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A TEST OF EXPECTATIONAL PROCESSES USING INFORMATION FROM THE CAPITAL MARKETS–THE ISRAELI CASE*

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1. INTRODUCTION

The purpose of this paper is to confront the various expectational processes that are used in many econometric studies with some independent evidence about expectations. Various expectational processes have been used in the literature: adaptive expectations, regressive expectations, rational expectations, and various combinations of some of them.[2] The main question that is investigated here is, which of those processes best describes the formation of expectations concerning the future proportional rate of increase in some general index of prices. Several attempts to answer such a question by using survey data (Turnosvky [33], De Menil and Bhalla [7], Kane and Malkiel [12], Lahiri [15] for the U.S. and Carlson and Parkin [5] for the U.K.) may be found in the recent literature.

In this paper another attempt to test empirically the way inflationary expectations are formed is made by using independent evidence from the Israeli capital market in which the existence of a wide market of bonds that are indexed to the Consumer Price Index (C.P.I.) provides a direct measure of the rate of interest net of the premium for inflationary expectations.[3] By comparing this rate with the nominal yield obtained from non-indexed financial instruments, it is possible, after adjusting for other elements that cause this differential, to get a measure of inflationary expectations without assuming any dependence of expectations

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[2] For example, adaptive expectations have been used by Nerlove in the context of single markets [23, 24] and by various others such as Cagan [4] and Solow [30] to describe the formation of expectations concerning the future behavior of the rate of increase in the general level of prices. Regressive and extrapolative expectations have been used by Kesselman [13] to describe the formation of expectations about the rate of exchange of the Canadian dollar. A blend of the two was used by DeLeeuw [6]. Rational expectations were first introduced by Muth [21] and recently revived in works by Lucas [16], Sargent [28], Modigliani and Shiller [19] and several others. Frenkel [10] and Mussa [22] suggest a combined adaptive-regressive process.

[3] This rate will be referred to in the future as "the real rate of interest." The rate of interest on regular bonds will be referred to as "the nominal rate of interest."

on distributed lags of past actual inflation rates.[4] The resulting series of inflationary expectations is then used to test various hypotheses regarding price expectations. This method compares favorably with the previously mentioned studies which are based on survey data, since it derives inflationary expectations from the outcomes of actions taken by market participants rather than from survey answers which contain an element of error. But the need to disentangle the part of the differential between the real and the nominal rate of interest which is caused by inflationary expectations from the part caused by other elements requires some other assumptions. Therefore this paper should be viewed as an empirical test of expectations hypotheses by means of independently obtained, but somewhat inaccurate, data on expectations. However, since direct data on expectations is scarce and the data available from surveys is subject to error as well, I believe that the results of the tests made with those expectations do provide pertinent information. Furthermore some sensitivity analysis with the assumptions did not change the qualitative nature of the results. In addition the existence of a measurement error in true expectations does not invalidate the tests per se.

2. INFLATIONARY EXPECTATIONS DERIVED FROM THE CAPITAL MARKET

According to Fisher's [9] theory of interest, the difference between the nominal rate (n_t) and the real rate of interest (r_t) is equal to the rate of inflation expected by participants in the capital market (q_t) provided that 1) both rates are measured on financial instruments which are identical in every respect and differ only in that one instrument is fully indexed while the other is not; 2) investors are predominantly risk neutral in real terms. When those assumptions are not fulfilled, the differential $n_t - r_t$ includes, besides the expected rate of inflation, other elements like a risk premium to compensate holders of nominal bonds for the uncertainty in their real return, and premiums or discounts which are related to differences in other financial features of these two instruments.

r_t is measured as an average of real yields to maturity on fully indexed government bonds. n_t is measured as the average nominal rate on bill brokerage transactions.[5] Since these two instruments differ in their financial risk and a few other characteristics, q_t cannot be solved without further assumptions which essentially maintain that over long periods, people's expectations are right and that the ratio between the elements of the differential $n_t - r_t$, which are not caused by the expected rate of inflation to the real rate of interest, was constant during the period under investigation.[6] Some sensitivity analysis with the exact form of the assumptions did not change the qualitative nature of the results. n_t is

[4] The only other attempt to test hypotheses about expectations formation by using the differential between indexed and non-indexed bonds that I know of is by Paunio and Suvanto [25] who use Finnish data.

