return expected from governmental
expenditures used to rebuild the physical and social infrastructure of the
country.

It seems therefore that a majority of individuals in many post World
War I European countries wanted to maintain a substantial investment in
public goods as well as levels of private consumption that were high in
relation to the wealth they privately owned at the time. The governments of
those countries responded by maintaining reasonably high budgets (by the
standards of the time) and by financing a sizable part of those budgets
through deficits. Those policies were particularly in evidence in Germany,
Austria, and Hungary that lost the war. But they appeared, although to a
lesser extent, in France as well.

The argument up to now explains why there was a preference towards
deficit finance in many of the war-ravaged European countries following
World War I. It does not explain why some of those countries, like Germany,
chose to finance those deficits almost exclusively by money creation while
others, like France, used some combination of bond and of inflationary
finance. In general, given the level of the deficit, individual attitudes
towards bond-versus-money financing of the deficit are determined by the
general equilibrium effects of bond-financing on the stock of capital and
by the loss in utility due to the inflation tax. Given the deficit, a shift
towards more bond-financing and less inflationary finance affects
individual welfare through several channels. First, if the decrease in the
portion of the deficit that is financed by money creation is associated
with a decrease in inflation, there is an increase in individual welfare
because the cost of holding the medium of exchange is lower. Second, the
increase in bond-financing induces some crowding out of private capital
since the consumption of wealth-constrained individuals increases. To the
extent that labor and capital are complements, this causes an increase in
the return to capital and a decrease in real wages. Individuals whose major
source of income is from wages dislike this change, whereas those who
derive their income mostly from capital like it. If the first type of
individual is the majority, some combination of bond and of inflationary
finance is to be expected, particularly if the costs of crowding out and of
inflationary finance in terms of individual welfare increase at the margin.

---

34This paradigm has been used previously to explain intratemporal redistribution (Meltzer
and Richard (1981)), progressive taxation (Cukierman and Meltzer (1986a)), and the existence
of deficits in a neo-Ricardian world (Cukierman and Meltzer (1987)).

35This expectation actually turned out to be correct for Germany and several other


37A detailed analysis of this mechanism in the context of a Neo Ricardian economy appears
as bond-financing and the rate of inflation increase, respectively. Hence, when the decisive voter relies mostly on wage income, some inflationary finance is to be expected given that there is a budgetary deficit.  

The above discussion seems to provide an appropriate vehicle for understanding the motivation for mild or intermediate inflations of the type experienced in France prior to Poincaré.  

But it is hard to believe that the above forces alone were sufficient to unleash the heavy reliance on money creation and the consequent fury of hyperinflation experienced by the countries whose monetary dynamics are analyzed in Cagan's (1956) seminal study. I believe that at least for the countries which lost the war in 1918 there was an additional powerful motive for such heavy reliance on inflationary finance triggered by the burden of reparations imposed. Although the arguments supporting this hypothesis are developed using the specific experience of Germany as an illustration, its qualitative features probably apply to other countries that had to pay reparations, like Austria and Hungary, as well.

After several years of uncertainty about their magnitudes the reparations to be paid by Germany were fixed in May of 1921, at a level that was equal to almost three times that year's national income. But only 39.4 percent of this obligation represented a definitely set schedule of payments. The rest - 61.6 percent of the total, known as "C bonds" - was made contingent on Germany's ability to pay. The Reparations Commission was empowered to demand interest payments on this tranch when it considered Germany capable of paying more (Holtfrerich 1986, p. 143). Both tranches of reparations were to be paid in foreign exchange. The allies, and in particular the French and Belgians, were quite insistent on prompt payment and were dead serious about using force to implement those demands as the January 1923 invasion of the Ruhr later demonstrated. The "C bonds" stipulation meant that "...a financial "Sword of Damocles" was suspended over the German economy..." (Holtfrerich, op. cit.). Under those circumstances most Germans expected, quite realistically, that fiscal responsibility would be taken as a signal of an increase in Germany's ability to pay and would trigger total or partial activation of payments against the "C bonds." On the other hand a large dose of inflationary finance was expected to decrease the likelihood of such an event, since it would have been taken as a sign of Germany's inability to pay. (Holtfrerich 1986, p. 167) cites Reichsbank reports to the government according to which "...not until reparations obligations had been reduced to a level consonant with the taxable capacity of the population would the Reich be in a position to renounce this 'extremely injurious mode of financing'..." He concludes later (p. 171) that until the fixing of reparations in May 1921 the central bank had regarded the costs of stabilizing inflation acceptable, since it had assumed lower reparation figures. However, once the May 1921 high and contingent figures were announced the Reichsbank changed its view and no longer regarded the cost of stabilizing the currency by fiscal deflation as lower than the benefits of achieving it.

Although they do not identify the mechanism creating the negative link between reparations to be paid and the extent of inflationary finance explicitly, those statements make it clear that policymakers at the time perceived the existence of such a link. I believe several mechanisms contributed to the creation of this negative relation. The first and most important one was the belief that fiscal and monetary stability would trigger payments on the "C bonds." The second is related to the fact that reparations payments had to be made in foreign exchange. Since the allies would not accept paper marks, seigniorage was less likely to be expropriated for payment of reparations than receipts from taxes or issuance of...
Due to the unreasonably high total amount of reparations requested in May 1921, it is unlikely that Germany would have paid everything that was demanded no matter the domestic method of taxation used. However, it is likely that, owing to the reasons sketched above, the Allies would have demanded and obtained more resources with a smoothly functioning German fiscal system than under the system of inflationary finance actually used between 1921 and 1923.

Hence, Germans from all levels of income and wealth had a ceteris paribus preference for inflationary finance. In addition to the usual conflicts about the domestic distribution of tax burdens, German voters were highly concerned about the international distribution of resources between Germany and the Allies. With respect to this second conflict all German voters, whether capitalists or workers, had an obvious common interest. They all were willing to accept a high tax rate on real money balances, since the effective cost of financing the German public good by other means was even higher. This interpretation is consistent with the high fraction of the total German government's budget financed by money creation and the relatively small fraction of reparations paid during the hyperinflation period (Table 4).

The ideas of this section are illustrated more rigorously in the next section.

7. FINANCIAL WEALTH CONSTRAINTS AND REPARATIONS AVOIDANCE AS FACTORS THAT INDUCED INFLATIONARY FINANCE - A FORMAL ANALYSIS

The ideas of the previous section are demonstrated in this one by means of a two-period model in which the size of the government's budget as well as the composition of its financing between money, bonds, and lump-sum taxes is determined by the preferences of the decisive voter.

