

## WHEN IS THE MEDIAN VOTER PARADIGM A REASONABLE GUIDE FOR POLICY CHOICES IN A REPRESENTATIVE DEMOCRACY?

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The median voter paradigm (MVP) has been widely used to study the interactions between economic and political behavior. While this approach is easy to work with, it abstracts from institutional detail. This paper explores whether the MVP leads on average to the same policies that would be chosen in a two-party representative democracy (RD). When it does not, the paper fully characterizes the size and magnitude of the average divergence (or bias) between policy choices in MVP and in RD in terms of the degree of polarization between the parties, their relative electoral prospects, and the distribution of electoral uncertainty. The results are then applied to the influential Meltzer and Richard (1981) theory of the size of government.

### 1. INTRODUCTION

THE LAST two decades witnessed the development of political economy models in which the interaction between economic and political behavior is recognized explicitly. A non-negligible part of this literature utilizes the median voter theorem or paradigm (MVP) to derive predictions about the policies that would be chosen in a democratic system.<sup>1</sup> Early examples of this approach are the models in Romer (1975) and Roberts (1977) which were applied by Meltzer and Richard's (1981) paper to provide a political-economic explanation for the growth of transfer payments and taxes in the US during the twentieth century. Following this article, the MVP has been

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<sup>1</sup> This paradigm is sometimes referred to as the "direct democracy" paradigm (see, e.g., Drazen, 2000, chapter 3.4).

used to study the choice of public policies in various areas, such as the formation of tariffs (Mayer, 1984); the determinants of government debt and deficits (Cukierman and Meltzer, 1989; Tabellini and Alesina, 1990); intergenerational redistribution (Tabellini, 1991); the interaction between growth and income distribution (Perotti, 1993; Persson and Tabellini, 1994a); and the political economy of labor market institutions (Saint-Paul, 1996a, 1996b). The MVP also features prominently in a recent extensive survey of political economics and public finance by Persson and Tabellini (1999) and in Drazen's (2000) book on political economy and macroeconomics.

Owing to its simplicity and the fact that it offers a direct mapping from voters' preferences into policy choices, the MVP provides a compact and easy to comprehend characterization of policy choices in a democracy. But, since it abstracts from institutional detail, one may wonder what are the circumstances under which the MVP reasonably approximates policy choices under a well-specified model of representative democracy (RD) where parties compete in elections and the winning party chooses a policy. There are two views regarding this question. One view, due to Hotelling (1929) and Downs (1957), is that although real-life democracies are representative, the MVP provides a good approximation to policy choices under RD.<sup>2</sup> The argument is that in the presence of purely office-motivated political entrepreneurs, political competition forces politicians to converge towards the policy that would have been adopted by the decisive median voter in the population. The other view, whose early proponents are Shepsle and Weingast (1981), is that institutional detail matters for policy choices. In particular, when politicians are not purely office-motivated but also have ideological concerns, policy platforms in an RD need not converge to the median (Wittman, 1983; Hansson and Stuart, 1984; Calvert, 1985; Alesina, 1988; Roemer, 1997, 2001). Therefore, in general, policy choices predicted by the MVP need not coincide with those that would emerge when electoral competition is explicitly taken into account.

In view of the extensive literature that utilizes the MVP, it is surprising that practically no effort has been devoted to investigate the appropriateness of this paradigm given that in practice most policy choices are made in representative democracies. The main purpose of this paper is to open such an investigation and distinguish the set of circumstances under which the MVP provides useful guidance for policy choices under a fully specified model of representative democracy (RD), and when it does not, to

<sup>2</sup> In this sense the MVP is analogous to the competitive paradigm that also abstracts from institutional detail and relies on a fictitious Walrasian auctioneer. The rationale for this abstraction is that, although they disregard many features of reality, competitive equilibrium models are nonetheless useful for understanding a wide range of real-life markets in which participants have relatively little market power.

characterize factors that affect the sign and magnitude of the difference in policy choices between the MVP and RD.<sup>3</sup>

It is well known that, except for some special cases, the median voter theorem requires a unidimensional issue space and single-peaked preferences. Hence our investigation is naturally confined to political systems that satisfy those preconditions.<sup>4</sup> In practice, there are several types of RD that differ in various institutional details. We focus on an RD with two large parties, or party blocks, that cater to two constituencies whose ideologies are located on opposite sides of the center of the political spectrum; obvious examples for this pattern include the Republican and the Democratic parties in the US and the Conservative and the Labor parties in the UK. The parties are headed by party leaders who face electoral uncertainty (i.e. probabilistic voting) and compete in the elections by announcing their respective platforms which commit them to a policy if elected. We then ask how appropriate is the MVP for predicting policy choices in this kind of system.

The modern political economy literature has used various combinations of two extreme paradigms to conceptualize the objectives of candidates competing for office. One is that they are purely office-motivated. The other is that they are only ideologically motivated. We assume here that, although the competing candidates under RD do not have policy preferences of their own, they act as agents for well-organized constituencies (parties) that do have policy preferences. The support that these constituencies give the candidates, or party leaders, once they are elected is stronger if the policies that the leaders are trying to implement are closer to those preferred by their respective constituencies. This creates a tension between the tendency of the party leaders to converge in order to increase their electoral prospects and their tendency to diverge in order to get more support and thereby lower their personal costs of implementing policies once they are in office. This tension is in the spirit of recent work by Aldrich (1995, p. 291), who stresses the role of parties as a resource for candidates, and by Roemer (1997, pp. 480–481), who stresses the role of candidates as agents of particular constituencies.

Owing to electoral uncertainty about the location of the median voter, policy choices under both the MVP and RD are stochastic. Moreover, since under the MVP, policy is chosen by a median voter, whereas under RD it is chosen by elected officials who commit to platforms before the resolution of electoral uncertainty, it is obvious that actual policies under the MVP and

<sup>3</sup> An alternative interpretation of our results is that they provide information about how well electoral competition satisfies, on average, the desires of the median voter. Finding answers to this question may also be of interest for normative reasons.

<sup>4</sup> It should be noted that at least in the context of American politics, there is strong evidence that a one-dimensional policy space is an appropriate simplification. For a discussion of this, see Poole and Rosenthal (1994) and Alesina and Rosenthal (1995, pp. 34–35). More specific statistical evidence appears in Poole and Rosenthal (1991, 1997).

RD will almost never coincide. The paper compares the ex-ante distribution of policies generated by the MVP and RD by comparing their expected policies.

When there are systematic differences between policy choices under the MVP and in RD, we say that the MVP has a “policy bias” relative to the more realistic model of RD. The use of the deviation of the expected value of policy choices under the MVP from the expected value of policy choices under RD to evaluate the usefulness of the MVP is analogous to common practice in econometrics, in which a primary criterion to judge the quality of an estimator is whether it is unbiased. In the present context the magnitude of the bias is a natural first criterion to evaluate the performance of the MVP as an indicator for policy choices under RD.

The paper’s results fall into two groups. The first characterizes the circumstances under which the MVP does not give rise to policy biases. Generally, this occurs when the two parties have either a sufficiently strong tendency to converge towards the center of the political spectrum (full convergence), or when the political system is symmetric so the equilibrium platforms are symmetrically located around the center of the political spectrum (partial but symmetric convergence). In both cases, the MVP is clearly a useful simplification. The second group of results opens by observing that asymmetry in the distribution of electoral uncertainty generally produces a policy bias and proceeds with an in-depth investigation of the sign and magnitude of policy biases when the distribution of electoral uncertainty is symmetric. In particular, the paper fully characterizes the sign and magnitude of the policy bias in terms of the *degree of political polarization* between the two parties, their *relative tendencies to converge* towards the center of the political spectrum, and the distribution of electoral uncertainty. Inter alia, our results suggest that political-economic models that use the MVP provide reasonable guidance for policy outcomes under RD when the polarization between parties is not too large, when the party leaders are sufficiently office-motivated, or when the political system is characterized by strong symmetries. When none of those conditions holds, the paper characterizes the direction and magnitude of the resulting bias in terms of observables such as the degree of effective polarization between parties and the relative electoral odds of the competing candidates. This is done for any (symmetric) distribution of electoral uncertainty.

The paper is organized as follows. Section 2 lays down the basic structure of the model and invokes the median voter theorem to characterize the policy outcome under the MVP. Section 3 characterizes the institutional structure of a representative democracy and solves for the political equilibrium under an RD. Since there is electoral uncertainty, policy choices under the MVP and RD are stochastic. Section 4 compares expected policies under the MVP and RD and derives conditions for which these expected policies will coincide. When they do not, section 4 identifies the factors that

determine the size and direction of the resulting policy biases. Section 5 applies the main results of the paper in the context of a political economy model of tax policy using a variant of the classic Meltzer and Richard (1981) model. This section characterizes the divergence between the income tax rate and the provision of a public good under the MVP and RD in terms of political polarization, the electoral prospects of the parties, and the distribution of electoral uncertainty. Section 6 offers concluding remarks. All proofs appear in the Appendix.

## 2. THE POLICY OUTCOME UNDER THE MVP

The economy consists of a continuum of individual voters who differ with respect to their preferences over a single policy issue. The utility of a voter from policy  $x$  is given by  $U(x|c)$ , where  $c$  is the voter's innate taste parameter. We assume that  $U(x|c)$  is single-peaked and maximized at  $x = c$ . We refer to the voter whose innate taste parameter is larger than those of exactly half of the voters as the *median voter*, and index this voter by a subscript  $m$ .

The MVP states that when preferences are single-peaked, there exists a Condorcet winner, and the policy choice under simple majority rule coincides with the ideal policy of the median voter in the population. The MVP thus abstracts from institutional detail (who gets to propose policies, how many proposals can be made, when does the process end, etc.) and provides a direct mapping from voters' preferences into policy choices.<sup>5</sup> Single-peakedness of  $U(x|c)$  assures that the median voter is decisive in the sense that his most preferred policy,  $x_m \equiv c_m$ , can defeat any other policy under simple majority rule and will therefore be adopted.

## 3. THE POLICY OUTCOME UNDER REPRESENTATIVE DEMOCRACY

### 3.1 *A Model of RD with Ideological Parties and Downsian Candidates*

We consider a two-party system with a right-wing party whose ideal policy is  $c_R$ , and a left-wing party whose ideal policy is  $c_L$ , where  $c_R > c_L$ . These ideal policies represent the policy preferences of the median voter within relatively well-organized, particular constituencies in the population. The two parties are headed by party leaders who compete for office by announcing platforms,  $y_L$  and  $y_R$ , that commit them to the policies that they will carry out if elected.<sup>6</sup> Since we are ultimately interested in comparing

<sup>5</sup> This characterization of the MVP is standard in the spatial theory of voting and in the subsequent literature that has used this paradigm to analyze the interactions between political and economic behavior. See, e.g., chapter 4 in Enelow and Hinich (1984), Persson and Tabellini (1994b), and chapter 3 in Drazen (2000).

<sup>6</sup> Although we refer to a *single* leader within each party, it is also possible to think of the party leadership as consisting of a narrow group of individuals who are candidates for major cabinet positions (this interpretation is perhaps more appropriate for parliamentary democracies). The

the outcome under RD with that under the MVP where policy preferences of political leaders play no role, we assume, in the spirit of Downs (1957), that the leaders of both parties **do not have policy preferences of their own**.<sup>7</sup> Accordingly, we assume that if party  $j$  loses the elections, its leader gets a constant utility which, for simplicity, we normalize to 0. If party  $j$  wins the elections, the utility of its leader, after committing to a platform  $y_j$ , is given by

$$V_j(y_j) \equiv h_j - |y_j - c_j|, \quad j = L, R, \quad (1)$$

where  $h_j$  is the value that the leader assigns to holding office, and  $|y_j - c_j|$  is the personal cost of effort that the leader incurs while in office. This cost increases with the distance of the committed policy,  $y_j$ , from the ideal policy of the party's center,  $c_j$ , thereby reflecting the view that an elected leader is likely to get less political, economic, and moral support from the constituency he represents when trying to implement a policy which is further away from the constituency's ideal policy. Laver and Schofield (1990) and Laver and Shepsle (1990) have emphasized the importance of intra-party politics for the choice of national policies. One element of intra-party politics concerns the conflict between the personal ambition of party leaders to win the elections and reap the benefits from holding office and the insistence of party centers on "appropriate" ideologies.<sup>8</sup> This conflict is reflected in our model of RD by the tradeoff between electoral prospects and the party's support once in office.

To reflect the uncertainty inherent in any electoral competition, we assume that the two parties do not exactly know the taste parameter of the median voter,  $c_m$ , and believe that it is distributed on the interval  $[c_0, c_1]$  according to a twice differentiable distribution function  $G(c_m)$  and a strictly positive density function  $g(c_m)$ .<sup>9</sup> Define  $\hat{c}_m$  as the median of the distribution

assumption that campaign platforms commit the party leaders to policies if elected is common in the literature. It relies on the presumption that if an elected official fails to deliver on his campaign promises, voters may refrain from voting for him in the future, so that it never pays to renege on campaign promises.

<sup>7</sup> This assumption is not new: for instance, Roemer (1996) models electoral competition as a contest between two teams, each consisting of an ideological party and a Downsian leader. It should be noted that our framework differs from earlier models of electoral competition, where the candidates have ideological preferences of their own and therefore care about policies even when they lose the elections (e.g. Wittman, 1977, 1983; Hansson and Stuart, 1984; Calvert, 1985; Alesina, 1988; Roemer, 1997).

<sup>8</sup> A case in point is Benjamin Netanyahu's promise during the 1996 Israel elections to honor the Oslo accord that his party, the Likud, had opposed up to that point. After winning the elections, Netanyahu indeed had a hard time getting his party's support for implementing various elements of the Oslo accord like the partial withdrawal of Israeli troops from the city of Hebron.

<sup>9</sup> For a comprehensive treatment of electoral competition under electoral uncertainty (i.e. probabilistic voting models), see Coughlin (1992). Electoral uncertainty may be due, for instance, to the dependence of voters' turnout on an uncertain state of nature like the weather (Roemer, 2001, chapter 2).

of  $c_m$ . That is, the probability that  $c_m \leq \hat{c}_m$  is exactly  $\frac{1}{2}$ . Hence, it is natural to refer to  $\hat{c}_m$  as the “center of the political spectrum.” We make the following assumptions on the distribution of the median voter’s types and on the taste parameters of the two parties:

**Assumption A1.**  $M(c_m) \equiv G(c_m)/g(c_m)$  is increasing and  $H(c_m) \equiv (1 - G(c_m))/g(c_m)$  is decreasing in  $c_m$ .