[5] Non-indexed government bonds do not exist. For details on bill brokerage see Appendix.

[6] For further details and rationale for those assumptions and the derivation of the q_t series, see Appendix.

measured on financial instruments whose average maturity is around 18 months, while r_t is measured on indexed bonds whose average maturity is longer so that there is some ambiguity as to the period in the future to which q_t refers. However, during the period of the sample the real yield curve is relatively flat[7] over the range of maturities of whose yields r_t is an average. Therefore r_t may be taken as an estimate of the short end of the real yield curve and q_t as the expected average annual rate of increase in the CPI over a span of around 18 months. q_t may also be taken to reflect the beliefs of the relatively informed segment of the community, such as bankers and businessmen who are active in the financial markets, since it is mostly their actions which determine the differential $n_t - r_t$.

q_t was solved by using the method detailed in the Appendix, on a monthly basis, from January, 1958 to December 1967.

3. THE EXPECTATIONAL PROCESSES TO BE TESTED

The independently derived index of expectation, q_t, will now be confronted with various expectations processes which are frequently used in the empirical and theoretical literature when an inflationary expectation variable is called upon. This requires a criterion for the measurement of "nearness" of one time series to another. A natural candidate is the square root of the mean square deviation (M. S. S. D.) (adjusted for lost degrees of freedom as a result of different numbers of parameters in the various processes) of q_t from each of the expectational processes to be tested. The processes to be tested are:

3.1. *Adaptive expectations*:[8] $E_t = \theta P_{t-1} + (1-\theta)E_{t-1}$ where θ is usually assumed to be between 0 and 1 and E_t stands for the expected rate of increase in prices. In the case $\theta = 0$, $E_t = \text{const.}$ for every t and there is no way to find this constant by just observing past rates of increase in the C. P. I. But it is possible to derive a lower limit to the test criterion by computing min. M. S. S. D. $= \sqrt{\sum_t (q_t - \bar{q})^2/(T-1)}$ where $\bar{q}$ is the average of q_t. For $\theta \neq 0$ the test criterion for each $\theta = 0.1, 0.2 \cdots 1.0$ is taken as M. S. S. D. $(\theta) = \sqrt{\sum_t (q_t - E_t(\theta))^2/(T-1)}$,[9] where T is the number of observations.

3.2. *Extrapolative expectations*: The main idea in this process as applied to expectations about the C. P. I. is that people form their expectations by projecting the direction of change of former periods in order to form their expectations about the future. The two variants a. $E_t = P_{t-1} + \alpha_1(P_{t-1} - P_{t-2})$ and b. E_t

[7] See Ben-Shahar and Cukierman [3].

[8] For their use see, for example, Nerlove [23, 24] and Cagan [4].

[9] The initial E_t is assumed to equal P_t. Since the initial point is chosen in 1948 or in 1949 which is about ten years before the period of the test, this arbitrary assumption does not affect $E_t(\theta)$ much, for $\theta \neq 0$ by the time it reaches 1958 and beyond.

$=P_{t-1}+\alpha_2(P_{t-1}-\sum_{i=2}^{n} w_i P_{t-i})$ will be dealt with in here.[10] The two processes differ only w.r.t. the assumption concerning the extrapolation base. The M.S.S.D. of variant (a) is computed by running the regression $q_t-P_{t-1}=\alpha_1(P_{t-1}-P_{t-2})$. Variant (b) may be rewritten as

$$E_t = b_1 P_{t-1} + b_2 P_{t-2} + \cdots + b_n P_{t-n} + b_0 \tag{1}$$

where $b_1=1+\alpha_2$, $b_0=0$, and $b_i=-\alpha_2 w_i$ for $n \geq i \geq 2$. For a given length of lag and any process that can be represented in the form (1) the least squares property assures that the minimum M.S.S.D. between true expectations (q_t) and E_t is achieved for the parameters estimated from a least square regression of q_t on $P_{t-1},\ldots, P_{t-n}$. Hence min M.S.S.D. for process (b) is computed by running the regression; $q_t=\sum_{i=1}^{n} b_i P_{t-i}$.