A. INDIVIDUAL BEHAVIOR.

The economy is inhabited by L individuals who live for two periods. All individuals have the same time separable utility function that depends
on consumption and real money balances in each of the two periods. This function is

$$u(c_1) + \check{s}(m_1) + \check{s}(c_2) + \check{s}(m_2)$$  \hspace{1cm} (32)

where \(c_1\) and \(c_2\) are consumption in the first and second periods and \(m_1\) and \(m_2\) are real money balances in the two periods. The component functions \(u(\cdot), v(\cdot), \check{s}(\cdot)\) all have positive and decreasing first partial derivatives which become very large as the respective arguments go to zero. Each individual inelastically supplies one unit of labor in each of the two periods and obtains real wages \(w_1\) and \(w_2\) in the first and in the second period, respectively. There are three assets in the economy: money, government bonds, and productive capital. Each individual enters period 1 with a (possibly) different amount and a different composition of wealth. There is no uncertainty so that government bonds and productive capital are perfect substitutes in portfolios. Let \(M_1\) and \(B_1\) be the nominal amounts of money with which a representative individual starts the first period, and let \(k_1\) be the amount of physical capital in his possession at that time.\(^{45}\) Government imposes real lump-sum taxes \(T_1\) and \(T_2\) in the two periods respectively. In the first period the individual collects his wage, pays taxes and can use the remainder, as well as his total first-period wealth, to buy consumption, bonds, money, or capital to be transferred to the second period. The first period nominal budget constraint is therefore\(^{46}\)

$$P_1c_1 + B_2 + P_1k_2 = P_1(w_1 - T_1) + M_1 + B_1 + P_1k_1.$$  \hspace{1cm} (33)

where \(P_i\), \(i = 1, 2\), is period’s \(i\) price level. Dividing by \(P_1\) and rearranging

$$c_1 = A_1 - (1 + \gamma_2)(m_2 + b_2) - k_2$$  \hspace{1cm} (33a)

where

$$A_1 = w_1 - T_1 + B_1 + \gamma_1 + M_1$$

and \(b_i = \frac{B_i}{P_i}, i = 1, 2; \gamma_2 = \frac{P_2}{P_1} .\)

Here \(b_i\) is the real value of the individual’s bond portfolio in period \(i\) and \(\gamma_2\) is the rate of inflation between the first and the second period.

In the second period the individual collects a real rate of interest, \(r_2\), on the capital and the bonds that he has decided to transfer from the first to the second period. Since this is the last period, he consumes all the resources at his disposal in this period. Hence, the second period’s budget constraint is\(^{47}\)

$$c_2 = w_2 - T_2 + m_2 + (1 + \gamma_2)(1 + r_2)(m_2 + b_2) - k_2.$$  \hspace{1cm} (35)

Individuals cannot increase their consumption beyond the resources available to them in the first period because there are no markets in which they can borrow against their future labor income. Formally this constraint implies

$$m_2 \geq 0; \quad k_2 \geq 0; \quad b_2 \geq 0.$$  \hspace{1cm} (36)

Substituting (33a) and (35) into (32) the typical individual problem can be formulated as\(^{48}\)

$$\max_{(m_2, k_2, b_2)} (u[A_1 - (1 + \gamma_2)(m_2 + b_2) - k_2] + \beta v[w_2 - T_2 + m_2 + (1 + \gamma_2)(1 + r_2)(m_2 + b_2) + \check{s}(m_2)])$$  \hspace{1cm} (37)

subject to the constraints in (36). Forming the Lagrangean and differentiating with respect to the three choice variables \(m_2, k_2, \) and \(b_2\) we obtain the first order conditions

---

\(^{45}\)Money in the second period yields utility directly and also because it is eventually used to buy second-period consumption.

\(^{46}\)The first-period utility from money has been deleted since it is not affected by the choice of \(m_2, b_2,\) and \(k_2\).
\[ u'[\cdot] + \alpha(1+r_2)v'[\cdot]' + \lambda_1 = 0 \] (a)

\[ -u'[\cdot] + \beta(1+r_2)v'[\cdot] + \lambda_2 = 0 \] (b) (38)

\[ -u'[\cdot](1+r_2) + \eta(1+r_2)v'[\cdot] + \lambda_3 = 0 \] (c)

where \( u'(\cdot), v'(\cdot) \) and \( \lambda'(\cdot) \) denote first order partial derivatives and

\[ n_2 = (1+r_2)(1+r_2) - 1 \] (39)

is the nominal rate of interest on bonds held from the first to the second period. \( \lambda_1, \lambda_2 \), and \( \lambda_3 \) are the Lagrange multipliers associated with the constraints \( n_2 \geq 0, k_2 \geq 0 \) and \( b_2 \geq 0 \), respectively. Since \( \lambda'(n_2) \) goes to infinity as \( n_2 \) approaches zero the constraint \( n_2 \geq 0 \) is not binding and \( \lambda_1 = 0 \). Using (39) in (38c) the last two first order conditions may be rewritten

\[ (1+r_2)[-u' + \alpha(1+r_2)v'] = -\lambda_3 \]

which implies \( \lambda_3 = (1+r_2)\lambda_2 \) so that (38b) and (38c) collapse to one condition only. This is simply a reflection of the fact that since \( k_2 \) and \( b_2 \) are perfect substitutes, the individual is indifferent to the composition of his portfolio between capital and bonds as long as the total of those two assets is held fixed. Hence, we can relabel this total as

\[ a_2 = k_2 + b_2 \] (40)

and try to rewrite the maximization problem in (37) as a function of this aggregate asset instead of its two constituents. It would seem that since \( b_2 \) is multiplied by \( \pi_2 \) while \( k_2 \) is not, this is not feasible. The reason for this divergence is that bonds are denominated in nominal terms and depreciate in value because of inflation while capital does not. For any dollar invested in bonds in period 1 the individual obtains a real bond portfolio that is worth only \( 1/(1+r_2) \) units of consumption in the second period. But since he gets the nominal interest rate in (39), his final (gross) real return is

\[ (1+r_2) \frac{1}{1+r_2} = 1+r_2 \]

which is the market real rate of return. The component \( \pi_2(1+r_2) \) fully compensates the individual for inflation on both the principal and the interest. For the problem as written in (37), this compensation is paid in the second period. If instead it had been paid in the first period while holding its present value constant, neither the utility of the individual nor his choices would be affected since he can always undo this change by buying less bonds. But the present value of the compensation for inflation is \( \pi_2 b_2 \), so that shifting of the compensation from the second to the first period amounts to adding \( \pi_2 b_2 \) to the individual's resources in the first period and subtracting \((1+r_2)\pi_2 b_2\) from his resources in the second period. The upshot of this discussion is that the maximization problem in (37) is equivalent to the following simpler problem

\[ \max \{ u(A_1^* - (1+r_2)A_2^*) + \pi(v_2^* - T_2^* + A_2^* + (1+r_2)A_1^*) + \lambda(n_2) \} \] (41)

subject to the wealth constraint

\[ a_2 \geq 0. \] (42)

The first order conditions for this equivalent problem are

\[ -(1+r_2)u'[\cdot] + \alpha(v'[\cdot] + \pi'[\cdot]) = 0 \] (a)

\[ -u'[\cdot] + \beta(1+r_2)v'[\cdot] + \lambda = 0 \] (b)

where \( \lambda \geq 0 \) is the Lagrange multiplier associated with the constraint in (42). For a financially constrained individual, \( \lambda > 0 \) and \( a_2 = 0 \). For an unconstrained individual, \( a_2 > 0 \) and \( \lambda = 0 \).

B. Production And The Public Sector

I turn now to a description of the production opportunities of the economy and of government's budget constraints. The first and second periods' aggregate production functions are, respectively

\[ Y_1 = f(K_1, L) \] (a)

\[ Y_2 = H(G_1) f(K_2, L) \] (b)
where
\[ K_1 = \sum_{i=1}^{L} k_{1i}; \quad K_2 = \sum_{i=1}^{L} k_{2i}. \]  
(45)

\( f(\cdot) \) is homogeneous of degree one in its arguments and it satisfies the usual properties: positive and decreasing marginal productivities. Labor and capital are complements, so \( f_{KL} > 0 \). \( G_1 \) is the total amount of public expenditures in the first period. It can be thought of as a productive investment designed to rebuild the economy's infrastructure. Its essential feature is that it augments the economy's productive capacity in the second period. Formally
\[ H'(G_1) > 0, \quad H(0) = 1. \]  
(46)

If government does not invest in rebuilding the infrastructure, the production function in period 2 remains the same as in period 1. Positive amounts of public outlays augment the productive capacity of the economy in period 2 as shown by equations (44b) and (46).