**Assumption A2.**  $h_L > 2M((c_L + c_R)/2)$  and  $h_R > 2H((c_L + c_R)/2)$ .

**Assumption A3.**  $c_0 < c_L < \hat{c}_m < c_R < c_1$ .

Assumption A1 ensures that the objective functions of the party leaders are nicely behaved. This assumption is satisfied by standard continuous distributions (e.g. uniform, exponential, and normal). Assumption A2 ensures that the values that the party leaders assign to holding office are sufficiently large so that, in equilibrium, both parties converge at least somewhat towards the center of the political spectrum. Assumption A3 ensures that the support of  $g(\cdot)$  is “sufficiently wide.” Moreover, it implies that more than half of the median voters’ types are more right-wing than the left-wing party and more than half of them are more left-wing than the right-wing party. This assumption seems consistent with the casual observation which suggests that the political centers of organized parties are at least somewhat away from the center of the political spectrum. It is also consistent with the view that, because they are located further away from the center, individuals with more extreme preferences are more likely to incur the costs of collective action needed to set up the organizational machinery of a party.

Since the party leaders are uncertain about the position of the median voter, the outcome of the election from their point of view is random. Let  $P_j(y_L, y_R)$  denote the probability that party  $j$  ( $j = R, L$ ) wins the elections given the pair of platforms that was announced. Then, using equation (1) and recalling that we normalized the utility of party leaders when they are out of office to 0, the expected payoff of party  $j$ ’s leader, before committing himself to a given platform, is

$$\pi_j(y_L, y_R) = P_j(y_L, y_R)[h_j - |y_j - c_j|], \quad j = L, R. \tag{2}$$

Equation (2) reveals that each leader has two considerations when choosing a platform. First, the leader takes into account the impact of the platform on his chances to be elected. Second, conditional on winning the elections, the leader wishes to minimize the deviation of the platform from the party’s ideal position,  $c_j$ .

### 3.2 Political Equilibrium under RD

In a political equilibrium, party leaders choose the platforms of their parties with the objective of maximizing their expected payoffs, taking the platforms of their rival as given. The equilibrium platforms are denoted by  $y_L^*$  and  $y_R^*$ . In order to characterize the equilibrium platforms, we first need to consider the outcome of the elections. Given  $y_j$ , the utility of a voter whose innate taste parameter is  $c$ , if party  $j$  is elected, is  $U(y_j|c)$ . If  $y_R = y_L$ , all voters are indifferent between the two parties, so they randomize their votes and  $P_R(y_L, y_R) = P_L(y_L, y_R) = \frac{1}{2}$ . Otherwise, assuming for simplicity that  $U(\cdot|c)$  is symmetric around  $c$ , each voter votes for the party whose platform is closer to his innate taste parameter. The ideal policy of the voter who is just indifferent between the two parties is  $\hat{y} = (y_L + y_R)/2$ .<sup>10</sup> All voters with taste parameters  $c < \hat{y}$  vote for the left-wing party, while all voters with taste parameters  $c > \hat{y}$  vote for the right-wing party. Since the taste parameter of the median voter is  $c_m$ , it follows that if  $\hat{y} > c_m$ , more than 50 percent of the voters prefer the left-wing party so this party wins the elections. If  $\hat{y} < c_m$ , then more than 50 percent of the voters prefer the right-wing party so this party wins the elections, and if  $\hat{y} = c_m$ , each party gets 50 percent of the votes.

**Lemma 1.** The equilibrium platforms,  $y_L^*$  and  $y_R^*$ , are such that either  $c_L \leq y_L^* < y_R^* \leq c_R$ , or  $y_L^* = y_R^* = \hat{c}_m$ .

*Proof.* See the Appendix.

Lemma 1 shows that, in a political equilibrium, the left-wing party never adopts a more right-wing platform than the right-wing party, and vice versa. Hence we can restrict attention to cases where  $y_L \leq y_R$ . In addition, Lemma 1 shows that, in equilibrium, the platforms of the two parties fully converge only if both parties choose the platform  $\hat{c}_m$ , which represents the exact center of the political spectrum. It should be noted that Lemma 1 does not depend on the particular functional form of the utility function of the party leaders; it only requires that the utility of each leader decreases as the party's platform shifts away from the party's ideal policy.

To find conditions for the existence of either type of political equilibrium, recall that the party whose platform is closest to  $c_m$  wins the elections. When the party leaders choose their parties' platforms they do not yet know the exact realization of  $c_m$ . Hence, by choosing their platforms they can only affect their chances of being elected. Since the cumulative distribution of  $c_m$  is  $G(c_m)$ , it follows that if  $y_L < y_R$ , then the probability that the left-wing party will win the elections is  $P_L(y_L, y_R) = G(\hat{y})$  and the probability that the right-wing party will win is  $P_R(y_L, y_R) = 1 - G(\hat{y})$ . If  $y_L = y_R$ , then the

<sup>10</sup> If  $U(\cdot|c)$  is not symmetric, then the indifferent voter will not be at the midpoint between  $y_L$  and  $y_R$  but at some other point between the two platforms.

probability of each party to win is  $\frac{1}{2}$ . Substituting for  $P_L(y_L, y_R)$  and  $P_R(y_L, y_R)$  into equation (2), the expected payoffs of the party leaders can be written as

$$\pi_L(y_L, y_R) = \begin{cases} G(\hat{y})[h_L - (y_L - c_L)], & \text{if } y_L < y_R, \\ \frac{1}{2}[h_L - (y_R - c_L)], & \text{if } y_L = y_R, \end{cases} \tag{3}$$

and

$$\pi_R(y_L, y_R) = \begin{cases} (1 - G(\hat{y}))[h_R - (c_R - y_R)], & \text{if } y_L < y_R, \\ \frac{1}{2}[h_R - (c_R - y_L)], & \text{if } y_L = y_R. \end{cases} \tag{4}$$

The equilibrium platforms,  $y_L^*$  and  $y_R^*$ , are given by the intersection of the best-response functions associated with the two expected payoff functions. But, since the expected payoff functions are discontinuous at  $y_L = y_R$  (unless  $y_L = y_R = \hat{c}_m$ ), an equilibrium in pure strategies may fail to exist.<sup>11</sup> Proposition 1 provides sufficient conditions for the existence of (pure strategy) equilibrium.

**Proposition 1.** Let  $z_L(y_L, y_R) \equiv h_L - (y_L - c_L) - 2M(\hat{y})$  and  $z_R(y_L, y_R) \equiv -h_R + (c_R - y_R) + 2H(\hat{y})$ . Then,

- (i) If  $z_L(\hat{c}_m, \hat{c}_m) < 0 < z_R(\hat{c}_m, \hat{c}_m)$ , there exists a unique political equilibrium in which  $c_L < y_L^* < \hat{c}_m < y_R^* < c_R$ .
- (ii) If  $z_L(\hat{c}_m, \hat{c}_m) > 0 > z_R(\hat{c}_m, \hat{c}_m)$ , there exists a unique political equilibrium in which  $y_L^* = y_R^* = \hat{c}_m$ .

*Proof.* See the Appendix.

Proposition 1 establishes sufficient conditions for the existence of a unique political equilibrium with either partial convergence in which the two platforms are located at opposite sides of the center of the political spectrum [part (i)] or with full convergence in which both platforms are located at the exact center of the political spectrum [part (ii)]. To understand these conditions suppose that  $y_L \leq y_R$ . Then, the best-response function of the left-wing party’s leader against  $y_R$  is implicitly defined by  $z_L(y_L, y_R) = 0$  and the best-response function of the right-wing party’s leader against  $y_L$  is implicitly defined by  $z_R(y_L, y_R) = 0$ . The condition in part (i) of the proposition ensures that the left-wing party wishes to adopt a “left-wing” platform with  $y_L < \hat{c}_m$  when the right-wing party adopts a centrist platform

<sup>11</sup> Ball (1999) shows that a similar non-existence problem arises in a unidimensional, two-candidate probabilistic spatial voting model with ideological party leaders. He shows, however, that for a large class of probability of winning functions, there exists an equilibrium in mixed strategies.

with  $y_R = \hat{c}_m$ , and the right-wing party wishes to adopt a “right-wing” platform with  $y_R > \hat{c}_m$  when the left-wing party adopts a centrist platform with  $y_L = \hat{c}_m$ . The condition in part (ii) of the proposition ensures that each party wishes to move all the way to the center of the political spectrum when the rival party adopts a centrist platform. Hence, under this condition we have full convergence.

It is worth noting that the conditions stated in Proposition 1 are only sufficient (but not necessary) conditions. If these conditions fail [i.e. both  $z_L(\hat{c}_m, \hat{c}_m)$  and  $z_R(\hat{c}_m, \hat{c}_m)$  are positive or both are negative], there may or may not exist a political equilibrium in pure strategies. In what follows we shall focus on cases where either the condition in part (i) or in part (ii) of Proposition 1 holds.

### 3.3 Convergence Parameters

Equations (3) and (4) show that the choices of  $y_L^*$  and  $y_R^*$  involve a tradeoff between the electoral concerns of party leaders that push the platforms closer to one another, and the ideological concerns of party members that induce each leader to limit the distance between the party’s platform and the party’s ideal policy. These two factors are fully captured by the parameters

$$a_L \equiv h_L - (\hat{c}_m - c_L), \quad a_R \equiv h_R - (c_R - \hat{c}_m). \quad (5)$$

These parameters reflect the combined impact of the intensity of each leader’s love of office and the distance of his party’s ideal policy from the center of the political spectrum. In what follows, we shall refer to  $a_L$  and  $a_R$  as the “convergence parameters” of the two parties. The convergence parameters together with the shape of the distribution of electoral uncertainty,  $g(\cdot)$ , determine the political equilibrium under RD.

**Proposition 2.** Suppose that  $z_L(\hat{c}_m, \hat{c}_m) < 0 < z_R(\hat{c}_m, \hat{c}_m)$  so in equilibrium  $c_L < y_L^* < \hat{c}_m < y_R^* < c_R$ . Then:

- (i) An increase in  $a_j$  causes both platforms to shift closer to one another (i.e.  $y_L^*$  increases and  $y_R^*$  decreases), although the shift of  $y_j^*$  is bigger than the shift of  $y_i^*$ ; consequently, the chances of party  $j$  to win the elections increase.
- (ii) Let  $d_L^* \equiv \hat{c}_m - y_L^*$  and  $d_R^* \equiv y_R^* - \hat{c}_m$  be the distances of  $y_L^*$  and  $y_R^*$  from the center of the political spectrum. Then  $d_L^* \cong d_R^*$  as  $a_L \cong a_R$  and  $G(y^*) \cong \frac{1}{2}$  as  $a_L \cong a_R$ .

*Proof.* See the Appendix.

Proposition 2 indicates that the party with the bigger convergence parameter (either because the party’s leader is more office-motivated than his rival or

because the party's ideological position is closer to the center than the ideological position of the rival party) will adopt the more centrist platform and will therefore be a favorite to win the elections.

Recalling that  $d_L^* \equiv \hat{c}_m - y_L^*$  and  $d_R^* \equiv y_R^* - \hat{c}_m$ , we can express the probability that the left-wing party will win the elections as  $G(\hat{y}^*) = (y_L^* + y_R^*)/2 = G(\hat{c}_m + (d_R^* - d_L^*)/2)$ . Part (ii) of Proposition 2 shows that the relative sizes of  $d_L^*$  and  $d_R^*$  depend only on the relative sizes of the convergence parameters,  $a_L$  and  $a_R$ , but not on their absolute sizes. This implies in turn that the relative electoral prospects of the two parties depend only on the difference between  $a_L$  and  $a_R$ , that will be denoted by  $\Delta \equiv a_L - a_R$ . On the other hand, part (i) of Proposition 2 shows that as  $a_L$  and  $a_R$  increase in absolute size,  $y_L^*$  and  $y_R^*$  move closer to one another. This suggests in turn that political polarization (the gap between  $y_L^*$  and  $y_R^*$ ) depends on the sum of the convergence parameters, denoted  $\Sigma \equiv a_L + a_R$ . In what follows, we will refer to  $\Sigma$  as the "aggregate convergence parameter," and to  $\Delta$  as the "relative convergence parameter."<sup>12</sup> The next proposition examines how the political equilibrium depends on  $\Sigma$  and  $\Delta$ .

**Proposition 3.** Suppose that  $z_L(\hat{c}_m, \hat{c}_m) < 0 < z_R(\hat{c}_m, \hat{c}_m)$  so in equilibrium  $c_L < y_L^* < \hat{c}_m < y_R^* < c_R$ . Then:

(i) holding  $\Sigma$  constant,

$$\frac{\partial y_L^*}{\partial \Delta} = \frac{g^2(\hat{y}^*) + g'(\hat{y}^*)}{8|J(y_L^*, y_R^*)|}, \quad \frac{\partial y_R^*}{\partial \Delta} = \frac{g^2(\hat{y}^*) - g'(\hat{y}^*)}{8|J(y_L^*, y_R^*)|},$$

$$\frac{\partial \hat{y}^*}{\partial \Delta} = \frac{g^2(\hat{y}^*)}{8|J(y_L^*, y_R^*)|} > 0,$$

where  $|J(y_L^*, y_R^*)| > 0$  is the determinant of the Jacobian matrix corresponding to equations (A3) and (A4) and is defined in equation (A6) in the Appendix;

(ii) holding  $\Delta$  constant,

$$\frac{\partial y_L^*}{\partial \Sigma} = \frac{1}{2}, \quad \frac{\partial y_R^*}{\partial \Sigma} = -\frac{1}{2}, \quad \frac{\partial \hat{y}^*}{\partial \Sigma} = 0.$$

*Proof.* See the Appendix.

Several conclusions emerge from Proposition 3. First, recall that under full symmetry,  $y_L^*$  and  $y_R^*$  are equally-distant from the center of the political

<sup>12</sup> Note that there is a one-to-one correspondence between  $(a_L, a_R)$  and  $(\Sigma, \Delta)$ . In particular, using the definitions of  $\Sigma$  and  $\Delta$ , we can write  $a_L = (\Sigma + \Delta)/2$  and  $a_R = (\Sigma - \Delta)/2$ .

spectrum, and each party has a 50 percent chance to win the elections. When the equilibrium is asymmetric, one party adopts a more centrist platform than its rival and is a favorite to win the elections. Since  $d_R^* - d_L^* = 2(\hat{y}^* - \hat{c}_m)$ , the equilibrium is fully symmetric if  $\hat{y}^* = \hat{c}_m$ ; as  $\hat{y}^*$  increases above  $\hat{c}_m$  or falls below it, the political equilibrium becomes more asymmetric and the lead margin of the favorite party increases. Proposition 3 shows that, since  $\hat{y}^*$  is increasing with  $\Delta$  but is independent of  $\Sigma$ , the degree of asymmetry of the political equilibrium depends only on  $\Delta$ . In particular, part (ii) of Proposition 2 implies that the equilibrium is fully symmetric when  $\Delta = 0$  (i.e.  $a_L = a_R$ ) because then  $d_L^* = d_R^*$  and  $G(\hat{y}^*) = \frac{1}{2}$ . Together with Proposition 3 this means that as  $|\Delta|$  increases, the equilibrium becomes increasingly more asymmetric. This is intuitive because an increase in  $|\Delta|$  means that the relative tendencies of the parties to converge become more dissimilar.