3.3. *Regressive expectations*:[11] The process may be written as $E_t=\gamma\bar{E}_t+(1-\gamma)P_{t-1}=P_{t-1}+\gamma(\bar{E}_t-P_{t-1})$ where $\bar{E}_t$ is the long run "normal" value as perceived at period t. Many hypotheses as to how $\bar{E}_t$ is formed are possible. Three are explored here:

(a) $\bar{E}_t$ is formed adaptively with an adaptive expectation coefficient $0<\theta\leq 1$.

(b) $\bar{E}_t$ is a general linear combination with non-negative weights of past rates of increase in the C.P.I.

(c) $\bar{E}_t=$constant for every t (will be dealt with as a particular case of the generalized process).

In case (a) the M.S.S.D. is obtained by running the regression $q_t-P_{t-1}=\gamma\{\bar{E}_t(\theta)-P_{t-1}\}$ for $\theta=0.1,\ldots, 1.0$ where $\bar{E}_t(\theta)$ are regular adaptive expectations. For case (b) $\bar{E}_t=\sum_{i=1}^{m} a_i P_{t-i}$, and the computation of M.S.S.D. is the same as for variant (b) of extrapolative expectations.

3.4. *A Synthesized regressive extrapolative process* (Generalized linear process): Such processes have been used in the context of expectations by DeLeeuw and Modigliani and Sutch and suggested in a more generalized context by Solow.[12] They usually impose no restrictions on the weights of the lag structure.[13] The basic idea is that final expectations synthesize both extrapolative and regressive elements. Formally the combined extrapolative-regressive process will be:

[10] Extrapolative expectations have been proposed and investigated by Goodwin **[11]** and Duesenberry **[8]**. Variant (a) has been used by Kesselman **[13]** to describe speculative expectations under the Canadian flexible exchange regime. Variant (b) appears as one of the components of the process proposed by DeLeeuw **[6]**.

[11] The concept goes back to Keynes **[14]** who used it in the context of expectations about bond prices.

[12] See DeLeeuw **[6]**, Modigliani and Sutch **[19]** and **[20]**, and Solow **[31]**.

[13] For some cases a non-negativity constraint is imposed on the weights of the lag structure as for the generalized versions of the regressive expectations process.

$$(2)\qquad E_t = P_{t-1} + \alpha_1\{\overbrace{V_R \sum_{i=1}^{m} u_i P_{t-i} + (1 - V_R)C}^{\bar{E}_t^R} - P_{t-1}\}$$
$$+ \alpha_2\{P_{t-1} - \overbrace{(V_E \sum_{i=1}^{n} w_i P_{t-i} + (1 - V_E)C)}^{\bar{E}_t^E}\}$$

where $1>\alpha_1, \alpha_2 \geq 0$, $u_i, w_i \geq 0$ for $i=1, 2, \ldots \max(m, n)$, and $\bar{E}_t^R$, $\bar{E}_t^E$ are some "normal" long run expected rates of increase of the regressive and extrapolative components respectively. The normal long run notion of the rate of increase in the C. P. I. is composed of two components: One — $\sum_{i=1}^{m} u_i P_{t-i}$ — varies with past history. The second — C — is a constant presumably determined by the general notions of people about the kind of policies the government is able and willing to pursure.[14] The "normal" rate of increase is a weighted average of those two components. The base for extrapolation in the extrapolative component is in general different from the normal long run expected rate of increase from the regressive component and is also a weighted average of C and of the recent price inflation history. Since it is a shorter run phenomena the extrapolation base is believed to be based on a more recent past. Namely m is believed to be larger than n. For estimation purposes (2) may be rewritten in the form (1) where $b_0=[\alpha_1(1-V_R)-\alpha_2(1-V_E)]C$, $b_1=1+\alpha_2-\alpha_1+\alpha_1 V_R u_1-\alpha_2 V_E w_1$, $b_i=\alpha_1 V_R u_i-\alpha_2 w_i V_E$, $i\geq 2$, and the best M. S. S. D. for each length of lag can be obtained from a regression of q_t on $P_{t-1}\cdots P_{t-m}$.