Public expenditure in the first period can be financed by either lump-sum taxes, issuance of bonds that are repaid in the second period, or by printing money. There is no public investment in infrastructure in the second period. In this period lump-sum taxes are imposed only in order to repay the national debt issued in the first period. The government budget constraints in periods 1 and 2 are given, respectively, by

\[ G_1 = \sum_{i=1}^{L} \left[ (1+r_{1b}) \frac{b_{2i}}{m_{2i}} \right] + \left[ \sum_{i=1}^{L} \frac{m_{2i}}{m_{1i}} \right] - \sum_{i=1}^{L} b_{1i} \]  
\[ G_2 = \sum_{i=1}^{L} \left[ (1+r_{2b}) \frac{b_{2i}}{m_{2i}} \right] \]  
(47a)

(47b)

where \( R \) is a reparations factor between zero and one which reflects the fact that part or all of the receipts from lump-sum taxes and issuance of bonds are confiscated for payment of war reparations. Seigniorage revenues are free from this confiscatory payment.\(^{49}\)

Assuming competitive factor markets, the real wage rate and the return to capital in the second period are given by
\[ w_2 = H(G_1) f_L(K_2, L) \]  
(48a)

\[ r_2 = H(G_1) f_K(K_2, L) \]  
(48b)

where \( f_L \) and \( f_K \) are the partial derivatives of \( f(\cdot) \) with respect to labor input and capital, respectively. Inserting (47b) and (48b) into (47a) and rearranging we obtain
\[ G_1 = L\left[ \left( \frac{T_2}{1 + H(G_1) f_K(K_2, L)} \right) + (1+r_{2b}) \frac{m_2}{m_1} \right] \]  
(49)

where bars over variables denote per capita values of those variables. Equation (49) incorporates the first and second periods' budget constraints from (47) as well as the effect of a change in government expenditures on the return to capital. It implies that given \( m_1 \) and \( b_1 \) which are inherited from the past only three out of the four fiscal variables \( T_1, T_2, r_2 \) and \( G_1 \) are independent. Once the political system produces a decision about \( T_1, T_2 \) and \( r_2 \), \( G_1 \) is determined by (49).

C. General Characterization of Individuals' Preferences Towards the Size of the Public Sector and the Structure of Taxation

In order to determine what the fiscal policy of a government that satisfies the desires of the majority of voters is likely to be, it is necessary to evaluate the effect of this policy on the welfare of voters. Since \( G_1 \) is fully determined once a choice of the vector \( (T_1, T_2, r_2) \) has been made, it suffices to evaluate the effect of each of the components in this vector on individual welfare, subject to the constraint in (49). Let \( V(G_1, T_1, T_2, r_2) \) be the maximum value of utility that an individual who solves the maximization problem in (41) achieves for given values of \( G_1, T_1, T_2, \) and \( r_2 \). \( V(\cdot) \) is the indirect utility of the individual as a function of the four fiscal policy variables chosen by government. In order to evaluate the effect of changes in the components of the vector \( (T_1, T_2, r_2) \) on this indirect utility, differentiate \( V(\cdot) \) totally with respect to each of these components. Using the envelope theorem this

\(^{49}\) For simplicity the reparations factor for seigniorage revenues is assumed to be zero. The main qualitative results of this section would be unaffected if it had been positive but smaller than the fraction of reparations extracted from ordinary taxes.
\[
\frac{dG_1}{dt_1} = -\lambda \frac{\partial V}{\partial t_1} \left[ H^t(f_L + a z K f_k) \frac{dG_1}{dt_1} + H(f_L a z^2 f_k) \frac{dK_2}{dt_1} \right] - (1 + r_2)
\]  
(a)

\[
\frac{dG_1}{dt_2} = \alpha V \left[ H^t(f_L + a z f_k) \frac{dG_1}{dt_2} + H(f_L a z^2 f_k) \frac{dK_2}{dt_2} - 1 \right]
\]  
(b) (50)

\[
\frac{dK_2}{dt_2} = -n_2 u + \nu^* \left[ H^t(f_L + a z f_k) \frac{dG_1}{dt_2} + H(f_L a z^2 f_k) \frac{dK_2}{dt_2} \right]
\]  
(c) (51)

where \( dG_1/dt \) and \( dK_2/dt \) (I=\( T_1, T_2, x_2 \)) are the total general equilibrium changes in \( G_1 \) and in \( K_2 \) as a result of a change in \( I \). \( dG_1/dt \) summarizes the total change in \( G_1 \) as a result of a change in \( I \) because of three types of effects. First, because any change in \( I \) (I=\( T_1, T_2, x_2 \)) affects \( G_1 \) directly through the government budget constraint in (49). Second, because the changes in \( I \) and in \( G_1 \) trigger general equilibrium effects that are due to capital stock, \( K_2 \) changes. As a consequence the returns to capital and labor change as can be seen from (48). Hence, \( G_1 \) changes not only because of the direct change in either of \( T_1, T_2, \) or \( x_2 \) but also because of the induced change in \( r_2 \). Finally, the changes in returns to factors as well as in the parameters of governmental financial policy induce adjustments in real money balances, \( \bar{m}_2 \), which (as can be seen from (49)) also affect \( G_1 \).

Similarly \( dK_2/dt \) summarizes the total effect on \( K_2 \) of two distinct changes induced by the change in \( I \). First, \( K_2 \) changes because of the direct effects that a change in either of \( T_1, T_2 \) or \( x_2 \) has on the demand for capital by individuals. Second, the change in \( K_2 \) as well as the change in \( G_1 \) trigger changes in returns to factors of production, as can be seen from (48). This causes an additional simultaneous realignment of the capital stock desired by individuals. This discussion also suggests that \( dG_1/dt \) and \( dK_2/dt \) are determined simultaneously. It is shown in part 4 of the Appendix that

\[
\frac{dg_1}{dt_1} = \frac{g_1}{d1} \left[ A(1 + r_2) \frac{a_m}{1} + 1 - R \right] + B \frac{a_m}{a_l}
\]

\[
\frac{dg_1}{dt_2} = \frac{g_1}{d2} \left[ A(1 + r_2) \frac{a_m}{1} + 1 - R \right] + B \frac{a_m}{a_l}
\]

\[
\frac{dg_1}{dt_2} = \frac{g_1}{d2} \left[ A(1 + r_2) \frac{a_m}{1} + 1 - R \right] + B \frac{a_m}{a_l}
\]

and

\[
\frac{dg_1}{dt_2} = \frac{g_1}{d2} \left[ A(1 + r_2) \frac{a_m}{1} + 1 - R \right] + B \frac{a_m}{a_l}
\]

where \( g_1 \) is the short-run elasticity of \( \bar{m}_2 \) with respect to \( 1 + r_2 \), \( \frac{a_m}{a_l} \) (I=\( T_1, T_2 \)) are the direct effects of changes in current and future taxes on average money balances in the second period, \( \frac{a_m}{a_l} \) (I=\( T_1, T_2, x_2 \)) are the direct effects of changes in current and future taxes and inflation on the capital stock in the second period, and \( ^{51} \)

\[
A = L(1 + r_2)^2 \left[ 1 - H(f_L \frac{aK_2}{aw_2} + f_k \frac{aK_2}{ar_2}) \right]
\]

\[
B = L(1 + r_2)^2 \left[ 1 - H(f_L \frac{aK_2}{aw_2} + f_k \frac{aK_2}{ar_2}) \right]
\]