Second, Proposition 3 implies that increasing  $\Delta$  (i.e. increasing  $a_L$  relative to  $a_R$ ), boosts the electoral prospects of the left-wing party at the expense of the right-wing party. Although this result is related to part (i) of Proposition 2, it is not quite the same because here the exercise involves a simultaneous increase in  $a_L$  and a decrease in  $a_R$  (to ensure that  $\Delta$  increases while  $\Sigma$  remains constant), whereas Proposition 2 examines the impact of changes in only one of the convergence parameters.

Third, raising the aggregate convergence parameter,  $\Sigma$ , while holding the relative convergence parameter,  $\Delta$ , constant, pushes  $y_L^*$  and  $y_R^*$  closer to one another. Hence, the political system becomes less polarized. Therefore, variations in  $\Sigma$  can be interpreted as reflecting changes in the degree of political polarization, with higher values of  $\Sigma$  being associated with less polarization.

Fourth, Proposition 3 shows that the gap between  $y_L^*$  and  $y_R^*$  can either increase or decrease with  $\Delta$ , depending on the sign of  $g'(\hat{y}^*)$ . This implies that, in general,  $\Delta$  has an ambiguous effect on the degree of political polarization. However, there are two special cases in which the impact of  $\Delta$  on political polarization is unambiguous. First, when  $g(\cdot)$  is uniform,  $g'(\cdot) = 0$ , so changes in  $\Delta$  do not affect the degree of polarization. Second, when  $g(\cdot)$  is symmetric and unimodal,  $-g'(\hat{y}^*) \geq 0$  as  $G(\hat{y}^*) \geq \frac{1}{2}$ . Since part (ii) of Proposition 2 states that  $G(\hat{y}^*) \geq \frac{1}{2}$  as  $\Delta \geq 0$ , it follows from part (ii) of Proposition 3 that the degree of polarization is a U-shaped function of  $\Delta$  that attains a minimum at  $\Delta = 0$ . Since the political equilibrium becomes more asymmetric as  $|\Delta|$  increases, it follows that there is more polarization when the political equilibrium is more asymmetric.

#### 4. COMPARISON OF POLICY CHOICES UNDER THE MVP AND RD

This section examines the two main questions posed in this paper. First, it identifies conditions under which the policy outcomes predicted by the

sizable literature that uses the MVP also emerge under the more realistic setting of RD. Second, when this is not the case, it characterizes the factors that determine the direction and magnitude of systematic differences between the policies predicted by the median voter theorem and a two-parties RD.

It should be noted that since the location of the median voter in our model is uncertain, policy choices under both the MVP and RD are stochastic. Moreover, since under the MVP, policy is chosen by a median voter, whereas under RD it is chosen by elected officials who commit to platforms before the resolution of electoral uncertainty, it is obvious that actual policies under the MVP and RD will almost never coincide. We focus, therefore, on a comparison of expected policies under the MVP and under RD and identify the circumstances under which they do and do not coincide. When they do not, we will say that the expected policy under the MVP has a “policy bias” relative to the expected policy under RD, and we shall examine the determinants of the direction and magnitude of these policy biases.

#### 4.1 Policy Biases

The policy adopted under the MVP is  $x_m = c_m$ , which is the policy that maximizes the utility of the median voter. Hence, expected policy under the MVP is equal to

$$Ex_{MV} = Ec_m = \bar{c}_m, \tag{6}$$

which is the mean of the distribution of the median voter’s taste parameter.

Under RD, the actual policy choice is  $y_L$  if the left-wing party wins the elections and  $y_R$  if the right-wing party wins. The expected policy under an RD is therefore given by:

$$Ex_{RD} = G(\hat{y}^*)y_L^* + (1 - G(\hat{y}^*))y_R^*, \tag{7}$$

where  $\hat{y}^* \equiv (y_L^* + y_R^*)/2$  and  $G(\hat{y}^*)$  is the equilibrium probability that the left-wing party wins the elections.

Policy biases arise when  $Ex_{MV} \neq Ex_{RD}$ ; we will say that  $Ex_{MV}$  has a right-wing bias when  $Ex_{MV} > Ex_{RD}$ , and a left-wing bias when  $Ex_{MV} < Ex_{RD}$ . Recalling that  $d_L^* \equiv \hat{c}_m - y_L^*$  and  $d_R^* \equiv y_R^* - \hat{c}_m$ , it follows that

$$\begin{aligned} Ex_{MV} - Ex_{RD} &= \bar{c}_m - G(\hat{y}^*)y_L^* - (1 - G(\hat{y}^*))y_R^* \\ &= (\bar{c}_m - \hat{c}_m) + (d_L^* + d_R^*) \left[ G(\hat{y}^*) - \frac{d_R^*}{d_L^* + d_R^*} \right]. \end{aligned} \tag{8}$$

Equation (8) reveals that there are two potential sources for policy biases. The first source depends on the mean–median spread of the distribution of median voter types,  $g(c_m)$ , and it arises because  $Ex_{MV}$  depends on the mean

of  $c_m$  whereas  $Ex_{RD}$  depends on the median of  $c_m$ . This implies for instance that if  $g(\cdot)$  is skewed to the right so that  $\bar{c}_m > \hat{c}_m$  (i.e. the extreme right wing is more extreme than the extreme left wing), then  $Ex_{MV}$  tends to have a right-wing bias. The second source of policy bias is captured by the square-bracketed term in equation (8) and is due to differences in the convergence parameters of the two parties,  $a_L$  and  $a_R$ . These reflect differences in (i) the office motivations of the two party leaders and (ii) the distances of the constituencies of the two parties from the center of the political spectrum. Part (ii) of Proposition 2 implies that the square-bracketed term in equation (8) vanishes if  $a_L = a_R$ ; otherwise this term may be either positive or negative. This leads to the following proposition.

**Proposition 4.** A necessary condition for the existence of policy biases is that the political system has one of the following types of asymmetries:

- (i) the distribution of the median voter's taste parameter,  $g(\cdot)$  is skewed;
- (ii) the convergence parameters of the two parties are not equal (either because the party leaders have unequal office-motivation or because the ideological positions of the party constituencies are not equally-distant from the center of the political spectrum or because of some combination of those).

The impact of the first type of asymmetry on policy biases is straightforward. Hence, we shall focus on asymmetries in convergence parameters of the two parties and study how policy biases depend on the aggregate and relative convergence parameters.

#### 4.2 The Effects of $\Delta$ and $\Sigma$ on Expected Policy under RD

We begin by looking at the impact of  $\Delta$  and  $\Sigma$  on  $Ex_{RD}$  and hence on the direction and magnitude of policy biases.

**Proposition 5.** Suppose that  $z_L(\hat{c}_m, \hat{c}_m) < 0 < z_R(\hat{c}_m, \hat{c}_m)$  so in equilibrium  $c_L < y_L^* < \hat{c}_m < y_R^* < c_R$ . Then:

- (i)  $\partial Ex_{RD} / \partial \Sigma \cong 0$  as  $\Delta \cong 0$ , implying that a small increase in  $\Sigma$  shifts  $Ex_{RD}$  towards the ideological position of the party which is an underdog in the political race.
- (ii) If  $g(\cdot)$  is symmetric and unimodal, then  $\Sigma < 1/g(\hat{y}^*)$  is sufficient for  $\partial Ex_{RD} / \partial \Delta < 0$ , implying that a small increase in  $\Delta$  (raising  $a_L$  relative to  $a_R$ ) shifts  $Ex_{RD}$  to the left. Moreover, starting from  $\Delta = 0$  (in which case the equilibrium is symmetric and  $\hat{y}^* = \hat{c}_m$ ), a small increase in  $\Delta$  shifts  $Ex_{RD}$  to the left if  $\Sigma < 1/g(\hat{y}^*)$  and to the right if  $\Sigma > 1/g(\hat{y}^*)$ .

*Proof.* See the Appendix.

To understand part (i) of Proposition 5, suppose that  $\Delta > 0$ . Then,  $y_L^*$  is closer to the center than  $y_R^*$  and the left-wing party is a favorite to win the elections. Now, Proposition 3 shows that a small increase in  $\Sigma$  pushes  $y_L^*$  to the right and  $y_R^*$  to the left without changing the electoral prospects of the two parties. But, since the left-wing party is a favorite to win, the shift of  $y_L^*$  to the right has a greater impact on  $Ex_{RD}$  than the shift in  $y_R^*$  to the left, so  $Ex_{RD}$  moves in the direction of the underdog right-wing party. The case where  $\Delta < 0$  is completely analogous.

The impact of a small increase in  $\Delta$  (i.e. raising  $a_L$  and lowering  $a_R$ ) on  $Ex_{RD}$  is more complex since it creates two conflicting effects. The first is a “position” effect that arises because the increase in  $\Delta$  causes shifts in both  $y_L$  and  $y_R$  to the right. Given the two parties’ relative odds of winning, the position effect raises  $Ex_{RD}$ . The second effect is a “probability to win” effect. It arises because an increase in  $\Delta$  also raises the odds that  $y_L$  will be implemented and reduces the odds that  $y_R$  will be implemented. Since  $y_L < y_R$ , the probability effect reduces  $Ex_{RD}$ . The proposition shows, that for sufficiently high levels of polarization [i.e. when  $\Sigma < 1/g(y^*)$ ], the probability effect dominates, whereas for low levels of polarization [i.e. when  $\Sigma > 1/g(y^*)$ ], the position effect dominates. Intuitively, high (low) polarization means that  $y_L^*$  and  $y_R^*$  are relatively far apart (close). As a consequence, a given increase in the relative winning odds of the left induces a stronger (weaker) probability effect on  $Ex_{RD}$  at high (low) levels of polarization. Hence, the negative probability effect overtakes the positive position effect at high levels of polarization and conversely at low levels of polarization. Further discussion of the interplay between those two effects and the distribution of electoral uncertainty appears at the end of the next subsection.

### 4.3 *When is the MVP a Reasonable Approximation for RD and When is it Not?*

Thus far we have seen that in the completely symmetric case where  $g(c_m)$  is symmetric and  $\Delta = a_L - a_R = 0$ , there is no policy bias on average because  $Ex_{MV} = Ex_{RD}$ . Although complete symmetry is sufficient to ensure that  $Ex_{MV} = Ex_{RD}$ , it is not a necessary condition for this “no bias” result. To obtain a more complete view on the comparison between the MVP and RD, we shall now fully characterize the conditions under which there is no policy bias on average. Taking this set as a benchmark, we then determine the set of parameters for which the MVP is a reasonable approximation for RD in the sense that the policy bias is small. Since the impact of the shape of the distribution of electoral uncertainty,  $g(c_m)$ , on the policy bias is already well-understood [see the discussion following equation (8)], we shall assume in

what follows that  $g(\cdot)$  is symmetric, i.e.  $\bar{c}_m = \hat{c}_m$ . Given this assumption, the first term in equation (8) vanishes and the direction and magnitude of policy biases depend only on the convergence parameters.

We begin with two special cases in which the political equilibrium is symmetric in the sense that  $d_L^* = d_R^*$  and  $G(\hat{y}^*) = \frac{1}{2}$ . Then, given that  $g(\cdot)$  is symmetric, equation (8) implies that  $Ex_{RD} = Ex_{MV}$ . In the first case,  $z_L(\hat{c}_m, \hat{c}_m) > 0 > z_R(\hat{c}_m, \hat{c}_m)$ , so by part (ii) of Proposition 1,  $y_L^* = y_R^* = \hat{c}_m$ . Using equation (5) and the definitions of  $z_L(\cdot, \cdot)$  and  $z_R(\cdot, \cdot)$  the two inequalities can be written as  $a_L > 2M(\hat{c}_m)$  and  $a_R > 2H(\hat{c}_m)$ . In the second case,  $z_L(\hat{c}_m, \hat{c}_m) < 0 < z_R(\hat{c}_m, \hat{c}_m)$ , or  $a_L < 2M(\hat{c}_m)$  and  $a_R < 2H(\hat{c}_m)$ , and  $\Delta = 0$  (i.e.  $a_L = a_R$ ). Then, part (i) of Proposition 1 and part (ii) of Proposition 2 imply that  $d_L^* = d_R^* > 0$  and  $G(\hat{y}^*) = \frac{1}{2}$ . The two cases differ in that in the first case there is full convergence whereas in the second there is only partial convergence. Yet in both cases the equilibrium under RD is symmetric so  $Ex_{MV} = Ex_{RD}$ . This leads to the following proposition.

**Proposition 6.** Suppose that  $g(\cdot)$  is symmetric. Then there is no policy bias if one of the following conditions holds:

- (i)  $a_L > 2M(\hat{c}_m)$  and  $a_R > 2H(\hat{c}_m)$  so that  $d_L^* = d_R^* = 0$ ;
- (ii)  $a_L < 2M(\hat{c}_m)$ ,  $a_R < 2H(\hat{c}_m)$ , and  $\Delta = 0$ , so that  $d_L^* = d_R^* > 0$ .

Next we consider cases in which the political equilibrium is asymmetric. Now, the left-wing party is a favorite to win the elections if  $\Delta > 0$  (in which case  $d_L^* < d_R^*$ ), and the right-wing party is a favorite to win if  $\Delta < 0$  (in which case  $d_L^* > d_R^*$ ).

**Proposition 7.** Suppose that  $g(\cdot)$  is symmetric and  $z_L(\hat{c}_m, \hat{c}_m) < 0 < z_R(\hat{c}_m, \hat{c}_m)$  so in equilibrium  $c_L < y_L^* < \hat{c}_m < y_R^* < c_R$ . Then:

- (i) If  $g(\hat{c}_m) < 1/(c_1 - c_0)$ , then  $Ex_{RD} \cong Ex_{MV} = \hat{c}_m$  as  $\Delta \cong 0$ .
- (ii) If  $g(\hat{c}_m) > 1/(c_1 - c_0)$  and  $|\Delta|$  is sufficiently small (i.e. whenever the equilibrium is not too asymmetric), there exists for each  $\Delta$  a unique value of  $\Sigma$ , denoted  $\Sigma^{NB}(\Delta)$ , for which  $Ex_{RD} = Ex_{MV} = \hat{c}_m$ . When  $|\Delta|$  is sufficiently large, the situation is as in part (i).
- (iii) When  $\Sigma^{NB}(\Delta)$  exists and  $g(\cdot)$  is unimodal and symmetric, then  $\Sigma^{NB}(\Delta)$  is a symmetric, U-shaped, and smooth function that attains a minimum at  $\Delta = 0$ . Moreover,  $\Sigma^{NB}(0) = 1/g(\hat{c}_m)$ .