The test criteria for three subcases of this general process may be obtained in a similar way. They are: (1) Only regressive elements are active, $\alpha_1>0$, $\alpha_2=0$. In that case $b_0=\alpha_1(1-V_R)C$, $b_1=1-\alpha_1+\alpha_1 V_R u_1$, $b_i=\alpha_1 V_R u_i$, $i\geq 2$. This implies that $b_i\geq 0$ for $i\geq 2$ since $u_i\geq 0$ for $i\geq 2$. (2) Only extrapolative elements are active, $\alpha_1=0, \alpha_2>0$ which implies $b_0=-\alpha_2(1-V_E)C$, $b_1=1+\alpha_2-\alpha_2 V_E w_1$, $b_i=-\alpha_2 w_i V_E$ for $i\geq 2$ which implies that $b_i\leq 0$ for $i\geq 2$. (3) Extreme regressiveness: $\bar{E}_t^R=C$ and $\alpha_2=0$, $\alpha_1\neq 0$ from which it follows that $E_t=P_{t-1}+\alpha_1(C-P_{t-1})$ for which the minimum M. S. S. D. can be found from a regression of q_t on a constant and P_{t-1}. For all the processes that reduce at the estimation stage to a very long flexible distributed lag, the Almon polynomial distributed lag[15] procedure is used in order to avoid the multicollinearity problem that is raised by such lags. The degree of the polynomial and the length of the lag structure are obtained by experimentation.

3.5. *Rational expectations*: The original concept is due to Muth [**14**]. A more general macroeconomic conception following Muth has been suggested in a series of papers by Lucas, Sargent, Modigliani and Shiller and Barro. (See [**16**], [**28**], [**18**], [**2**] respectively). The basic idea of this approach is that the public knows the systematic part of the economic model which generates inflation and

[14] Modigliani and Sutch [**20**, (185)] call it "a very long run normal level."

[15] See Almon [**1**].

forms its expectations concerning the rate of inflation by using the model and all the information about the exogeneous and endogenous variables it has available at the time the expectation is formed. In this paper I shall restrict myself to cases in which the reduced form of actual inflation can be described as a distributed lag on past rates of inflation. In the recent literature on rational expectations there have been several attempts to find conditions under which different distributed lags, expectational processes, like adaptive and regressive expectations, are rational in the sense of Muth (See Sargent and Wallace [**29**] and Mussa [**22**]). Sargent in particular shows that a generalized linear expectational process whose general form is as in equation (1), will be a minimum-mean squared error (rational) forecast of inflation if the *actual* rate of inflation evolves according to the autoregressive process.

$$P_t = \sum_{i=1}^{n} b_i P_{t-i} + U_t \tag{3}$$

where U_t is an independently (of P_{t-i}), identically distributed random variable with zero mean, constant variance and no serial correlation.[16] Therefore the generalized linear process, or for that matter most of the other processes which are particular cases of it, may be interpreted as rational expectations as well. However, because of these characteristics of the generalized linear process it is impossible to *discriminate* between its interpretation as a rational expectations process or as a synthesized process by using the M.S.S.D. test alone.[17] Such a discrimination may be achieved by using a test proposed and used by Turnovsky [**33**] and Pensando [**26**]. Expectations will be rational if their formation follows the same autoregressive scheme as the subsequent realization of the rate of inflation. Consequently a test for the rationality of a particular scheme is obtained by regressing both the expected and the realized rates of inflation on the same distributed lags of past inflation and by comparing the magnitudes of the corresponding coefficients.

3.6. *Frenkel's combined adaptive-regressive process*: Frenkel [**10**] has recently introduced a process of expectations formation which tries to explain the impact of money supply on nominal interest rates in the short run as well as in the long run. The basic idea is that the rate of change in expected inflation is influenced by two components: the first is the usual adaptive discrepancy between the previous period short term actual and expected rates of inflation; the second is a longer term regressive element. Formally this process may be expressed as

$$E_t - E_{t-1} = \delta(\bar{E}_t^R - P_{t-1}) + \beta(P_{t-1} - E_{t-1}) \tag{4}$$

[16] See Sargent [**27**, (722–723)].