The crucial ultimate expressions for evaluating individual attitudes to different types of taxes and to public expenditures are in equations (50). Each of those expressions is composed of at least three qualitatively similar (across equations) terms. First are the direct negative effects of increased taxes on welfare. Those terms are \( -p'v(1 + r_2) \) for an increase in \( T_1 \), \( -p'v \) for an increase in \( T_2 \), and \( -q'v \) for an increase in \( x_2 \). Second are the contributions of the changes in governmental expenditures on welfare through the consequent changes in productivity. This element is represented by the terms in \( dG_1/dt \), I=\( T_1, T_2, x_2 \). Third, there are induced general equilibrium effects on capital which affect individual welfare by changing returns to capital and labor. This is captured by the terms

\[ ^{50} \text{In deriving (50a) use has been made of the fact that either } \lambda = 0 \text{ or } \partial x_2/\partial t_1 = 0 \text{ and of the first order condition in (43b). Here } f_L \text{ and } f_K \text{ denote the second partial derivatives of } f(\cdot) \text{ with respect to its arguments.} \]

\[ ^{51} \text{In general I follow the convention of using partial derivative symbols to denote direct effects. The explicit expression for } D \text{ is given in part 4 of the Appendix.} \]
Finally, an increase in current taxes, \( T_1 \), forces financially constrained individuals to reduce their current consumption even further, decreasing their welfare. This is represented by \(-1\). Obviously, for financially unconstrained individuals \( \lambda = 0 \).

It is shown in part 5 of the Appendix that \( \lambda \frac{k_2}{\mu_2} \) is negative.\(^{52}\) Since \( f_{1K} > 0 \) this implies \( f_{1K}(\lambda \frac{k_2}{\mu_2}) < 0 \). The sign of \( \lambda \frac{k_2}{\mu_2} \) depends on whether the substitution effect of an increase in \( r_2 \) dominates the income effect. Assuming that the substitution effect dominates, \( \lambda \frac{k_2}{\mu_2} \) is positive and \( f_{KK}(\lambda \frac{k_2}{\mu_2}) < 0 \). According to part 5 of the Appendix, it is shown that if the behavior of aggregate money balances is dominated by unconstrained individuals, \( \lambda \frac{\mu_2}{\mu_2} \) is positive.\(^{53}\) Assuming that when the real rate goes up people decrease their holdings of money balances, \( \lambda \frac{\mu_2}{\mu_2} \) is negative. Since \( f_{1K} > 0 \) and \( f_{KK} < 0 \) this implies that \( B \) is positive. It is also shown in part 4 of the Appendix that under a weak sufficient condition \( D \) is positive as well. The rest of the discussion is conducted, therefore, under the presumption that \( A, B, \) and \( D \) are all positive.

Consider next the sign of \( d\lambda \frac{k_2}{d1} \) in equation (52). Since \( \lambda \frac{k_2}{\mu_2} \) and \( \lambda \frac{k_2}{r_2} \) have opposite signs, the sign of \( f_{1}(\lambda \frac{k_2}{r_2}) + f_{K}(\lambda \frac{k_2}{r_2}) \) is ambiguous. I assume that whatever the sign of this expression, it does not dominate the sign of the entire expression in brackets on the right-hand side of (52). Since \( A > 0 \) this implies that the sign of \( d\lambda \frac{k_2}{d1} \) is dominated by the direct effects of changes in the tax rate \( T_1, T_2, \) and \( \lambda_2 \) on the capital stock.\(^{54}\) If consumption smoothing is the major force determining the demand for assets, the direct effect of an increase in \( T_1 \) on \( k_2 \) is negative since unconstrained individuals decrease their capital stock in order to maintain first-period consumption. Since the direct effect dominates and \( A > 0 \), this implies \( d\lambda \frac{k_2}{d1} < 0 \). It is shown in part 5 of the Appendix that \( \lambda \frac{k_2}{r_2} \) is positive. Since this effect dominates and \( A > 0 \), this implies \( d\lambda \frac{k_2}{d1} > 0 \). Finally, if substitution effects dominate portfolio behavior \( \lambda \frac{k_2}{r_2} \) is positive implying, since \( A > 0 \) and since direct effects dominate, that \( d\lambda \frac{k_2}{d1} > 0 \) as well. The upshot is that if direct effects dominate

\[
\frac{d\lambda}{dt_1} < 0; \quad \frac{d\lambda}{dt_2} > 0; \quad \frac{d\lambda}{dx_2} > 0. \tag{54}
\]

D. The Effect Of Reparations On Current Taxes

Consider a ceteris paribus increase in current taxes that is meant to increase future productivity by increasing \( G_1 \). Since \( \lambda \frac{k_2}{r_1} \) and \( \lambda \frac{\mu_2}{r_1} \) are both negative,\(^{55}\) the expression for \( dG_1/dT_1 \) in equation (51a) is negative unless \( R \) is sufficiently small than one. If \( R \) is sufficiently large to make \( dG_1/dT_1 \) negative an increase in current taxes, taking all the readjustments in the economy into consideration, actually decreases government investments in infrastructure.\(^{56}\) This effect obviously decreases the welfare of all individuals. This is confirmed formally by the fact that the coefficient of \( dG_1/dT_1 \) in (50a) is always positive so that a negative overall response of \( G_1 \) to an increase in \( T_1 \) contributes a negative term to \( dV/dT_1 \).

If in addition the individual is financially constrained, \( \lambda > 0 \) and an increase in \( T_1 \) decreases the individual's welfare also by forcing him to decrease his first-period consumption. Since from (54) \( d\lambda \frac{k_2}{d1} < 0 \), the only term that contributes a positive quantity to \( dV/dT_1 \) is \( \lambda \frac{\mu_1}{a_2} \frac{d\lambda}{dT_1} \). It reflects the fact that the welfare of wealthy individuals goes up due to the increase in the rate of return on wealth triggered by the decrease in the capital stock. For financially constrained individuals this effect is obviously zero (since \( a_2 = 0 \)), so they are always unambiguously worse off when current taxes are increased if \( dG_1/dT_1 \leq 0 \). Actually, it is likely that for most individuals in the economy, except perhaps the very wealthy, the positive effect of an increase in \( T_1 \) on welfare through a higher return will be dominated by the decrease in \( G_1 \) as well as by the other negative terms in (50a).

In conclusion when (as was the case in Germany after May 1921) the

\[\text{52}The\ intuitions\ is\ that\ when\ current\ resources\ are\ larger,\ unconstrained\ individuals\ want\ to\ transfer\ more\ resources\ to\ the\ future.\]

\[\text{53This\ just\ confirms\ that\ money\ is\ a\ normal\ good.}\]

\[\text{54The\ effects\ which\ are\ dominated\ are\ the\ changes\ in\ the\ capital\ stock\ that\ are\ due\ to\ changes\ in\ factor\ returns\ because\ of\ the\ change\ in}\ G_1.\]

\[\text{55The\ negativity\ of}\ \lambda \frac{\mu_2}{r_1}\ \text{is\ demonstrated\ in\ part}\ 5\ \text{of\ the\ Appendix.}\]

\[\text{56Since}\ \lambda \frac{k_2}{r_1}\ \text{and}\ \lambda \frac{\mu_2}{r_1}\ \text{are\ both\ negative,\ this\ obviously\ occurs\ for}\ R\ \text{smaller\ than}\ \text{one.}\]

\[\text{57}The\ calculation\ is\ the\ same\ as\ the\ above.\]

\[\text{58}The\ calculation\ is\ the\ same\ as\ the\ above.\]
expected reparations factor, R, is relatively high all individuals in the
Economy, except perhaps the very wealthy, strictly prefer no current taxes
at all. I believe this result goes a long way in explaining the low
proportion of taxes in Germany during the hyperinflation. Moreover, it can
be used to rationalize the further decrease in this proportion in 1922 and
1923 (Table 4) as the consequence of an increase in the anticipated value
of R following the announcement, in May 1921, of a reparations sum that was
several times the size of GNP. A lot has been made of the fact that right-
wing parties in Germany at the time managed to block taxes on capital while
left-wing parties managed to block taxes on labor.\footnote{The analysis of this
subsection is consistent with the view that blocking was so easy because of
the underlying common interest of both parties to minimize current non-
inflationary tax payments.}

Even in the absence of reparations it is likely that some deficit
financing would have been created because of the relatively large number of
financially constrained individuals after the war. But the reparations
transformed this tendency from a stream into a torrent of deficits.