*Proof.* See the Appendix.

Proposition 7 shows that we need to distinguish between two cases depending on whether  $g(\hat{c}_m) > 1/(c_1 - c_0)$ , or vice versa. Let us first consider the case where  $g(\hat{c}_m) > 1/(c_1 - c_0)$ . Since  $1/(c_1 - c_0)$  is the density of a uniform distribution on the interval  $[c_0, c_1]$ , this case arises if  $g(\cdot)$  has more weight

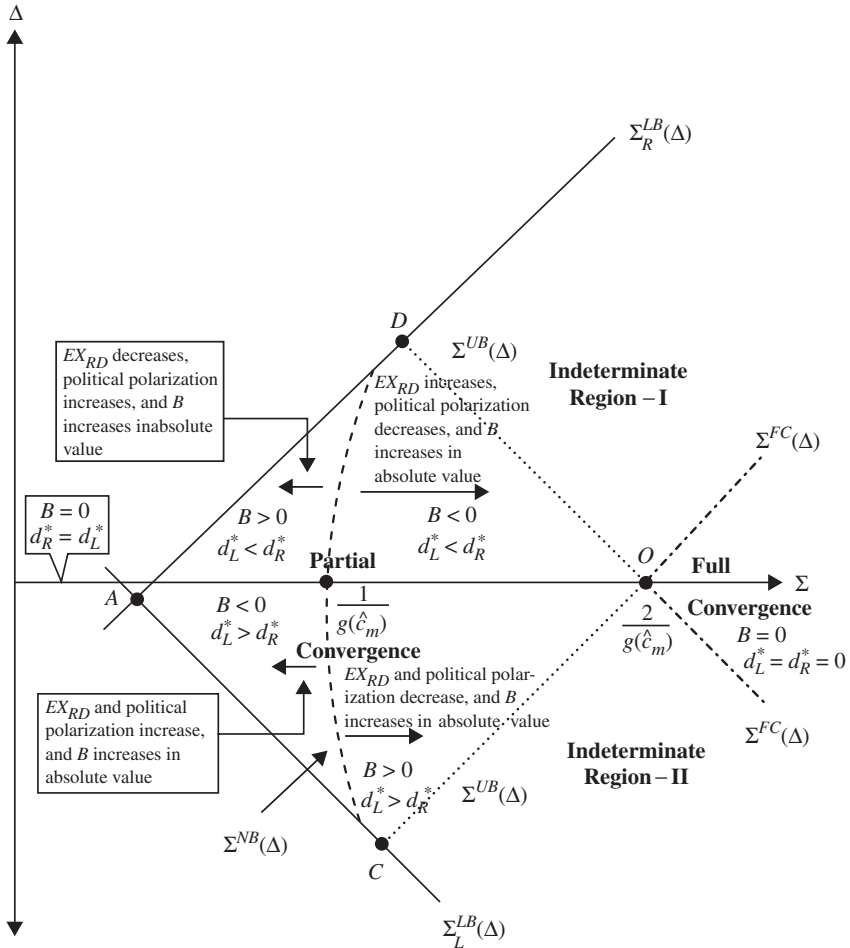


Figure 1. Comparison of  $EX_{RD}$  and  $EX_{MV}$  when  $g(\hat{c}_m) > 1/(c_1 - c_0)$ .

Notes:  $B = EX_{MV} - EX_{RD}$  is the policy bias. There is a right-wing bias when  $B > 0$ , a left-wing bias when  $B < 0$ , and no bias when  $B = 0$ . The right-wing party is a favorite to win the elections when  $d_R^* < d_L^*$ , the left-wing party is a favorite to win when  $d_R^* > d_L^*$ , and when  $d_R^* = d_L^*$ , the political race is tied. When  $d_R^* = d_L^* = 0$ , the equilibrium platforms fully converge on the center of the political spectrum.

around its mean than a uniform distribution [say, as is the case where  $g(\cdot)$  is symmetric and unimodal]. This case is illustrated in Figure 1 in the  $(\Sigma, \Delta)$  space. To interpret the figure, recall that Assumption A2 and the conditions in Proposition 1 place restrictions on the parameter values that we consider. The lines  $\Sigma_L^{LB}(\Delta)$  and  $\Sigma_R^{LB}(\Delta)$ , respectively, represent for each  $\Delta < 0$  and each  $\Delta > 0$ , the lowest value of  $\Sigma$  permitted by Assumption A2.

$\Sigma^{UB}(\Delta)$  represent, for each  $\Delta$ , the largest value of  $\Sigma$  that is consistent with the condition in part (i) of Proposition 1, while the lines defined by  $\Sigma^{FC}(\Delta)$  represent, for each  $\Delta$ , the lowest value of  $\Sigma$  which is consistent with the condition in part (ii) of Proposition 1. We establish the shapes of  $\Sigma_L^{LB}(\Delta)$ ,  $\Sigma_R^{LB}(\Delta)$ ,  $\Sigma^{UB}(\Delta)$ , and  $\Sigma^{FC}(\Delta)$  in the Appendix. Given these curves, parameter values in the area delimited by the rectangular area  $ADCO$  give rise to a political equilibria with partial convergence. On the other hand, parameter values in the area enclosed to the right of the two vertices, labeled  $\Sigma^{FC}(\Delta)$ , with base at  $2/g(\hat{c}_m)$ , give rise to equilibria with full convergence. For parameter values in the two regions enclosed between the areas of full and partial convergence, a (pure strategy) political equilibrium may or may not exist.

As Figure 1 shows, when  $|\Delta|$  is sufficiently small (in which case the favorite party has a small lead margin), there exists a curve,  $\Sigma^{NB}(\Delta)$ , along which the policy bias vanishes even though the equilibrium is asymmetric. This curve, together with the line  $\Delta=0$ , split the partial convergence into four regions. Whenever  $\Sigma > \Sigma^{NB}(\Delta)$ ,  $Ex_{MV}$  is biased in the direction of the favorite party (a left-wing bias if  $\Delta > 0$  and a right-wing bias if  $\Delta < 0$ ). However, when  $\Sigma < \Sigma^{NB}(\Delta)$ ,  $Ex_{MV}$  is biased in the direction of the party which is an underdog in the political race (a right-wing bias if  $\Delta > 0$  and a left-wing bias if  $\Delta < 0$ ). Either way, the absolute value of the policy bias grows as  $\Sigma$  moves further away from  $\Sigma^{NB}(\Delta)$ .

When  $|\Delta|$  is sufficiently large (in which case one party is a clear favorite to win), then  $Ex_{MV}$  has a right-wing bias if  $\Delta < 0$  (the right-wing party is a favorite to win the elections), and a left-wing bias if  $\Delta > 0$  (the left-wing party is a favorite to win). That is,  $Ex_{MV}$  is always biased in the direction of the favorite party. This result is somewhat counterintuitive because it might be thought that if one party is a favorite to win, then  $Ex_{RD}$  will lean in its direction more than  $Ex_{MV}$  which only reflects the preferences of the median voter. This intuition, however, fails to take into account the fact that a party can be a favorite to win only if it adopts a more centrist platform than its rival party; this in turn has a moderating effect on  $Ex_{RD}$  in comparison with  $Ex_{MV}$ . For instance, when  $\Delta > 0$ , the left-wing party adopts a more centrist platform than the right-wing party and is therefore a favorite to win. As a result, the policy under RD is more likely to be selected by the left-wing party. But since this policy is closer to the center,  $Ex_{RD}$  can very well be less left-wing than  $Ex_{MV}$ , implying that the MVP may have a left-wing bias. The size of the policy bias increases in absolute value as  $\Sigma$  increases. Since an increase in  $\Sigma$  also means that there is less political polarization, we can conclude that the policy bias and the degree of political polarization are inversely related in this case. In contrast, when  $|\Delta|$  is sufficiently small, the relationship between the policy bias and the degree of political polarization is non-monotonic because when  $\Sigma$  increases, the absolute value of the policy bias shrinks when  $\Sigma < \Sigma^{NB}(\Delta)$  but increases when  $\Sigma > \Sigma^{NB}(\Delta)$ .

The second case arises when  $g(\hat{c}_m) < 1/(c_1 - c_0)$  so that  $g(\cdot)$  has a smaller mass around its median than a uniform distribution. Proposition 7 shows that this case is similar to the case where  $g(\hat{c}_m) > 1/(c_1 - c_0)$  and  $|\Delta|$  is large. In particular,  $Ex_{MV}$  is always biased in this case in the direction of the favorite party and the policy bias and the degree of political polarization are inversely related.

The main conclusions from the above discussion are summarized as follows:

**Proposition 8.** Suppose that  $g(\cdot)$  is symmetric and  $z_L(\hat{c}_m, \hat{c}_m) < 0 < z_R(\hat{c}_m, \hat{c}_m)$  so in equilibrium  $c_L < y_L^* < \hat{c}_m < y_R^* < c_R$ . Then:

- (i) If either  $g(\hat{c}_m) < 1/(c_1 - c_0)$ , or  $g(\hat{c}_m) > 1/(c_1 - c_0)$  and  $|\Delta|$  is relatively large,  $Ex_{MV}$  is biased in the direction of the party that is a favorite to win the elections. Moreover, the absolute value of the bias and the degree of political polarization are inversely related.
- (ii) If  $g(\hat{c}_m) > 1/(c_1 - c_0)$  and  $|\Delta|$  is not too large,  $Ex_{MV}$  is biased in the direction of the party that is the favorite to win the elections provided the degree of political polarization is sufficiently small [i.e.  $\Sigma > \Sigma^{NB}(\Delta)$ ]. Moreover, the policy bias and the degree of political polarization are inversely related. In contrast, when there is a sufficiently large degree of political polarization [i.e.  $\Sigma < \Sigma^{NB}(\Delta)$ ],  $Ex_{MV}$  is biased in the direction of the party that is an underdog in the political race and the policy bias and the degree of political polarization are positively related.

The general message from Proposition 8 is that while the set of parameters for which there is no policy bias is rather small, there is a considerably larger set of parameters for which the absolute value of the policy bias is not too large. The factors that determine the absolute value of the policy bias include the shape of the distribution of electoral uncertainty, the degree of political polarization, and the degree to which the political equilibrium is symmetric, which in turn determines the electoral prospects of the two parties. Propositions 7 and 8 provide a full characterization of the policy bias for the case in which the distribution of electoral uncertainty is symmetric. In particular, when this distribution has less weight around its mean than a uniform distribution, or when it has more weight and one party is a clear favorite to win the elections, the bias decreases as the degree of political polarization increases. When the distribution of electoral uncertainty has more weight around its mean than the uniform and no party is a clear favorite, the bias tends to zero in absolute value as  $\Sigma$  (which determines the degree of political polarization), tends to the “asymmetric no-bias” locus,  $\Sigma^{NB}(\Delta)$ , either from above or from below.

Intuitively, the basic difference between the case where  $g(\hat{c}_m) < 1/(c_1 - c_0)$  and the case where  $g(\hat{c}_m) > 1/(c_1 - c_0)$ , can be understood in terms of the factors that affect the relative magnitudes of the position and the probability effects. Note first that, taking full symmetry ( $\Delta = 0$ ) as a benchmark, whenever  $g(\hat{c}_m) < 1/(c_1 - c_0)$ , the position effect dominates the probability effect for all parameter values for which there is partial convergence. By contrast, when  $g(\hat{c}_m) > 1/(c_1 - c_0)$ , the position effect dominates the probability-to-win effect for  $\Sigma > \Sigma^{NB}(\Delta)$ , but the opposite is true when  $\Sigma < \Sigma^{NB}(\Delta)$ . We saw at the end of the previous subsection that when polarization is sufficiently low, the position effect dominates the probability-to-win effect and conversely when polarization is sufficiently high. At  $\Sigma^{NB}(\Delta)$ , the two effects exactly offset each other producing the asymmetric no bias locus and a **region of low bias around it**.

The preceding considerations still leave open the question why the probability-to-win effect never dominates the position effect when  $g(\cdot)$  has a smaller mass around its median than a uniform distribution [i.e.  $g(\hat{c}_m) < 1/(c_1 - c_0)$ ] while regions of reverse domination do arise for sufficiently high levels of polarization when  $g(\cdot)$  is sufficiently centered around its median [i.e.  $g(\hat{c}_m) > 1/(c_1 - c_0)$ ]. To understand the reasons for this basic difference, it is useful to trace the effects of an increase in  $\Delta$ , starting from a symmetric political system ( $\Delta = 0$ ) at which  $y^* = \hat{c}_m$ . We saw at the end of the previous subsection that the negative probability effect on  $EX_{RD}$  is stronger when there is a high level of polarization. Holding  $\Sigma$  and hence the degree of polarization constant, an increase in  $\Delta$  leads to transfer of probability mass from  $y_R$  to  $y_L$ . This transfer is larger the higher the density of  $g(\cdot)$  at  $\hat{c}_m$ . Hence, the negative probability effect becomes stronger. When  $g(\cdot)$  is sufficiently centered around its median, there exist sufficiently high values of polarization for which there is partial convergence and the probability effect dominates the position effect. But when  $g(\cdot)$  is not sufficiently centered around its median, the position effect dominates for all values of  $\Sigma$  and  $\Delta$  for which there is partial convergence.

#### 5. AN APPLICATION: COMPARISON OF TAX POLICIES UNDER THE MVP AND UNDER RD

This section illustrates the main results of the paper by applying them within the context of the following variant of the classic model of Meltzer and Richard (1981). The economy consists of a continuum of individuals whose total mass is 1. Individual preferences are defined over a private good,  $c$ , leisure,  $l$ , and a public good,  $G$ , and are represented by the following utility function:

$$U(c, l, G) = cl + \frac{G}{\alpha}, \quad (9)$$

where  $\alpha > 0$  is a stochastic parameter that characterizes the relative preferences of **all** individuals for private versus public goods. The fact that  $\alpha$  is stochastic will introduce electoral uncertainty into the model. Each individual has 1 unit of time that can be allocated to leisure,  $l$ , and labor,  $n = 1 - l$ . Individuals have access to a constant returns to scale technology so the pretax income of an individual who works  $n$  hours is  $\theta n$ , where  $\theta$  is the individual's productivity. The productivity parameter  $\theta$  is distributed in the population on the support  $[\theta_0, \theta_1]$  according to a commonly known density function  $h(\theta)$ . To economize on notation we normalize the mean of  $\theta$  to 1.