[17] Another version of rational expectations based on a simple fixed velocity quantity theory of money gave very poor results in terms of the M. S. S. D. criterion. Since this may be caused by a failure of rational expectations as well as by the fact that changes in the money supply do not constitute a reasonable set of "economic structure" to describe the rate of inflation, this version is not described in detail.

where it is assumed $1 \geq \delta > \beta > 0$[18] and $\bar{E}_t^R$ is the expected long run rate of inflation. The first term states that if the actual inflation rate was above the expected average rate in the immediate past this will tend to decrease the short run expected rate of inflation. The second term states that a positive difference between the previous period actual and expected rates of inflation contributes a positive element to the change in the short run expected rate of inflation. Two versions of this process will be tested here: (a) The long run notion of the expected rate of inflation is formed adaptively; $\bar{E}_t^R - \bar{E}_{t-1}^R = \gamma[P_{t-1} - \bar{E}_{t-1}^R]$. By recursive substitution we obtain

$$(5) \qquad E_t = (\beta - \delta + \delta\gamma)P_{t-1} + \sum_{i=2}^{\infty} [\beta(1-\beta)^i + \delta\gamma(1-\gamma)^{i-1} + \sum_{j=1}^{i-1}(1-\beta)^j(1-\gamma)^{i-j-1}]P_{t-i}$$

so that this process may be considered as a generalized linear process of the form 1 with certain restrictions on the coefficients.[19] Its M. S. S. D. will therefore be the same as that of extrapolative expectations (b). (b) $\bar{E}_t^R$ is a general distributed lag of past rates of increase in prices, namely $\bar{E}_t^R = \sum_{i=1}^{\infty} u_i P_{t-i}$, $u_i \geq 0$ for all i.[20] Recursive substitution yields

$$(6) \qquad E_t = \theta_2 P_{t-1} + (\theta_2\theta_1 + u_2)P_{t-2} + (\theta_2\theta_1^2 + \theta_1 u_2 + u_3)P_{t-3} + (\theta_2\theta_1^3 + \theta_1^2 u_2 + \theta_1 u_3 + u_4)P_{t-4} + \cdots$$

where $\theta_1 \equiv 1 - \beta$, $\theta_2 \equiv \beta - \delta + \delta u_1$. This again is of the form 1 and has the same M. S. S. D. as the first variant. Therefore discrimination between the two variants may not be achieved by the M. S. S. D. test alone and we will have to consider the degree of compatibility of the restrictions imposed on the lag structure by each variant, with the estimated lag structure.

Before presentation of the results a few words are in order regarding the effect of the possible presence of a measurement error in the series q_t on the validity of the M. S. S. D. test. It is well known that the presence of errors in the dependent variables does not lead to any bias *per se*.[21] However, the disturbances of the particular sample chosen may introduce an element of randomness into the ranking by the M. S. S. D. criterion, particularly when it ranks two processes very near to each other. To get a better idea about the significance of such differences a statistical test of significance is carried out in these doubtful cases.

[18] This assumption is required if this expectations process is to produce behavior which is consistent with the empirical evidence on the short run effects of changes in the rate of monetary expansion.

[19] For example the coefficient of P_{t-1} may be negative and the coefficients of P_{t-i} for $i \geq 2$ must all be non-negative. Note that those coefficients may first increase and then decrease.

[20] Presumably all u_i become 0 past a certain large enough lag period.

[21] On this point see for example Lahiri [**15**, (127)].