E. The Effect of Reparations on Bond Financing of the Deficit

Consider a ceteris paribus increase in current bond issues designed to
increase future productivity by increasing $G_1$. Since this is equivalent to
an increase in future taxes, $T_2$, the effect of such a change on individual
welfare can be analyzed by focusing on $dW/dT_2$ in equation (50b). Since
$dK_2/dT_2$ is positive and $dG_1/dT_2$ is negative,\footnote{The evidence on the behavior of seigniorage during the German hyperinflation is
consistent with the view that it is smaller than one. See Section 2.}
the sign of the expression for $dW/dT_2$ in (51b) is ambiguous in general.
But the larger is $R$, the smaller $1-R$ and the more likely it is that $dG_1/dT_2$ is negative or positive
but small. Hence, a high reparations factor makes it more likely that
$dW/dT_2$ is either positive but small or negative. Individuals with small
wealth are likely to have, ceteris paribus, a preference for future taxes
since such taxes increase the real wage by increasing the capital stock.
On the other hand individuals with large amounts of wealth are likely to
dislike future taxes since the increase that such taxes induce in the
capital stock lowers the return to their wealth. Hence, it is likely that
individuals in the economy will be opposed or in favor of bond financing of
public investments depending on whether they are wealthy or not. The larger
is $R$ the more likely it is that the majority will be opposed to bond
floating as a method for financing the deficit. This conclusion is
consistent with the observation that in 1922 and 1923 Germany financed only
negligible parts of its deficit by borrowing from the public.

F. Voters Preferences for Inflationary Finance
In the Presence of Reparations

If $\sigma$, the short run elasticity of money demand with respect to
inflation, is smaller than one in absolute value,\footnote{Examples from Bresciani-Turroni (1937) pp. 57-60, Maier (1975), and Alesina (1987).
This is demonstrated in part 6 of the Appendix which confirms the usual presumption
that money is a normal good.} the effect of an increase in $\sigma$ on governmental investment in infrastructure and therefore on
productivity is unambiguously positive (equation (51c)). Hence, the term
involving $dG_1/dT_2$ contributes a positive element to the expression for
$dW/dT_2$ in equation (50c). On the other hand the tax on real money balances
taken alone decreases $\sigma$. The magnitude of this decrease is larger for
individuals with higher real money balances as can be seen from the term
$-\phi_0\mu$. Finally, as before, the increase in the capital stock triggered by
substitution away from money is liked or disliked by the individual
depending on whether he has a small or a large amount of wealth. Thus, in
general, more wealthy individuals who also have higher real balances are
more likely to be against inflationary finance and less wealthy individuals
are likely to be for it. But even the more wealthy may be in favor of
inflationary finance if the overall effect of an increase in such finance
on the rate of return to wealth is positive. This total effect is given by

$$
\frac{dG_1}{dt_2} + \frac{dK_2}{dK_t} \frac{dG_1}{dt_2} 
$$

and is positive if the marginal productivity of governmental investments in
infrastructure, $H_f K_s$, is large. Due to social and other disruptions caused
by the war, this marginal productivity is likely to have been large after
the war in most European countries and in Germany in particular.
8. CONCLUDING REMARKS

The evidence presented in this paper is consistent with the surprising conclusion that during most of the German hyperinflation, government possessed the ability to increase seigniorage temporarily by stepping up the rate of monetary growth. Moreover, expectations seem to have lagged quite substantially behind actual development particularly when inflation and depreciation shifted from one range to another. This is consistent with the view that the public had difficulties in separating persistent from transitory shifts in government's objectives. This informational limitation as well as lags in the adjustment of money-saving institutions enabled the Reichsbank to maintain and even increase seigniorage in spite of the fact that real balances were on an almost continuously downward course during the hyperinflation. It is therefore reasonable to expect that under lower inflationary circumstances there are similar, and most likely longer, lags in the full adjustment of real balances and of expected inflation. Those lags make it possible and rational for governments with strong time preference to increase the rate of inflation in order to temporarily increase seigniorage revenues. As expectations and institutions adjust, money demand gradually decreases further, making it necessary for such governments to speed up the rate of inflation in order to prevent seigniorage from decreasing.

This process is self-limiting since after a while real money balances have decreased so much that the present value of inflation tax revenues becomes sufficiently small, in comparison to the present value of the costs of inflation to make a change in monetary regime worthwhile. At this point hyperinflation is stabilized. Thus, the entire process of accelerating inflation followed by an ultimate stabilization can be viewed as the consequence of an attempt by a government with a strong desire for seigniorage revenues to extract them for as long as the lags in the adjustment of expectations and of institutions make the present value of such taxes net of the costs of inflation, sufficiently large. This observation also suggests that substantial amounts of revenues from seigniorage can be obtained only temporarily. The evolution of hyperinflation in Germany and its stabilization at the end of 1923 conformed to this pattern. In the German case, an additional incentive for stabilizing at that particular time was the creation of the Dawes plan and the not-unrelated decrease in government's financing of passive resistance. The plan reduced the immediate menace of reparations on receipts from ordinary taxes, making it politically feasible to return to conventional methods of financing the budget.

The German central bank was also very slow in adjusting its discount rate in the face of rising inflation. As a consequence it shared the revenue from seigniorage with those parties in the private economy, like industrialists, who had access to the discount window. The discount rate (measured per year) was 5% until July 1922 and rose to 10% at the end of that year. It then gradually rose from 10% at the beginning of 1923 to 90% at its end. The smallness of those rates by comparison to inflation and to market rates in 1923 is rather surprising. It is noteworthy that since the end of May 1922 the Reichsbank became legally independent of the central government and was responsible to a board of directors, many of whom were private bankers and industrialists. Contrary to the Allies' expectation the Reichsbank did not use its newly acquired power to curb the rate of monetary expansion. Instead, from June 1922, it discounted private commercial bills along with Treasury bills, thus giving the business sector a share of seigniorage revenues. This raises the possibility that the very slow adjustment of the discount rate was the consequence of a deliberate policy designed to transfer resources to sectors that were heavily

---

60 Seigniorage did not systematically decrease as inflation accelerated and sometimes increased during the Austrian, Hungarian, and Polish hyperinflations in the twenties (Bental and Eckstein (1987)). This finding supports the view that in other hyperinflations, too, there were lags either in the adjustment of expectations or in the adjustment of money demand or in both.


62 The average monthly discount rates in 1923 (in percentages per year) were

<table>
<thead>
<tr>
<th>January</th>
<th>February</th>
<th>March</th>
<th>April</th>
<th>May</th>
<th>June</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>12</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>July</th>
<th>August</th>
<th>September</th>
<th>October</th>
<th>November</th>
<th>December</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>30</td>
<td>75</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
</tbody>
</table>

During 1924 it was 108.