The government finances the public good,  $G$ , by means of a uniform income tax,  $t$ . Assuming that individuals spend their entire net income on the private good,  $c$ , the budget constraint of an individual whose productivity is  $\theta$  is given by  $c = (1 - t)n\theta$ . Given his budget constraint, each individual chooses  $c$  and  $l$  to maximize his utility. The solution to this maximization problem is given by:

$$c^* = \frac{\theta(1 - t)}{2}, \quad l^* = \frac{1}{2}. \tag{10}$$

Recalling that the population size and the mean of  $\theta$  have been both normalized to one, total tax revenues, when the tax rate is  $t$ , are given by:

$$T(t) = \int_{\theta_0}^{\theta_1} t\theta(1 - l^*)h(\theta)d\theta = \frac{t}{2}. \tag{11}$$

The government uses  $T(t)$  to finance the public good whose production function is quadratic and given by

$$G(t) = AT(t) - (T(t))^2, \quad A > 0. \tag{12}$$

Substituting from equations (10)–(12) into equation (9) and rearranging, the utility of an individual with productivity  $\theta$ , as a function of the income tax rate is  $t$ , is given by

$$V(t|\theta) = \frac{\theta(1 - t)}{4} + \frac{At}{2\alpha} - \frac{t^2}{4\alpha}. \tag{13}$$

The ideal tax rate from the point of view of an individual with productivity  $\theta$  is given by:

$$t^* \equiv A - \frac{\alpha\theta}{2}. \tag{14}$$

That is, all individuals agree that the tax rate should be higher when the productivity in the public sector is higher, but high-productivity individuals who earn more money prefer lower tax rates. Substituting from (14) into (13) and rearranging terms, the utility function of each individual can be

expressed in terms of his ideal tax rate,  $t^*$ , as follows:

$$V(t|t^*) = \frac{t^{*2} - 2t^* + 2A}{4\alpha} - \frac{(t - t^*)^2}{4\alpha}. \quad (15)$$

This utility function is single-peaked and symmetric around  $t^*$  as assumed in section 2.

Since  $t^*$  is monotonically decreasing in  $\theta$ , the median voter is the individual whose productivity parameter is equal to  $\theta_m$  which is the median of the distribution of  $\theta$ . Since the utility functions are single-peaked, the MVP states that the ideal income tax rate of the median voter,  $t_m \equiv A - \alpha\theta_m/2$  will be adopted. The expected policy under MVP then is  $Et_{MV} = A - \bar{\alpha}\theta_m/2$ , where  $\bar{\alpha}$  is the mean of  $\alpha$ . Since the mean of  $\theta$  was normalized to 1 and the labor supply of each individual is  $\frac{1}{2}$ ,  $\theta_m$  is equal to the inverse of the ratio of the mean to median income. Hence, consistent with the Meltzer and Richard model,  $Et_{MV}$  is an increasing function of the ratio of mean to median income.

Next, we turn to the choice of policy under RD. Since the parameter  $\alpha$  that reflects the relative intensity of individual preferences for the public good is stochastic, the two candidates that compete for office face electoral uncertainty. This uncertainty means that although the candidates know the productivity parameter of the median voter,  $\theta_m$ , they do not know the precise realization of the median voter's ideal tax rate,  $t_m$ . The distribution of  $\alpha$  induces a distribution for  $t_m$ . We denote the density function of  $t_m$  by  $g(t_m)$  and assume that it satisfies Assumptions A1–A3. This setup maps into the general framework of the paper with the tax rate  $t$  mapping into the platform space and  $t_m$  mapping into  $c_m$ . The only difference is that in the context of taxation, it is natural to define the right-wing party as the party that supports lower tax rates, and hence, the ideal policy of its constituency,  $t_R$ , will be below  $\hat{t}_m$ , which is the median of the distribution of  $t_m$  (i.e. the center of the political spectrum), whereas the ideal policy of the constituency of the left-wing party,  $t_L$ , will be above  $\hat{t}_m$ .

Now we can readily use the results of the paper and compare the expected tax rates under the MVP,  $Et_{MV}$ , and under RD,  $Et_{RD}$ . To simplify matters we restrict attention to cases in which  $g(t_m)$  is symmetric and unimodal.<sup>13</sup>  $Et_{MV}$  coincides with  $Et_{RD}$  when the equilibrium is symmetric. Symmetry can arise either when (i) the two parties advocate exactly the same tax rate, or (ii) the tax rates advocated by the two parties are equally-distant from the center of the political spectrum,  $\hat{t}_m$ . Part (ii) of Proposition 2 shows that case (i) arises when the parties have sufficiently strong tendencies to converge towards the center, while part (ii) of Proposition 1 shows that case (ii) arises

<sup>13</sup> Note that since the distribution of  $t_m$  is induced by the distribution of  $\alpha$ ,  $g(t_m)$  could be symmetric even if the distribution of  $\theta$  is skewed to the right as Meltzer and Richard argue.

when the two parties have more moderate but equal tendencies to converge towards the center.

When the equilibrium is asymmetric,  $Et_{MV}$  is in general biased relative to  $Et_{RD}$ . To illustrate, suppose that we start from a symmetric equilibrium with partial convergence at which  $Et_{MV} = Et_{RD}$ , and now, perhaps due to an increase in the political contributions of rich individuals,  $t_R$  moves further away from  $\hat{t}_m$  than  $t_L$ . At first blush it seems that this should lower  $Et_{RD}$  and create a positive bias in  $Et_{MV}$  relative to  $Et_{RD}$ . But as we saw earlier, this intuition should be qualified since  $t_L$  will be now closer to  $\hat{t}_m$  and hence the left-wing party will be a favorite to win the election and implement relatively high taxes. If the shift in  $t_R$  is so big that the left-wing party becomes a strong favorite to win the elections (i.e.  $\Delta$  is large), then the reduction in  $t_R$  will have a dominating negative effect on  $Et_{RD}$  and, as a result,  $Et_{MV}$  will have a positive bias. In other words, a model of taxation that uses the MVP would over-predict the average tax rate under an RD. The magnitude of this over-prediction shrinks as the gap between the tax rates advocated by the two parties becomes wider (i.e. as  $\Sigma$  falls).

Things are more complex when the left-wing party has a more moderate lead (i.e.  $\Delta$  is not too large). Then there exists a curve,  $\Sigma^{NB}(\Delta)$ , such that to its right where the tax rates advocated by the two parties are not too far apart ( $\Sigma$  is large),  $Et_{MV}$  would still have a positive bias as before. However, to the left of  $\Sigma^{NB}(\Delta)$  where the tax rates advocated by the two parties are sufficiently far apart ( $\Sigma$  is small),  $Et_{MV}$  would have a negative bias. Now a model of taxation based on the MVP can either over- or under-predict the average tax rates under RD. The magnitude of this bias depends on the gap between the tax rates advocated by the two parties. As this gap grows from 0, the absolute value of the bias falls towards 0, and once  $\Sigma < \Sigma^{NB}(\Delta)$ , the absolute value of the bias begins to increase from 0.

The main empirical implication of the Meltzer and Richard (1981) model is that redistribution and income tax rates are increasing functions of the ratio of mean to median income. This hypothesis was tested empirically by Gouveia and Masia (1998) using panel data from US states between 1979 and 1991, and by Rodriguez (1999) using panel data from US states between 1984 and 1994, and a time series with US data from 1947–1992. Neither paper finds support for the Meltzer–Richard hypothesis. Our analysis suggests that this lack of support might be due to omitted variables that reflect political polarization and the lead margin of the favorite party (these variables in turn reflect the convergence parameters of the parties). For instance, assuming that electoral uncertainty is symmetric and unimodal, our analysis suggests that in states in which the Democratic party is a favorite to win the elections but its lead is relatively narrow (i.e.  $\Delta$  is positive but small), the ratio of mean to median income should be adjusted downward if there is a low degree of political polarization (i.e.  $Et_{MV}$  has a positive bias relative to  $Et_{RD}$ ), and upward if there is a high degree of

polarization (i.e.  $Et_{MV}$  has a negative bias relative to  $Et_{RD}$ ). Likewise, in states in which the Republicans have a relatively narrow lead (i.e.  $\Delta$  is negative but small in absolute value), the ratio of mean to median income should be adjusted upward if there is a low degree of polarization, and downward if there is a high degree of polarization.

## 6. CONCLUDING REMARKS

There are several general lessons that emerge from this paper. First, the Downsian benchmark in which the platforms of the two parties fully converge on the center of the political spectrum arises only when party leaders are sufficiently office-motivated and/or if the parties are not too polarized and neither party is a clear favorite to win the elections.<sup>14</sup> In this benchmark case, there are no systematic differences between policy choices under the MVP and under RD. Second, when convergence is partial, systematic policy differences between the MVP and RD are likely to be the rule rather than the exception. The sign and the size of the resulting policy bias depend on the degree of asymmetry in the ideal positions of the parties in relation to the center of the political spectrum, on differences in the extent to which the party leaders are office-motivated, and on the skewness in the distribution of electoral uncertainty. Even when this distribution is symmetric, the set of parameters for which there is no bias is rather narrow. Third, this set consists of a symmetric no-bias locus, along which the political equilibrium is symmetric in the sense that the equilibrium platforms are at equal distances from the center of the political spectrum and have equal chances to win the elections, and (for distributions of electoral uncertainty with sufficiently salient modes) of an asymmetric no-bias locus along which the political equilibrium is asymmetric. Along the asymmetric no-bias locus, the position and the probability-to-win effects triggered by differences in convergence tendencies exactly offset each other.

Fourth, given the relative tendency of the parties to converge, the magnitude of the policy bias is monotonically related to the divergence of the aggregate convergence parameter of the political system from the asymmetric no-bias locus. The results of the paper thus suggest under what conditions and where to look for “correction factors” for the results of politico-economic models that utilize the MVP.

It would appear at first blush that if one party moves closer to the center of the political spectrum, the expected policy under RD should also shift in the same direction. This intuition, however, abstracts from the fact that

<sup>14</sup> A somewhat related result is obtained by Lindbeck and Wiebull (1993) in the context of a two-party representative democracy in which the parties are both office- and policy-motivated and the voters trade-off their policy preferences against their “party identity.” They show that in equilibrium (if it exists) the parties will converge on a utilitarian optimum only if they are equally popular. Otherwise, the two parties will adopt different platforms.

when a party is closer to the center, it is also a favorite to win the elections and implement its policy. This effect pushes the expected policy under RD in the direction of that party and away from the center.

A main contribution of the paper is to characterize the combinations of effective political polarization and of effective political asymmetry for which the position effect dominates the probability-to-win effect as well as the combinations of those parameters for which the opposite is true. The paper shows that, in the first case, the bias is in the direction of the party whose candidate is a favorite to win the election, whereas in the second case the bias is in the direction of the underdog party. The first case arises when polarization is sufficiently low and the convergence tendency of the left is stronger than that of the right or when polarization is sufficiently high and the convergence tendency of the right is stronger than that of the left. The second case arises when polarization is sufficiently high and the convergence tendency of the left is larger than that of the right or when polarization is sufficiently low and the convergence tendency of the right is relatively stronger. We believe the qualitative nature of this result is likely to carry over to a framework in which the candidates running for office are also ideologically motivated as in the literature initiated by Wittman (1977, 1983).

A primitive of our model is the distribution of electoral uncertainty which may be difficult to observe in practice. On the other hand, the paper characterizes the size and the sign of the bias in terms of observable such as the degree of effective polarization and the electoral odds of the two parties for many types of symmetric distribution of electoral uncertainty. Thus, it is not necessary to exactly pinpoint the distribution of electoral uncertainty in order to relate the predicted size of the bias to observable such as the effective degree of political polarization and the electoral odds of the parties.

#### APPENDIX

Following are the proofs of Lemma 1, and Propositions 1, 2, 3, 5, and 7.

*Proof of Lemma 1.* The proof proceeds in three steps.

- (i) Assume by way of negation that  $y_R^* < y_L^*$ . Then, there are three possible cases. First, if  $y_L^* \leq \hat{c}_m$ , the chances of the right-wing party to win the elections are less than  $\frac{1}{2}$  (all median voter types whose taste parameters exceed  $\hat{c}_m$  surely vote for the left-wing party). But, by deviating to  $y_L^*$ , the leader of the right-wing party increases his chances to be elected to  $\frac{1}{2}$ , and moreover, his party's platform shifts closer to  $c_R$ ; hence, the deviation upsets the putative equilibrium. Second, if  $y_R^* \geq \hat{c}_m$ , the leader of the left-wing party can increase his expected payoff by deviating to  $y_R^*$ , because then his chances to be elected increase to  $\frac{1}{2}$ , and the party's platform is closer to  $c_L$ . Finally,

if  $y_R^* < \hat{c}_m < y_L^*$ , both party leaders can increase their expected payoffs by moving slightly towards  $\hat{c}_m$ , thereby increasing their chances of being elected, while moving closer to their party's ideal policies. Thus, in equilibrium (if it exists) it must be the case that  $y_L^* \leq y_R^*$ .

- (ii) Suppose by way of negation that  $y_L^* = y_R^* \neq \hat{c}_m$ . Since  $y_L^* = y_R^*$ , the probability of each party to win the elections is  $\frac{1}{2}$ . If  $y_L^* = y_R^* < \hat{c}_m$ , the leader of the right-wing party can increase his expected payoff by deviating to  $\hat{c}_m$ , in which case his chances to be elected exceed  $\frac{1}{2}$ , and his party's platform shifts closer to  $c_R$ . Similarly, if  $y_L^* = y_R^* > \hat{c}_m$ , the leader of the left-wing party can increase his chances to be elected to more than  $\frac{1}{2}$  by moving to  $\hat{c}_m$  while moving closer to  $c_L$ . Hence, the only possible equilibrium with  $y_L^* = y_R^*$  is such that  $y_L^* = y_R^* = \hat{c}_m$ .
- (iii) Suppose by way of negation that  $y_L^* < c_L$ . Since by Assumption A3,  $c_L < \hat{c}_m$ , it is clear that  $y_L^* < y_R^*$  [we know from step (i) that  $y_L^* \leq y_R^*$ , and from step (ii) that  $y_L^* = y_R^*$  only if  $y_L^* = y_R^* = \hat{c}_m$ ; hence the assumption that  $y_L^* < c_L$  rules out the possibility that  $y_L^* = y_R^*$ ]. But now, the leader of the left-wing party can increase his chances to be elected by moving to  $c_L$ , while maximizing the support he gets from his party after being elected. Consequently,  $y_L^* < c_L$  cannot be an equilibrium choice for the leader of the left-wing party. Similar argument establishes that  $y_R^* > c_R$  cannot be an equilibrium choice for the leader of the right-wing party. □

*Proof of Proposition 1.* By Lemma 1, either  $y_L^* < y_R^*$  or  $y_L^* = y_R^* = \hat{c}_m$ . If  $y_L < y_R$ , then the payoffs of the two party leaders are given by the top lines in equations (3) and (4). The derivatives of these payoffs are given by

$$\frac{\partial \pi_L(y_L, y_R)}{\partial y_L} = \frac{g(\hat{y})z_L(y_L, y_R)}{2}, \tag{A1}$$

where  $z_L(y_L, y_R) \equiv h_L - (y_L - c_L) - 2M(\hat{y})$ , and

$$\frac{\partial \pi_R(y_L, y_R)}{\partial y_R} = \frac{g(\hat{y})z_R(y_L, y_R)}{2}, \tag{A2}$$

where  $z_R(y_L, y_R) \equiv -h_R + (c_R - y_R) + 2H(\hat{y})$ .

Since  $g(\cdot) > 0$ , the signs of the two derivatives depend on the signs of  $z_L(\cdot, \cdot)$  and  $z_R(\cdot, \cdot)$ . Given that  $g(\cdot) > 0$ ,  $z_L(\cdot, \cdot)$  and  $z_R(\cdot, \cdot)$  are continuous functions of  $y_L$  and  $y_R$  and by Assumption A1 they are strictly decreasing with both  $y_L$  and  $y_R$ . Hence, fixing  $y_R$ ,  $z_L(\cdot, y_R)$  can change signs only once, from positive to negative, and fixing  $y_L$ ,  $z_R(y_L, \cdot)$  can change signs only once, from positive to negative. Consequently, for all  $y_R$ ,  $\pi_L(y_L, y_R)$  attains a local maximum when  $z_L(y_L, y_R) = 0$ . This local maximum is above  $c_L$  since

Assumptions A1 and A2 ensure that  $h_L > 2M((c_L + c_R)/2) > 2M((c_L + y_R)/2)$  for all  $y_R < c_R$ , thereby implying that  $z_L(c_L, y_R) > 0$  for all  $y_R < c_R$ . The reason why the local maximum may not be global is that  $\pi_L(y_L, y_R)$  has a discontinuity point at  $y_L = y_R$  and may have an upward jump at that point.

A sufficient condition for the local maximum to be a global maximum is that  $y_R \geq \hat{c}_m$  because then  $\pi_L(y_L, y_R)$  does not jump upward at  $y_L = y_R$  as the left-wing party's chance to win the elections falls from above  $\frac{1}{2}$  when  $y_L$  is just below  $y_R$ , to  $\frac{1}{2}$  when  $y_L = y_R$ , to less than  $\frac{1}{2}$  when  $y_L > y_R$  [moving  $y_L$  further above  $y_R$  only lowers  $\pi_L(y_L, y_R)$  since then the electoral prospects of the left-wing party decline further while its platform shifts away from the party's ideal policy,  $c_L$ ]. Hence, under Assumptions A1 and A2 the local maximum characterized by the condition  $z_L(y_L, y_R) = 0$  is also a global maximum provided that  $y_R \geq \hat{c}_m$ .

Likewise, for all  $y_L \leq \hat{c}_m$ ,  $\pi_R(y_L, y_R)$  reaches a global maximum when  $z_R(y_L, y_R) = 0$ . This maximum is below  $c_R$  since Assumptions A1 and A2 ensure that  $h_R > 2H((c_L + c_R)/2) > 2H((y_L + c_R)/2)$  for all  $y_L > c_L$ , thus implying that  $z_R(y_L, c_R) < 0$  for all  $y_L > c_L$ .

Now, suppose that  $z_L(\hat{c}_m, \hat{c}_m) < 0 < z_R(\hat{c}_m, \hat{c}_m)$ . Then, the best response of the left-wing party's leader against  $y_R = \hat{c}_m$  is to choose  $y_L < \hat{c}_m$  and the best response of the right-wing party's leader against  $y_L = \hat{c}_m$  is to choose  $y_R > \hat{c}_m$ . Since  $z_L(\cdot, \cdot)$  is decreasing with  $y_R$ ,  $z_L(\hat{c}_m, \hat{c}_m) < 0$  implies that  $z_L(\hat{c}_m, y_R) < 0$  for all  $y_R \geq \hat{c}_m$ . Hence, if  $z_L(\hat{c}_m, \hat{c}_m) < 0$ , then the best response of the left-wing party's leader against any  $y_R \geq \hat{c}_m$  is to choose  $y_L < \hat{c}_m$ . Likewise, since  $z_R(\cdot, \cdot)$  is decreasing with  $y_L$ ,  $z_R(\hat{c}_m, \hat{c}_m) > 0$  implies that  $z_R(y_L, \hat{c}_m) > 0$  for all  $y_L \leq \hat{c}_m$ . Hence,  $z_R(\hat{c}_m, \hat{c}_m) > 0$  ensures that the best response of the right-wing party's leader against any  $y_L \leq \hat{c}_m$  is to choose  $y_R > \hat{c}_m$ . Consequently,  $z_L(\hat{c}_m, \hat{c}_m) < 0 < z_R(\hat{c}_m, \hat{c}_m)$  is a sufficient condition for the existence of a unique political equilibrium in which  $c_L < y_L^* < \hat{c}_m < y_R^* < c_R$ .

Next, suppose that  $z_L(\hat{c}_m, \hat{c}_m) > 0 > z_R(\hat{c}_m, \hat{c}_m)$ . Then, the best response of the left-wing party's leader against  $y_R = \hat{c}_m$  is to choose  $y_L = \hat{c}_m$ : given that  $z_L(\hat{c}_m, \hat{c}_m) > 0$ ,  $y_L = \hat{c}_m$  gives the leader of the left-wing party a higher payoff than any  $y_L < \hat{c}_m$ . Setting  $y_L > \hat{c}_m$  will only lower  $\pi_L(y_L, y_R)$  since the left-wing party's chance to win the elections falls below  $\frac{1}{2}$  and the party's platform shifts further away from  $c_L$ . For similar reasons, if  $z_R(\hat{c}_m, \hat{c}_m) < 0$ , then the best response of the right-wing party's leader against  $y_L = \hat{c}_m$  is to choose  $y_R = \hat{c}_m$ . Hence,  $z_L(\hat{c}_m, \hat{c}_m) > 0 > z_R(\hat{c}_m, \hat{c}_m)$  is a sufficient condition for an equilibrium with  $y_L^* = y_R^* = \hat{c}_m$ .  $\square$

*Proof of Proposition 2.* (i) Since  $c_L < y_L^* < y_R^* < c_R$ , the equilibrium platforms are given by the solution to the following pair of equations:

$$\frac{\partial \pi_L(y_L, y_R)}{\partial y_L} = \frac{g(\hat{y})}{2} [h_L - (y_L - c_L)] - G(\hat{y}) = 0, \tag{A3}$$

and

$$\frac{\partial \pi_R(y_L, y_R)}{\partial y_R} = -\frac{g(\hat{y})}{2} [h_R - (c_R - y_R)] + (1 - G(\hat{y})) = 0. \tag{A4}$$

The comparative statics matrix associated with these equations is given by

$$J(y_L^*, y_R^*) \times \begin{vmatrix} \frac{\partial y_L^*}{\partial y_R^*} \\ \frac{\partial y_R^*}{\partial y_L^*} \end{vmatrix} = \begin{vmatrix} -g(\hat{y}^*)/2 \\ 0 \end{vmatrix} \times \partial a_L, \tag{A5}$$

where  $\hat{y}^* \equiv (y_L^* + y_R^*)/2$  and

$$J(y_L^*, y_R^*) \equiv \begin{vmatrix} \frac{\partial^2 \pi_L(y_L^*, y_R^*)}{\partial y_L^2} & \frac{\partial^2 \pi_L(y_L^*, y_R^*)}{\partial y_L \partial y_R} \\ \frac{\partial^2 \pi_R(y_L^*, y_R^*)}{\partial y_R \partial y_L} & \frac{\partial^2 \pi_R(y_L^*, y_R^*)}{\partial y_R^2} \end{vmatrix}. \tag{A6}$$

is the Jacobian matrix corresponding to equations (A3) and (A4). Taking the second derivatives of the expected payoff functions and substituting for  $h_L - (y_L - c_L)$  from (A3) and  $h_R - (c_R - y_R)$  from (A4) reveals that the diagonal terms in  $J(\cdot)$  are

$$\frac{\partial^2 \pi_L(y_L^*, y_R^*)}{\partial y_L^2} = -\frac{g(\hat{y}^*)}{2} [1 + M'(\hat{y}^*)], \tag{A7}$$

$$\frac{\partial^2 \pi_R(y_L^*, y_R^*)}{\partial y_R^2} = -\frac{g(\hat{y}^*)}{2} [1 - H'(\hat{y}^*)].$$

The off-diagonal terms in  $J(\cdot)$  are

$$\frac{\partial^2 \pi_L(y_L^*, y_R^*)}{\partial y_L \partial y_R} = -\frac{g(\hat{y}^*)}{2} M'(\hat{y}^*), \tag{A8}$$

$$\frac{\partial^2 \pi_R(y_L^*, y_R^*)}{\partial y_R \partial y_L} = \frac{g(\hat{y}^*)}{2} H'(\hat{y}^*).$$

By Assumption A1, all four derivatives in  $J(\cdot)$  are negative. Using equations (A7) and (A8), the determinant of  $J(y_L^*, y_R^*)$ , is given by

$$\begin{aligned} |J(y_L^*, y_R^*)| &= \frac{3g^2(\hat{y}^*) + g'(\hat{y}^*)(1 - 2G(\hat{y}^*))}{4} \\ &= -\frac{g(\hat{y}^*)}{2} \left( \frac{\partial^2 \pi_L(y_L^*, y_R^*)}{\partial y_L \partial y_R} + \frac{\partial^2 \pi_R(y_L^*, y_R^*)}{\partial y_R \partial y_L} \right) + \frac{g^2(\hat{y}^*)}{4} > 0, \end{aligned} \tag{A9}$$

where the inequality follows since the two cross-partial derivatives are negative.

Using equation (A5) and applying Cramer’s rule, yields:

$$\frac{\partial y_L^*}{\partial a_L} = -\frac{g(\hat{y}^*) \frac{\partial \pi_R^2(y_L^*, y_R^*)}{\partial y_R^2}}{2|J(y_L^*, y_R^*)|} > 0,$$

$$\frac{\partial y_R^*}{\partial a_L} = \frac{g(\hat{y}^*) \frac{\partial \pi_R^2(y_L^*, y_R^*)}{\partial y_R \partial y_L}}{2|J(y_L^*, y_R^*)|} < 0.$$
(A10)

Moreover, equations (A7) and (A8) reveal that  $\partial \pi_R^2(y_L^*, y_R^*)/\partial y_R^2 < \partial \pi_R^2(y_L^*, y_R^*)/\partial y_R \partial y_L$ . Thus, an increase in  $y_L^*$  outweighs the decrease in  $y_R^*$ . Hence  $P_L(\cdot)$  increases. The comparative static result regarding an increase in  $a_R$  is analogous.

(ii) Adding equations (A3) and (A4), recalling that  $d_L^* \equiv \hat{c}_m - y_L^*$ ,  $a_L \equiv h_L - (\hat{c}_m - c_L)$ ,  $d_R^* \equiv y_R^* - \hat{c}_m$ ,  $a_R \equiv h_R - (c_R - \hat{c}_m)$ , and rearranging terms yields,

$$\frac{g(\hat{y}^*)}{2} [(a_L - a_R) + (d_L^* - d_R^*)] = 2G(\hat{y}^*) - 1.$$
(A11)

Now let  $a_L > a_R$  and assume by way of negation that  $d_L^* > d_R^*$ . Then the left side of (A11) is positive, while the right side is negative because  $d_L^* > d_R^*$  implies  $y_L^* + y_R^* < 2\hat{c}_m$ , so  $G(\hat{y}^*) < G(\hat{c}_m) = \frac{1}{2}$ , a contradiction. Hence,  $a_L > a_R$  implies  $d_L^* < d_R^*$ . A similar proof establishes that  $a_L < a_R$  implies  $d_L^* > d_R^*$ . Finally, when  $a_L = a_R$ , equation (A11) can hold only if  $d_L^* = d_R^*$ , in which case  $y_L^* + y_R^* = 2\hat{c}_m$ , so  $G(\hat{y}^*) = G(\hat{c}_m) = \frac{1}{2}$ . □

*Proof of Proposition 3.* Noting that  $\Sigma \equiv a_L + a_R = h_L + h_R + c_L - c_R$  and  $\Delta \equiv a_L - a_R = h_L - h_R + c_L + c_R - 2\hat{c}_m$ , we can rewrite equations (A3) and (A4) as

$$\frac{g(\hat{y})}{2} \left[ \frac{\Sigma + \Delta}{2} + \hat{c}_m - y_L \right] - G(\hat{y}) = 0,$$
(A12)

and

$$-\frac{g(\hat{y})}{2} \left[ \frac{\Sigma - \Delta}{2} + y_R - \hat{c}_m \right] + (1 - G(\hat{y})) = 0.$$
(A13)

The two comparative statics matrices that correspond to this pair of equations are given by

$$J(y_L^*, y_R^*) \times \begin{vmatrix} \partial y_L^* \\ \partial y_R^* \end{vmatrix} = \begin{vmatrix} g(\hat{y}^*)/4 \\ -g(\hat{y}^*)/4 \end{vmatrix} \times \partial \Delta,$$

$$J(y_L^*, y_R^*) \times \begin{vmatrix} \partial y_L^* \\ \partial y_R^* \end{vmatrix} = \begin{vmatrix} -g(\hat{y}^*)/4 \\ -g(\hat{y}^*)/4 \end{vmatrix} \times \partial \Sigma.$$
(A14)

Using Cramer’s rule and recalling that  $|J(y_L^*, y_R^*)| > 0$  yields the comparative static results regarding  $y_L^*$ ,  $y_R^*$ , and  $y^*$ .  $\square$