APPENDIX

1. DERIVATION OF A LINEAR APPROXIMATION FOR $g_1$.
Inserting (10) in (7)

$$g_1 = \mu_{t+1} e^{-\alpha_{t+1} \psi} \equiv \psi(\mu_{t+1}, \psi_{t+1}).$$

Equations (16) and (17) in the text are obtained by expanding $\psi(\cdot)$ linearly around the point $\mu_{t+1} = \psi_{t+1} = \mu_M$. Note that this point corresponds to a deterministic nonstochastic steady state with no real growth in which the money supply grows at the constant rate $\mu_M$.

2. DERIVATION OF EQUATION (19).
Taking expected values, conditional on $I_t$, of both sides of (12) and rearranging

$$\psi_{t+1} = E[\psi_{t+1} | I_t] = \frac{1}{1+\alpha} I_t \psi_{t+1} + \alpha E[\psi_{t+2} | I_t]$$

where

$$E[\psi_{t+2} | I_t], \ j \geq 1.$$ (A2)

Leading (12) by $j$ periods, taking expected values as of period $t$ (conditional on $I_t$) we, similarly, obtain

$$E[\psi_{t+j} | I_t] = \frac{1}{1+\alpha} \left[ \psi_{t+1} + \alpha E[\psi_{t+j} | I_t] \right], \ j \geq 1.$$ (A3)

Using (A3) recursively in (A1) add infinitum

$$\psi_{t+1} = \frac{1}{1+\alpha} \sum_{j=1}^{\infty} \left( \frac{\alpha}{1+\alpha} \right)^{j-1} t^{*+j}.$$ (A4)

Since the public knows that the behavior of money growth is given by (18) and since $E[\psi_{t+j} | I_t] = 0$ for all $j \geq 1$

$$\psi_{t+j} = 0 + BE[\dot{p}_{t+j} | I_t], \ j \geq 1.$$ (A5)

Cukierman and Meitzer (1986b, p. 1105) show that given (14), (15), (18), and the distributions of $v$ and $e$.
\[ E[p_{t+1}|I_t] = \frac{\beta}{\beta^k - 1} \sum_{s=0}^{\infty} \lambda^s [\nu_{t-s} - \theta] A \]  \hspace{1cm} (A6)

where \( \lambda \) is given by (21a). Due to (15)

\[ E[p_{t+1}|I_t] = \sigma \beta E[p_{t+1}|I_t], j \geq 1. \]  \hspace{1cm} (A7)

Substituting (A6) into (A7), substituting the resulting expression into (A5) and using the resulting expression in (A4)

\[ s_{t+1} = \frac{1}{1+\alpha} \left( \frac{p_{t+2}}{(1+\alpha)^2} + (\alpha - \lambda) \sum_{j=1}^{\infty} \frac{\beta^j}{(1+\alpha)^j} \sum_{s=0}^{\infty} \lambda^s [\nu_{t-s} - \theta] A \right). \]  \hspace{1cm} (A8)

Equation (19) in the text is obtained by summing up one of the infinite geometric progressions in (A8) and by rearranging.

3. Derivation Of Equation (27) And Demonstration Of The Uniqueness Of The Solution For \( B_0 \) And \( B \).

Since \( 0 < \rho < 1 \) and \( 0 \leq \sigma \leq 1 \),

\[ \beta \sigma^2 < 1. \]  \hspace{1cm} (A9)

Footnote 11 of Cukierman and Meltzer (1986b) implies \( 1 \geq \rho - \lambda \geq 0 \). In view of this, and (A9)

\[ 0 \leq \frac{\beta \sigma (\rho - 1)}{1 - \rho \beta} < 1 \]  \hspace{1cm} (A10)

which implies in turn that

\[ 0 \leq \frac{\beta \sigma (\rho - 1)}{(1 - \rho \beta)(1 + \alpha(1 - \rho))} < 1. \]  \hspace{1cm} (A11)

If, on average, government operates in the range in which the elasticity of money demand is not larger than unity, it makes sense to approximate seigniorage around a value of \( \mu_m \) that is no larger than \( 1/\alpha \). Assuming this is the case, \( \alpha \mu_m \leq 1 \) which together with (A11) implies

\[ 0 \leq \frac{\beta \sigma (\rho - 1)}{(1 - \rho \beta)(1 + \alpha(1 - \rho))} < 1. \]  \hspace{1cm} (A12)

Equation (A12) implies that the right-hand side of (26b) is always positive. The discussion on page 1108 of Cukierman and Meltzer (1986b) implies that \( \lambda \) is monotonically decreasing in \( B \). Since the right-hand side of (26b) is increasing in \( \lambda \), it follows that it is also monotonically decreasing in \( B \). Since the right-hand side of (26b) is always positive and monotonically decreasing in \( B \), there is a unique value of \( B \) that satisfies this equation. Given this solution equation (26a) implies a unique solution for \( B_0 \).

Equations (A12) and (26b) also imply that \( 1 \geq B > 0 \).

Since \( \rho < 1 \) and \( \beta \sigma < 1 \)

\[ 0 \leq \frac{\beta \sigma (\rho - 1)}{(1 - \rho \beta)(1 + \alpha(1 - \rho))} < 1 \]  \hspace{1cm} (A13)

which implies together with the condition \( \alpha \mu_m < 1 \) that \( 1 \geq B > 0 \).

4. Derivation Of The Expressions For \( dG_1/dt \) (\( t = T_1, T_2, T_3 \)) In Equations (51).

Differentiating (41) totally with respect to \( T_1, T_2 \) and \( \tau_2 \) we obtain respectively

\[ \frac{dG_1}{dT_1} = L^2(1 - R) \frac{(1 - R)T_2}{(1 + r_2)^2} J_1 + (1 + r_2) \frac{d\bar{m}}{dt} \]  \hspace{1cm} (a)

\[ \frac{dG_1}{dT_2} = L^2(1 - R) \frac{(1 - R)T_2}{(1 + r_2)^2} J_2 + (1 + r_2) \frac{d\bar{m}}{dt} \]  \hspace{1cm} (b) (A14)

\[ \frac{dG_1}{dT_3} = L^2(1 - R) \frac{(1 - R)T_2}{(1 + r_2)^2} J_3 + (1 + r_2) \frac{d\bar{m}}{dt} \]  \hspace{1cm} (c)

where

\[ J_1 = H_f k \frac{dG_1}{dt} + H_f \frac{dK_1}{dt} \]  \hspace{1cm} (15)

Differentiating equations (48) totally with respect to the dummy variable \( t \).
\[ \frac{dw_2}{dt} = H' f_2 \frac{dg_1}{dt} + H f_{KL} \frac{dk_2}{dt} \]  

(a)

\[ \frac{dr_2}{dt} = H' f_k \frac{dg_1}{dt} + H f_{KK} \frac{dk_2}{dt} \]  

(b)

Differentiating \( K_2 \) and \( \tilde{m}_2 \) totally with respect to \( t \)

\[ \frac{dK_2}{dt} = \frac{ak_2}{\tilde{a}t} + \frac{ak_2}{\tilde{a}w_2} \frac{dw_2}{dt} + \frac{ak_2}{\tilde{a}r_2} \frac{dr_2}{dt} \]  

(a)

\[ \frac{d\tilde{m}_2}{dt} = \frac{a\tilde{m}_2}{\tilde{a}t} + \frac{a\tilde{m}_2}{\tilde{a}w_2} \frac{dw_2}{dt} + \frac{a\tilde{m}_2}{\tilde{a}r_2} \frac{dr_2}{dt} \]  