*Proof of Proposition 5.* Using equation (7) and Proposition 3, it follows that as long as  $y_L^* < y_R^*$ ,

$$\frac{\partial Ex_{RD}}{\partial \Sigma} = G(\hat{y}^*) - \frac{1}{2}, \tag{A15}$$

and

$$\frac{\partial Ex_{RD}}{\partial \Delta} = \frac{g^2(\hat{y}^*)(1 - g(\hat{y}^*)(y_R^* - y_L^*)) + 2g'(\hat{y}^*)(G(\hat{y}^*) - \frac{1}{2})}{8|J(y_L^*, y_R^*)|}. \tag{A16}$$

Recalling from Proposition 2 that  $G(\hat{y}^*) \cong \frac{1}{2}$  as  $\Delta \equiv a_L - a_R \cong 0$ , part (i) of the proposition follows immediately from equation (A15). To prove part (ii) of the proposition, we subtract equation (A4) from equation (A3), note that  $\Sigma \equiv a_L + a_R = h_L + h_R + c_L - c_R$ , and rearrange terms to obtain,

$$\frac{g(\hat{y}^*)}{2} [\Sigma + y_R^* - y_L^*] = 1. \tag{A17}$$

Substituting for  $y_R^* - y_L^*$  from equation (A17) into the numerator of equation (A16) and rearranging:

$$\frac{\partial Ex_{RD}}{\partial \Delta} = \frac{g^3(\hat{y}^*) \left( \Sigma - \frac{1}{g(\hat{y}^*)} \right) + 2g'(\hat{y}^*)(G(\hat{y}^*) - \frac{1}{2})}{8|J(y_L^*, y_R^*)|}. \tag{A18}$$

When  $g(\cdot)$  is symmetric and unimodal, the second term in the numerator vanishes if  $G(\hat{y}^*) = \frac{1}{2}$  and is negative otherwise because  $G(\hat{y}^*) - \frac{1}{2}$  and  $g'(\hat{y}^*)$  have opposite signs. Therefore,  $\Sigma < 1/g(\hat{y}^*)$  is sufficient for  $\Delta$  to have a negative impact on  $Ex_{RD}$ . If  $\Delta = 0$ , then  $G(\hat{y}^*) = \frac{1}{2}$  and  $y^* = \hat{c}_m$ . Hence, in this case,  $\partial Ex_{RD}/\partial \Delta < 0$  as  $\Sigma < 1/g(\hat{c}_m)$  and  $\partial Ex_{RD}/\partial \Delta > 0$  as  $\Sigma > 1/g(\hat{c}_m)$ .  $\square$

*Proof of Proposition 7.* To prove the proposition we first prove the following lemmas.

**Lemma 2.** At the highest possible value of  $\Sigma$  that is consistent with a partial convergence equilibrium in which  $y_L^* < \hat{c}_m < y_R^*$ ,  $Ex_{RD} \cong Ex_{MV} = \hat{c}_m$  as  $\Delta \cong 0$ .

*Proof of Lemma 2.* By Proposition 3, an increase in  $\Sigma$  shifts  $y_L^*$  to the right and  $y_R^*$  to the left. Hence, to ensure that  $y_L^* < \hat{c}_m < y_R^*$ ,  $\Sigma$  can be raised until either (i)  $d_L^* = 0 < d_R^*$  or (ii)  $d_L^* > 0 = d_R^*$ . Proposition 2 implies that in case (i),  $G(\hat{y}^*) > \frac{1}{2}$ , so by equation (7),  $Ex_{RD} > Ex_{MV} = \hat{c}_m$ . In case (ii),  $G(\hat{y}^*) < \frac{1}{2}$ , so by equation (7),  $Ex_{RD} < Ex_{MV} = \hat{c}_m$ . The proof follows by noting that

Proposition 2 also implies that  $d_L^* \cong d_R^*$  as  $a_L \cong a_R$  or  $d_L^* \cong d_R^*$  as  $\Delta \cong 0$ . Hence,  $\Delta > 0$  as  $Ex_{RD} > Ex_{MV} = \hat{c}_m$  and  $\Delta < 0$  as  $Ex_{RD} < Ex_{MV} = \hat{c}_m$ .  $\square$

**Lemma 3.** For each  $\Delta$  there exists a lowest value of  $\Sigma$  that is consistent with Assumption A3. At this value,

- (i) if  $\Delta > 0$ , then  $Ex_{RD} \cong Ex_{MV} = \hat{c}_m$  as  $2G(\hat{y}^*)(c_1 - \hat{y}^*) \cong (c_1 - \hat{c}_m)$ , and
- (ii) if  $\Delta < 0$  then  $Ex_{RD} \cong Ex_{MV} = \hat{c}_m$  as  $2(1 - G(\hat{y}^*))(\hat{y}^* - c_0) \cong (\hat{c}_m - c_0)$ .

*Proof of Lemma 3.* Since  $\partial y_L^* / \partial \Sigma = -\frac{1}{2}$  and  $\partial y_R^* / \partial \Sigma = \frac{1}{2}$ , then holding  $\Delta$  fixed,  $\Sigma$  can decrease until either  $y_R^*$  approaches  $c_1$  or  $y_L^*$  approaches  $c_0$  (note that since  $\Sigma \equiv h_L + h_R + c_L - c_R$ , we can continue to lower  $\Sigma$  until  $c_R$  approaches  $c_1$  and  $c_L$  approaches  $c_0$ ). Now there are two cases to consider.

First, if  $\Delta > 0$ , then by Proposition 2,  $d_L^* < d_R^*$ , so  $y_R^*$  is closer to  $c_1$  than  $y_L^*$  is to  $c_0$ . Hence  $\Sigma$  can decrease until  $y_R^*$  approaches  $c_1$ . Since  $y_L^*$  and  $y_R^*$  move away from  $\hat{y}^*$  at equal rates, then as  $y_R^*$  approaches  $c_1$ ,  $y_L^*$  approaches  $2\hat{y}^* - c_1$ . Hence, at the lower bound on  $\Sigma$ ,  $Ex_{RD} = G(\hat{y}^*)(2\hat{y}^* - c_1) + (1 - G(\hat{y}^*))c_1 = c_1 - 2G(\hat{y}^*)(c_1 - \hat{y}^*)$ . Hence, at the lower bound on  $\Sigma$ ,  $Ex_{RD} \cong Ex_{MV} = \hat{c}_m$  as  $2G(\hat{y}^*)(c_1 - \hat{y}^*) \cong (c_1 - \hat{c}_m)$ .

Second, if  $\Delta < 0$ , Proposition 2 implies that  $d_L^* > d_R^*$ , so  $y_L^*$  is closer to  $c_0$  than  $y_R^*$  is to  $c_1$ . Hence,  $\Sigma$  can decrease until  $y_L^*$  approaches  $c_0$ , at which point,  $y_R^*$  approaches  $2\hat{y}^* - c_0$ . Consequently,  $Ex_{RD} = c_0 + 2(1 - G(\hat{y}^*))(\hat{y}^* - c_0)$ . Hence, at the lower bound on  $\Sigma$ ,  $Ex_{RD} \cong Ex_{MV} = \hat{c}_m$  as  $2(1 - G(\hat{y}^*))(\hat{y}^* - c_0) \cong (\hat{c}_m - c_0)$ .  $\square$

**Lemma 4.** If  $|\Delta|$  is not too large and  $g(\hat{c}_m) > 1/(c_1 - c_0)$ , then at the lowest value of  $\Sigma$  that is consistent with Assumption A3,  $Ex_{RD} \cong Ex_{MV} = \hat{c}_m$  as  $\Delta \cong 0$ . Otherwise,  $Ex_{RD} \cong Ex_{MV} = \hat{c}_m$  as  $\Delta \cong 0$ .

*Proof of Lemma 4.* Since at  $\Delta = 0$ ,  $\hat{y}^* = \hat{c}_m$  and  $G(\hat{y}^*) = \frac{1}{2}$ , then  $G(\hat{y}^*)(c_1 - \hat{y}^*) = (c_1 - \hat{c}_m)/2$ . Evaluated at  $\Delta = 0$ , the derivative of the left side of this equality with respect to  $\Delta$  is equal to  $(g(\hat{c}_m)/2)[2(c_1 - \hat{c}_m) - 1/g(\hat{c}_m)](d\hat{y}^*/d\Delta)$ , where  $d\hat{y}^*/d\Delta > 0$  by Proposition 3. Since the distribution of  $c_m$  is symmetric,  $2(c_1 - \hat{c}_m) = c_1 - c_0$ , so the derivative is positive if and only if  $g(\hat{c}_m) > 1/(c_1 - c_0)$ . Hence, when  $\Delta > 0$ ,  $2G(\hat{y}^*)(c_1 - \hat{y}^*) \cong (c_1 - \hat{c}_m)/2$  as  $g(\hat{c}_m) \cong 1/(c_1 - c_0)$ . Lemma 3 therefore implies that when  $\Delta > 0$ ,  $Ex_{RD} \cong Ex_{MV} = \hat{c}_m$  as  $g(\hat{c}_m) \cong 1/(c_1 - c_0)$ .

Similarly, at  $\Delta = 0$ ,  $\hat{y}^* = \hat{c}_m$  and  $G(\hat{y}^*) = \frac{1}{2}$ ; hence,  $(1 - G(\hat{y}^*))(\hat{y}^* - c_0) = (\hat{c}_m - c_0)/2$ . Evaluated at  $\Delta = 0$ , the derivative of the left side of the equality with respect to  $\Delta$  is  $(g(\hat{c}_m)/2)[1/g(\hat{c}_m) - 2(\hat{c}_m^* - c_0)](d\hat{y}^*/d\Delta)$ . Since the distribution of  $c_m$  is symmetric,  $2(\hat{c}_m - c_0) = (c_1 - c_0)$ , so the derivative is negative if and only if  $g(\hat{c}_m) > 1/(c_1 - c_0)$ . Hence, when  $\Delta < 0$ ,  $(1 - G(\hat{y}^*))(\hat{y}^* - c_0) \cong (\hat{c}_m - c_0)/2$  as  $g(\hat{c}_m) \cong 1/(c_1 - c_0)$ . Lemma 3 therefore implies that for  $\Delta < 0$ ,  $Ex_{RD} \cong Ex_{MV} = \hat{c}_m$  as  $g(\hat{c}_m) \cong 1/(c_1 - c_0)$ .  $\square$

**Lemma 5.** If  $g(\hat{c}_m) < 1/(c_1 - c_0)$ , then  $Ex_{RD} \cong Ex_{MV} = \hat{c}_m$  as  $\Delta \cong 0$ .

*Proof of Lemma 5.* By Proposition 5,  $\partial Ex_{RD}/\partial \Sigma \cong 0$  as  $\Delta \cong 0$ . Now, Lemma 4 implies that if  $g(\hat{c}_m) < 1/(c_1 - c_0)$ , then at the lowest value of  $\Sigma$ ,  $Ex_{RD} \cong Ex_{MV} = \hat{c}_m$  as  $\Delta \cong 0$ . This implies in turn that if  $\Delta > 0$ , then  $Ex_{RD} > Ex_{MV} = \hat{c}_m$  for all  $\Sigma$ , whereas if  $\Delta < 0$ , then  $Ex_{RD} < Ex_{MV} = \hat{c}_m$  for all  $\Sigma$ .  $\square$

**Lemma 6.** If  $|\Delta|$  is not too large and  $g(\hat{c}_m) > 1/(c_1 - c_0)$ , there exists for each  $\Delta$  a unique value of  $\Sigma$ , denoted  $\Sigma^{NB}(\Delta)$ , for which  $Ex_{RD} = Ex_{MV} = \hat{c}_m$ .

*Proof of Lemma 6.* By Proposition 5,  $\partial Ex_{RD}/\partial \Sigma > 0$  if  $\Delta > 0$  and  $\partial Ex_{RD}/\partial \Sigma < 0$  if  $\Delta < 0$ . Now, Lemma 2 implies that at the upper bound on  $\Sigma$ ,  $Ex_{RD} > 0$  if  $\Delta > 0$  and  $Ex_{RD} < 0$  if  $\Delta < 0$ , while Lemma 3 implies that if  $g(\hat{c}_m) > 1/(c_1 - c_0)$ , then at the lower bound on  $\Sigma$ ,  $Ex_{RD} \cong Ex_{MV} = \hat{c}_m$  as  $\Delta \cong 0$ . Therefore, there exists a value of  $\Sigma$ , denoted  $\Sigma^{NB}(\Delta)$ , at which  $Ex_{RD} = Ex_{MV} = \hat{c}_m$ .  $\square$

Before turning to the properties of  $\Sigma^{NB}(\Delta)$ , we first prove the two following lemmas:

**Lemma 7.** Let  $\hat{y}^*(\Delta)$  be the value of  $\hat{y}^*$  given  $\Delta$  and define  $G(\Delta) \equiv G(\hat{y}^*(\Delta))$  and  $g(\Delta) \equiv g(\hat{y}^*(\Delta))$ . If  $g(\cdot)$  is symmetric and unimodal, then  $G(\Delta) = 1 - G(-\Delta)$  and  $g(\Delta) = g(-\Delta)$ .

*Proof of Lemma 7.* Adding equations (A12) and (A13) and rearranging, yields

$$\hat{y}^*(\Delta) - \hat{c}_m = \frac{2}{g(\Delta)} \left[ \frac{1}{2} - G(\Delta) \right] + \frac{\Delta}{2}. \tag{A19}$$

Evaluating the same equation at  $-\Delta$  and adding to (A19) yields:

$$\begin{aligned} [\hat{y}^*(\Delta) - \hat{c}_m] - [\hat{c}_m - \hat{y}^*(-\Delta)] &= \frac{2}{g(\Delta)} \left[ \frac{1}{2} - G(\Delta) \right] \\ &+ \frac{2}{g(-\Delta)} \left[ \frac{1}{2} - G(-\Delta) \right]. \end{aligned} \tag{A20}$$

Now, assume by way of negation that the expressions on both sides of equation (A20) are positive. This implies that  $\hat{y}^*(\Delta)$  is further away from the center than  $\hat{y}^*(-\Delta)$ . Since  $g(\cdot)$  is symmetric and unimodal, this implies that  $G(\Delta) > 1 - G(-\Delta)$  and  $g(\Delta) \leq g(-\Delta)$ . Hence,

$$\begin{aligned} \frac{2}{g(\Delta)} \left[ \frac{1}{2} - G(\Delta) \right] - \frac{2}{g(-\Delta)} \left[ \frac{1}{2} - G(-\Delta) \right] &< \frac{2}{g(\Delta)} [1 - G(\Delta) \\ &- G(-\Delta)] < 0, \end{aligned} \tag{A21}$$

thus contradicting the presumption that (A20) is positive. Similarly we can prove that the left side of equation (A20) cannot be negative. Hence,  $\hat{y}^*(\Delta) - \hat{c}_m = \hat{c}_m - \hat{y}^*(-\Delta)$ . Given that  $g(\cdot)$  is symmetric and unimodal, this implies in turn that  $G(\Delta) = 1 - G(-\Delta)$  and  $g(\Delta) = g(-\Delta)$ .  $\square$

**Lemma 8.** Let  $d_L^*(\Delta)$  be the value of  $d_L^*$  given  $\Delta$  and define  $d_R^*(\Delta)$  similarly. Then,  $d_L^*(\Delta) = d_R^*(-\Delta)$  and  $d_L^*(-\Delta) = d_R^*(\Delta)$ .