(b)

Substituting (A16) into (A17a) and rearranging

\[ \frac{dK_2}{dt} = \frac{1}{L} \frac{aK_2}{\tilde{a}t} + H' \left( f_L \frac{aK_2}{\tilde{a}w_2} f_k \frac{aK_2}{\tilde{a}r_2} \right) \frac{dg_1}{dt} \]  

(A18)

where

\[ Q = 1 - H \left( f_L \frac{aK_2}{\tilde{a}w_2} f_k \frac{aK_2}{\tilde{a}r_2} \right). \]  

(A19)

Substituting (A16) and (A18) into (A17b) and rearranging

\[ \frac{d\tilde{m}_2}{dt} = \frac{1}{Q} \left( a\tilde{m}_2 - \frac{a\tilde{m}_2}{\tilde{a}t} + H' \left( f_L \frac{aK_2}{\tilde{a}w_2} f_k \frac{aK_2}{\tilde{a}r_2} \right) \frac{dg_1}{dt} \right) \]  

\[ + H \left( f_L \frac{a\tilde{m}_2}{\tilde{a}w_2} f_k \frac{a\tilde{m}_2}{\tilde{a}r_2} \right) \frac{aK_2}{\tilde{a}t} + H' \left( f_L \frac{aK_2}{\tilde{a}w_2} f_k \frac{aK_2}{\tilde{a}r_2} \right) \frac{dg_2}{dt} \right). \]  

(A20)

Equations (51) in the text follow by inserting (A18) and (A20) into equations (A14) where

\[ D = Q \left( \frac{1}{1+\gamma} \right)^2 (1-L(1+\gamma)) \left( f_L \frac{a\tilde{m}_2}{\tilde{a}w_2} f_k \frac{a\tilde{m}_2}{\tilde{a}r_2} \right) + L(1-R) \frac{H' f_k}{aK_2 \frac{aK_2}{\tilde{a}r_2}} \]  

(A21)

Since \( H' > 0, f_k > 0, f_L > 0, f_{KL} < 0, \) and \( 1-R > 0 \) jointly sufficient (but not necessary) conditions for \( D \) to be positive are;

\[ \frac{aK_2}{aK_2} + \frac{aK_2}{aK_2} < 0. \]  

(a)

\[ f_L \frac{aK_2}{\tilde{a}w_2} f_k \frac{aK_2}{\tilde{a}r_2} < 0. \]  

(b)  

(A22)

\[ f_L \frac{aK_2}{\tilde{a}w_2} + f_k \frac{aK_2}{\tilde{a}r_2} < 0. \]

(c)

It is shown in part 5 of the Appendix that \( aK_2/a\tilde{w}_2 < 0 \) and that \( a\tilde{m}_2/a\tilde{w}_2 > 0 \). The sign of \( aK_2/\tilde{a}r_2 \) depends on whether the income or the substitution effect dominates. If those two effects make \( aK_2/\tilde{a}r_2 \) small in absolute value relatively to the absolute value of \( aK_2/a\tilde{w}_2 \), (A22a) and (A22b) are dominated by their first terms and are therefore negative as required. It is shown in part 5 of the Appendix that \( a\tilde{m}_2/a\tilde{w}_2 > 0 \). The sign of \( a\tilde{m}_2/\tilde{a}r_2 \) depends on whether the substitution or the wealth effect of an increase in \( \tilde{r}_2 \) dominates. If those two conflicting effects make \( a\tilde{m}_2/\tilde{a}r_2 \) relatively small in absolute value, (A22c) is dominated by its first term and is therefore negative.

Speaking loosely, the above discussion implies that a sufficient condition for \( D > 0 \) is that the signs of the expressions in (A22) be dominated by the terms whose signs are unequivocal.

5. Comparative Statics with Respect to \( w_2 \) and \( t_1 \).

a. Proof that \( aK_2/a\tilde{w}_2 < 0 \).

Since \( K_2 \equiv \sum_{i=1}^{L} k_{2i} \), it is enough to show that \( aK_2/a\tilde{w}_2 \leq 0 \) for all \( i \).
and that it is strictly negative for some i. For an individual who is not constrained, \( \lambda = 0 \) in equation (43b). Totally differentiating equations (39) with respect to \( w_2 \) and solving for \( \frac{\partial a_2}{\partial w_2} \)

\[
\frac{\partial a_2}{\partial w_1} = -\frac{8}{|S|} v^*(n_2^0(1+r_2)u^* + \delta(1+r_2)\delta^2z)
\]  
(A23)

where \(|S|\) is the determinant of the second order partial derivatives of the maximum problem in (41) and is positive by the second order condition for a maximum. The individual's index, i, has been deleted for simplicity. If the nominal rate of interest, \( n_2 \), is positive, (A23) is negative since the second partial derivatives of utility, \( v^*, u^* \), and \( z^* \), are all negative. But \( a_2 = k_2 + b_2 \). If the total quantity of bonds is constant, the decrease in \( a_2 \) takes the form of an equal size decrease in \( k_2 \). Hence, \( \frac{\partial k_2}{\partial w_2} < 0 \) for an unconstrained individual.

For a strictly constrained individual \( a_2 = 0 \), and an increase in \( w_2 \) does not affect \( a_2 \) since the individual desires to decrease \( a_2 \) even further but is unable to because of the constraint \( a_2 \geq 0 \). Hence, \( \frac{\partial a_2}{\partial w_2} = 0 \).

Assuming not all individuals are constrained

\[
\frac{\partial k_2}{\partial w_2} < 0 .
\]

Q.E.D.

b. Proof that \( \frac{\partial a_2}{\partial w_2} > 0 \) when unconstrained individuals dominate.

Since \( \bar{a}_2 = \sum_{i=1}^{L} a_{2i}/L \) and since unconstrained individuals dominate, it is enough to show that \( \frac{\partial m_{2i}}{\partial w_2} > 0 \) for such individuals. For unconstrained individuals \( \lambda = 0 \) and \( a_2 > 0 \). Inserting this constraint into (43), totally differentiating with respect to \( w_2 \) and solving for \( \frac{\partial m_2}{\partial w_2} \)

\[
\frac{\partial m_2}{\partial w_2} = \frac{6}{|S|} u^*v^*
\]

which is positive. Again the individual's index is deleted for simplicity.

Q.E.D.

c. Proof that \( \frac{\partial a_2}{\partial T_1} < 0 \) if \( n_2 > 0 \).

Differentiating (43) totally with respect to \( T_1 \) for an unconstrained individual \( (\lambda=0, a_2 > 0) \) and solving for \( \frac{\partial m_2}{\partial T_1} \)

\[
\frac{\partial m_2}{\partial T_1} = -\frac{3}{|S|} (1+r_2)u^*v^*n_2
\]

which is negative for \( n_2 > 0 \).

For a constrained individual \( a_2 = 0 \) and equation (43a) alone determines \( m_2 \). Differentiating it with respect to \( T_1 \)

\[
\frac{\partial m_2}{\partial T_1} = -\frac{u^*}{S_{mm}}
\]

where \( S_{mm} \) is the second partial of (41) with respect to \( m_2 \) and is negative by the second order condition for a maximum. Since \( u^* < 0 \), \( \frac{\partial m_2}{\partial T_1} \) is negative for a constrained individual as well. Hence,

\[
\frac{\partial a_2}{\partial T_1} < 0 .
\]

Q.E.D.
REFERENCES


Cukierman, A. and Meltzer, A.H.  

and  


and  


Dornbusch, R. and Fischer, S.  

Drazen, A. and Helpman, E.  

Eichengreen, B. and Wyplosz, C.  

Einzig, P.  

Fischer, S.  

Flood, R.P. and Garber, P.  

Flood, R.P. and Garber, P.  

Frenkel, J.A.  

Graham, F.D.  

Grossman, H.I. and V n Huyck, J.B.  

Helpman, E. and Sadka, E.  

and Leiderman, L.  

Holtfrerich, C.L.  

Kydland, F.E. and Prescott, E.  
Maier, C.  

Weltzer, A.H. and Richard, S.  

Morales, J.A.  

Offenbacher, A.  

Parkin, M.  

Piterman, S.  

Sachs, J.  

Sargent, T.J.  


Sargent, T.J.  

Sokoler, M.  

Webb, S.B.  

Young, J.P.  
RAPID INFLATION - DELIBERATE POLICY OR MISCALCULATION?
A COMMENT

ROBERT FLOOD
International Monetary Fund
and
Northwestern University

In his paper Alex Cukierman has taken on one of the important and systematically-ignored aspects of the German Hyperinflation of 1921-23. At issue is finding an explanation for the rising seigniorage toward the end of the inflation. Almost equivalently, we can seek an explanation for the surprisingly high levels of real money balances observed in the final months of the German experience.\(^1\) The latter way of stating the issue is more traditional. Cagan's (1956) inability to explain real money balances in Germany during this period led him to exclude the German observations from August through November 1923 from his famous study. The observations excluded by Cagan involved the most dramatic months of the whole experience and may contain valuable information about the hyperinflationary process. The extent of the difference between the excluded months and the rest of the hyperinflation is documented in Cukierman's Table 1 which shows that monthly inflation rates and money growth rates during these final months dwarf the rates experienced in the rest of the period.

What Cukierman is looking for in this paper is a unified explanation of the entire period. He seeks an explanation in terms of a theory of government preferences for seigniorage revenue. In the theory, the government's preferences are stochastic, and the government has an information advantage in the sense that it knows more about its preferences than the private sector knows. It follows that the government attempts to exploit this information advantage to gain additional seigniorage during periods when seigniorage is most precious to it. When combined with Cagan's model of money demand, the theory gives rise to a function by which private

\(^1\)The well-known relationship between seigniorage and real money balances is given by Cukierman in his equation (7).
agents calculate their expectation of inflation. That equation is numbered (19) in the text. By adding an appropriate unpredictable error to (19) we can get the model's inflation equation. Large seigniorage or surprisingly high real money balances, in this model, are coincident with unexpectedly high money printing and inflation. Cukierman's explanation of the unexpected events is that the German government experienced a surprisingly high desire to inflate in the final months of 1923.

I find the proposed explanation to be internally consistent. To be taken seriously, however, the explanation needs to be subject to testing in the data, which Cukierman has not yet undertaken. Currently the Cukierman explanation is not much different than Cagan's concern that something different was going on in the inflation's final months than was happening previously.

I have a particular interest in these data points and in the explanation of them. In a pair of papers I wrote with Peter Garber (Flood and Garber (1980), 1983) we also tried to explain the data at the end of the German Hyperinflation. Our story, which was first proposed by Cagan, was that toward the end of the period agents began to expect a reform of the money supply process. The possibility of a monetary reform lowered agents' (unconditional) expectation of inflation raising real balances above what they would have been without the reform thought possible. Because they were concerned about the possibility of reform, during the periods prior to the reform the money demanding agents would have experienced unexpectedly high inflation and money printing, which would generate high seigniorage and high real balances. Our papers were devoted largely to making the reform endogenous and calculating agents' probability in each period that the reform would take place in the subsequent period. In Appendix F to Flood and Garber (1980) we combined our probabilities with Cagan's model to estimate the combined model while incorporating the data Cagan discarded. We were, I think, reasonably successful, and until a better explanation of those data points is proposed and implemented I will stick with Cagan's more than thirty-year-old story. The Cukierman model is well thought through and might be a serious contender. I look forward to seeing the empirical implementation of the model.

REFERENCES

Cagan, P.

Flood, R. and Garber, P.

RAPID INFLATION - DELIBERATE POLICY OR MISCALCULATION?
REPLY TO FLOOD

ALEX CUKIERMAN
Tel Aviv University

In his comment Flood contrasts my explanation for the surprisingly high levels of real money balances during the last four months of the German hyperinflation with the explanation that is provided in his work with Garber. My explanation is based on the view that in those months the German government experienced an unexpectedly large increase in its desire for seigniorage revenue the full extent of which was realized by the public only after a while. Flood and Garber’s explanation relies on the idea that real money balances were high because the public already anticipated a monetary reform during those final months. On this view the expected rate of inflation was low not because of gradual learning but because the public assigned a non negligible positive probability to the event of a monetary reform that would eliminate inflation. In both cases the expected rate of inflation should be relatively low during August through November 1923. However, if expected inflation was low because of a strong belief that stabilization was imminent, expectations should have adjusted to the neighborhood of actual inflation as soon as stabilization took place. If, on the other hand, expected inflation was relatively low during the last months of the hyperinflation because of gradual learning, we should expect inflationary expectations to come down only gradually, even after the implementation of stabilization. As pointed out by Flood a definite discrimination between the two hypotheses must await more complete empirical verification. However, recent new data on daily and monthly interest rates in the aftermath of the stabilization provides useful preliminary discriminatory evidence.

Table 1 provides data on daily rates of interest from November 22, 1923 through December 22, 1923 at monthly rates in decimals (thus the 0.68 figure on December 22 represents a rate of interest of 68 percent per month). From November 20, 1923 on, inflation decelerated dramatically. Yet, as can be seen from the table, poststabilization daily rates are.
until the end of November, substantially higher than their counterparts until October 1923 (column 3 of Table 3 in the paper). This is consistent with the view that expected inflation was larger in the first ten days of stabilization than in the previous several months during which inflation was raging at astronomical rates. Under the Flood and Garber hypothesis the probability of stabilization should have gone up with the realization of stabilization, causing the immediate poststabilization expected inflation to be lower than its prestabilization counterpart. Hence the behavior of day money rates seems to be at variance with this hypothesis, but it is consistent with the gradual (but optimal) learning hypothesis. This hypothesis implies that when inflation accelerates it is underestimated for a while, and when it decelerates it is overestimated for a while. The data in Table 1 is broadly consistent with this view.

A possible objection to this conclusion is that expectations adjusted very quickly upon stabilization and that the colossal rates in Table 1 represent almost entirely huge ex ante real rates that were brought about by the poststabilization increased demand for liquidity (Dornbusch (1980)). Truth is probably somewhere in between. It is likely that nominal rates were so high in the immediate aftermath of stabilization because of the increase in money demand as well as the fact that inflationary expectations did not immediately credit the stabilization with full credibility. The data on the differential between nonindexed day rates and indexed month rates in Table 2 supports this view. It suggests that inflationary expectations were non-negligible as far down as March/April of 1924 since the differential is consistently positive until then. From the beginning of May 1924 on, the differential becomes negative creating a traditional positively sloped yield curve in spite of the fact that the month rate was indexed while the day rate was not. This is consistent with the view that inflationary expectations fully subsided only 4 to 5 months after stabilization. It is interesting to note, in this context, that data on inflationary expectations from the capital market suggests that expectations fully adjusted to the new regime following the 1985 Israeli stabilization after a similar length of time (Cukierman (1988)).
REFERENCES

Cukierman, A.

Dornbusch, R.