*Proof of Lemma 8.* Evaluating equation (A12) at  $\Delta$ , recalling that  $d_L^*(\Delta) \equiv \hat{c}_m - y_L^*$ , and rearranging,

$$\frac{g(\Delta)}{2} \left[ \frac{\Sigma + \Delta}{2} + d_L^*(\Delta) \right] = G(\Delta). \tag{A22}$$

Similarly, evaluating (A13) at  $-\Delta$ , recalling that  $d_R^*(\Delta) \equiv y_R^* - \hat{c}_m$ , and rearranging,

$$\frac{g(-\Delta)}{2} \left[ \frac{\Sigma + \Delta}{2} + d_R^*(-\Delta) \right] = (1 - G(-\Delta)). \tag{A23}$$

Subtracting equation (A23) from (A22), recalling from Lemma 7 that  $g(\Delta) = g(-\Delta)$ , and rearranging terms yields,

$$\frac{g(\Delta)}{2} [d_L^*(\Delta) + d_R^*(-\Delta)] = 0. \tag{A24}$$

Since  $g(\Delta) \neq 0$ , it follows that  $d_L^*(\Delta) = d_R^*(-\Delta)$ . The proof that  $d_L^*(-\Delta) = d_R^*(\Delta)$  is completely analogous.  $\square$

We are now ready to prove the properties of  $\Sigma^{NB}(\Delta)$ :

■ **Symmetry.** To prove symmetry we need to show that  $\Sigma^{NB}(\Delta) = \Sigma^{NB}(-\Delta)$ . To this end, let  $Ex_{RD}(\Delta, \Sigma)$  be the expected policy under RD given  $\Delta$  and  $\Sigma$ , and recall that  $\Sigma^{NB}(\Delta)$  is defined by  $Ex_{RD}(\Delta, \Sigma^{NB}(\Delta)) = \hat{c}_m$ . Hence, we can prove that  $\Sigma^{NB}(\Delta)$  is symmetric by showing that  $Ex_{RD}(\Delta, \Sigma^{NB}(\Delta)) = Ex_{RD}(-\Delta, \Sigma^{NB}(-\Delta))$ . Using equation (7),

$$Ex_{RD}(\Delta, \Sigma^{NB}(\Delta)) = G(\Delta)y_L^*(\Delta) + (1 - G(\Delta))y_R^*(\Delta). \tag{A25}$$

Recalling from Lemma 7 that  $G(\Delta) = 1 - G(-\Delta)$ , this equation becomes,

$$Ex_{RD}(\Delta, \Sigma^{NB}(\Delta)) = (1 - G(-\Delta))y_L^*(\Delta) + G(-\Delta)y_R^*(\Delta). \tag{A26}$$

By Lemma 8,  $d_L^*(\Delta) = d_R^*(-\Delta)$ . Using the definitions of  $d_L^*(\cdot)$  and  $d_R^*(\cdot)$ , it follows that  $y_L^*(\Delta) = 2\hat{c}_m - y_R^*(-\Delta)$  and  $y_R^*(\Delta) = 2\hat{c}_m - y_L^*(-\Delta)$ . Substituting

in equation (A26) we get,

$$\begin{aligned} Ex_{RD}(\Delta, \Sigma^{NB}(\Delta)) &= 2\hat{c}_m - [(G(-\Delta))y_L^*(-\Delta) + (1 - G(-\Delta))y_R^*(-\Delta)] \\ &= 2\hat{c}_m - Ex_{RD}(-\Delta, \Sigma^{NB}(-\Delta)). \end{aligned} \tag{A27}$$

Hence,

$$\hat{c}_m - Ex_{RD}(\Delta, \Sigma^{NB}(\Delta)) = Ex_{RD}(-\Delta, \Sigma^{NB}(-\Delta)) - \hat{c}_m. \tag{A28}$$

The left side of the equation vanishes by definition, so the right side must also vanish implying that  $Ex_{RD}(\Delta, \Sigma^{NB}(\Delta)) = Ex_{RD}(-\Delta, \Sigma^{NB}(-\Delta)) = \hat{c}_m$ .

■  **$\Sigma^{NB}(\Delta)$  is U-shaped.** Recall that  $\Sigma^{NB}(\Delta)$  is defined implicitly by  $Ex_{RD}(\Delta, \Sigma) = \hat{c}_m$ . Since  $\Sigma^{NB}(\Delta)$  is symmetric around 0, then if  $\Sigma^{NB}(\Delta)$  exists, the equation has exactly two solutions, one positive and one negative. Now, consider the positive solutions and differentiate the equation fully to obtain:

$$\begin{aligned} \frac{d\Delta}{d\Sigma} &= - \frac{\partial Ex_{RD} / \partial \Sigma}{\partial Ex_{RD} / \partial \Delta} \\ &= \frac{8(\frac{1}{2} - G(\hat{y}^*))|J(y_L^*, y_R^*)|}{-g^3(\hat{y}^*)(y_R^* - y_L^*) + g^2(\hat{y}^*) + 2g'(\hat{y}^*)(G(\hat{y}^*) - \frac{1}{2})}. \end{aligned} \tag{A29}$$

Differentiating this expression again with respect to  $\Sigma$  and using part (ii) of Proposition 3 yields

$$\frac{d^2\Delta}{d\Sigma^2} = \frac{8g^3(\hat{y}^*)(\frac{1}{2} - G(\hat{y}^*))|J(y_L^*, y_R^*)|}{(-g^3(\hat{y}^*)(y_R^* - y_L^*) + g^2(\hat{y}^*) + 2g'(\hat{y}^*)(G(\hat{y}^*) - \frac{1}{2}))^2}. \tag{A30}$$

Since we consider positive values of  $\Delta$ ,  $G(\hat{y}^*) > \frac{1}{2}$ , so (A29) and (A30) imply that along the  $\Sigma^{NB}(\Delta)$  curve,  $\Delta$  is increasing with  $\Sigma$  at a decreasing rate. When we take negative solutions of  $\Delta$ , then  $G(\hat{y}^*) < \frac{1}{2}$ , so (A29) and (A30) imply that along the  $\Sigma^{NB}(\Delta)$  curve,  $\Delta$  is decreasing with  $\Sigma$  at a decreasing rate. Together, these properties imply that  $\Sigma^{NB}(\Delta)$  is a U-shaped function of  $\Delta$  that attains a unique minimum at  $\Delta = 0$  [the proof that  $\Sigma^{NB}(\Delta)$  is differentiable at  $\Delta = 0$  appears directly below].

■  **$\Sigma^{NB}(\Delta)$  is smooth at  $\Delta = 0$ .** To prove smoothness, we need to show that the slope of  $\Sigma^{NB}(\Delta)$  is equal to 0 when  $\Delta = 0$ , or alternatively, that the derivative in (A29) goes to infinity as  $\Delta$  goes to 0. To this end, note from Proposition 2 that the numerator of equation (A29) vanishes as  $\Delta$  goes to 0, and assume by way of negation that the denominator of (A29) does not vanish as  $\Delta$  goes to 0. Given this assumption, the derivative in (A29) vanishes at  $\Delta = 0$ . But since both the numerator and the denominator of

(A29) are continuous functions and since by assumption the denominator does not vanish, the derivative in (A29) is also a continuous function. Together with the fact that  $\Sigma^{NB}(\Delta)$  is symmetric this means that the derivative in (A29) must go to infinity as  $\Delta$  goes to 0, a contradiction. Thus, the numerator of equation (A29) must vanish at  $\Delta=0$ . Since both the numerator and denominator of (A29) vanish at  $\Delta=0$ , we need to apply L'Hôpital's rule to determine the limit of this derivative as  $\Delta$  goes to 0. Recalling that  $y^* = \hat{c}_m$  when  $\Delta=0$ , and using the assumption that  $g(\cdot)$  is symmetric and unimodal so that  $G(\hat{c}_m) = \frac{1}{2}$  and  $g^*(\hat{c}_m) = 0$ , we obtain:

$$\lim_{\Delta \rightarrow 0} \left( \frac{d\Sigma^{NB}(\Sigma)}{d\Sigma} \right) = \frac{6g^2(\hat{c}_m)}{0} = \infty. \tag{A31}$$

This implies in turn that  $d\Sigma^{NB}(\Delta)/d\Delta = 0$  as  $\Delta$  goes to 0, so  $\Sigma^{NB}(\Delta)$  is smooth at  $\Delta = 0$ .

■  $\Sigma^{NB}(0) = 1/g(\hat{c}_m)$ . Recall that  $\Sigma^{NB}(\Delta)$  is defined implicitly by  $EX_{RD}(\Delta, \Sigma^{NB}(\Delta)) = \hat{c}_m$ . Differentiating the identity with respect to  $\Delta$ , evaluating at  $\Delta = 0$ , and using the fact that since  $g(\cdot)$  is symmetric and unimodal,  $G(\hat{c}_m) = \frac{1}{2}$  and  $g'(\hat{c}_m) = 0$ , we obtain:

$$\begin{aligned} \frac{d\Sigma^{NB}(0)}{d\Delta} &= \frac{-g^3(\hat{c}_m)(y_R^* - y_L^*) + g^2(\hat{c}_m)}{6g^2(\hat{c}_m)} \\ &= \frac{1 - g(\hat{c}_m)(y_R^* - y_L^*)}{6}. \end{aligned} \tag{A32}$$

Substituting from equation (A15) for  $y_R^* - y_L^*$ , this expression becomes,

$$\frac{d\Sigma^{NB}(0)}{d\Delta} = \frac{g(\hat{c}_m)}{6} \left[ \Sigma(0) - \frac{1}{g(\hat{c}_m)} \right]. \tag{A33}$$

Recalling that  $d\Sigma^{NB}(0)/d\Delta = 0$ , we get  $\Sigma^{NB}(0) = 1/g(\hat{c}_m)$ . □

**Establishing the shapes of  $\Sigma_L^{LB}(\Delta)$ ,  $\Sigma_R^{LB}(\Delta)$ ,  $\Sigma^{UB}(\Delta)$ , and  $\Sigma^{FC}(\Delta)$**

■ **Derivation of  $\Sigma_L^{LB}(\Delta)$  and  $\Sigma_R^{LB}(\Delta)$ .** These lines represent for each  $\Delta$ , the lowest value of  $\Sigma$  permitted by Assumption A2. Subtracting  $\hat{c}_m - c_L$  from both sides of the first inequality in Assumption A2, yields  $a_L \equiv h_L - (\hat{c}_m - c_L) > 2M((c_L + c_R)/2) - (\hat{c}_m - c_L)$ . But since the definitions of  $\Sigma$  and  $\Delta$  imply that  $a_L = (\Sigma + \Delta)/2$ , then

$$\Sigma > \Sigma_L^{LB}(\Delta) \equiv 2M\left(\frac{c_L + c_R}{2}\right) - (\hat{c}_m - c_L) - \Delta. \tag{A34}$$

Likewise, subtracting  $c_R - \hat{c}_m$  from both sides of the second inequality in Assumption A2, and noting that the definitions of  $\Sigma$  and  $\Delta$  imply that  $a_R = (\Sigma - \Delta)/2$ , yields

$$\Sigma > \Sigma_R^{LB}(\Delta) \equiv 2H\left(\frac{c_L + c_R}{2}\right) - (c_R - \hat{c}_m) + \Delta. \tag{A35}$$

Hence, Assumption A2 implies that for each  $\Delta$ , the lower bound on  $\Sigma$  is  $\max\{\Sigma_L^{LB}(\Delta), \Sigma_R^{LB}(\Delta)\}$ .

■ **Derivation of  $\Sigma^{UB}(\Delta)$ .** This line represents, for each  $\Delta$ , the largest value of  $\Sigma$  for which  $z_L(\hat{c}_m, \hat{c}_m) < 0 < z_R(\hat{c}_m, \hat{c}_m)$ . Using the definitions of  $a_L$  and  $a_R$ , and noting that  $H(\hat{c}_m) = M(\hat{c}_m) = \frac{1}{2}g(\hat{c}_m)$ , this condition is equivalent to  $z_L(\hat{c}_m, \hat{c}_m) = h_L - (\hat{c}_m - c_L) - 2M(\hat{c}_m) = a_L - 1/g(\hat{c}_m) < 0$  and  $z_R(\hat{c}_m, \hat{c}_m) = -h_R + (\hat{c}_m - c_R) + 2H(\hat{c}_m) = -a_R + 1/g(\hat{c}_m) > 0$ . Noting that the definitions of  $\Sigma$  and  $\Delta$  imply that  $a_L = (\Sigma + \Delta)/2$  and  $a_R = (\Sigma - \Delta)/2$ , the two inequalities can be written as  $(\Sigma + \Delta)/2 < 1/g(\hat{c}_m)$  and  $(\Sigma - \Delta)/2 < 1/g(\hat{c}_m)$ . Noting that the first inequality implies the second when  $\Delta > 0$  and conversely if  $\Delta < 0$ , we get

$$\Sigma^{FC}(\Delta) = \frac{2}{g(\hat{c}_m)} - |\Delta|. \tag{A36}$$

■ **Derivation of  $\Sigma^{FC}(\Delta)$ .** This line represents, for each  $\Delta$ , the lowest value of  $\Sigma$  for which  $z_L(\hat{c}_m, \hat{c}_m) > 0 > z_R(\hat{c}_m, \hat{c}_m)$ . Following the same steps as in the derivation of  $\Sigma^{UB}(\Delta)$  yields

$$\Sigma^{FC}(\Delta) = \frac{2}{g(\hat{c}_m)} + |\Delta|. \tag{A37}$$

Note that  $\Sigma^{UB}(0) = \Sigma^{FC}(0) = 2/g(\hat{c}_m)$ . Hence in Figure 1, the curves  $\Sigma^{UB}(\Delta)$  and  $\Sigma^{FC}(\Delta)$  intersect on the horizontal axis at  $2/g(\hat{c}_m)$ . □

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