4. RESULTS AND INTERPRETATION

The criterion of goodness of fit of the various processes to expectations derived from the capital market (q_t) as measured by M. S. S. D. is summarized in Table 1. The ranking of the different processes that emerges from the table is: *The synthesized extrapolative regressive processes come first.* In second place come extrapolative expectations (b), and Frenkel's expectations, then extreme regressive and constant expectations (Adaptive with $\theta=0$). Since the average value of q_t is in the neighborhood of 4.8 percent any process with M. S. S. D. above 2.4 percent (which is over 50 percent of the average value of true expectations) is a pretty bad approximation to true expectations. Consequently Table 1 seems to yield enough evidence to believe that the extrapolative (a), regressive (a) and adaptive (for $\theta \geq 1$) are not the right processes. They are by far clearly dominated by the generalized expectational process, extrapolative (b), Frenkel's extreme regressive and constant expectations over time. The *best overall* fit for *all* variants of assumptions 1 and 2 is achieved in the generalized process with a lag of 72 months. The corresponding global minimum value of M. S. S. D. is 1.1 percent, which is rather small in comparison to the mean values of true expectations.[22] It may be claimed that this result is in great part caused by the larger number of explanatory variables and the relatively weak constraints imposed on their coefficients by the Almon lag method which is used in the estimation of the generalized linear process. In order to appraise the extent to which the better fit of this process really reflects the fact that it is a better representation of q_t rather than overfitting, Chow tests of the best process against the extreme regressive process were performed.[23] Such a test gave very significant results suggesting that the probability that the better fit of the generalized process is caused by spurious overfitting is very small. However, when similar tests were replicated for pairwise comparisons of each of the different (and shorter) lengths of lags of the generalized process with the extreme regressive process, the same qualitative result managed to survive for all lags larger than 24 months. This suggests that a distributed lag on past inflation is significant but that the 72 months' lag picked by the M. S. S. D. criterion

[22] It is clear from Table 1 that except for some minor changes in ranking among the worst processes, this ranking remains virtually the same for the two additional variants of Assumption 2 which assume that over the period of the sample expectations overestimate and underestimate actual inflation by 20 percent respectively. (The true expectations indices derived from those two alternative assumptions are denoted by q_u and q_L respectively). Some sensitivity analysis with Assumption 1 was done as well, again without affecting the ranking appreciably and in any case leaving the identity and the internal ranking of the best processes as in Table 1.

[23] The extreme regressive process was chosen as the benchmark since it has the minimal M. S. S. D. among the processes that do not use a long tailed Almon distributed lag. Therefore if the distributed lag process reduces significantly the unexplained sum of squares of this process, a similar or stronger result may be expected when it is compared with the other even worse processes which have larger unexplained errors and about the same number of parameters as the extreme regressive process.

TABLE 1

GOODNESS OF FIT OF EXPECTATIONAL PROCESSES[24]

Expectational Process	M.S.S.D. q	q_u	q_L
(1) Adaptive Expectations			
$E_t=\theta P_{t-1}+(1-\theta)E_{t-1}$			
$\theta=0.0$	0.020	0.020	0.020
$\theta=0.1$	0.038	0.035	0.042
$\theta=0.2$	0.056	0.055	0.059
$\theta=0.3$	0.075	0.074	0.077
$\theta=0.4$	0.093	0.092	0.095
$\theta=0.5$	0.112	0.111	0.114
$\theta=0.6$	0.132	0.131	0.133
$\theta=0.7$	0.152	0.151	0.153
$\theta=0.8$	0.174	0.173	0.175
$\theta=0.9$	0.198	0.197	0.198
$\theta=1.0$	0.224	0.223	0.225
(2) Extrapolative expectations			
(a) $E_t=P_{t-1}+\alpha_1(P_{t-1}-P_{t-2})$	0.137	0.137	0.138
(b) $E_t=P_{t-1}+\alpha_2(P_{t-1}-\sum_{i=2}^{n} w_i P_{t-i})$			
12 months lag	0.038	0.026	0.022
24 months lag	0.017	0.018	0.017
30 months lag	0.016	0.017	0.016
36 months lag	0.017	0.017	0.016
48 months lag	0.016	0.016	0.016
60 months lag	0.016	0.016	0.016
(3) Regressive expectations			
$E_t=P_{t-1}+\gamma(\bar{E}_{t-1}-P_{t-1})$			
(a) $\bar{E}_t=\theta P_{t-1}+(1-\theta)E_{t-1}$			
$\theta=0.1$	0.034	0.031	0.039
$\theta=0.2$	0.048	0.046	0.051
$\theta=0.3$	0.062	0.061	0.065
$\theta=0.4$	0.076	0.074	0.077
$\theta=0.5$	0.088	0.087	0.090
$\theta=0.6$	0.100	0.098	0.101
$\theta=0.7$	0.111	0.110	0.112
$\theta=0.8$	0.122	0.121	0.124
$\theta=0.9$	0.143	0.141	0.144
$\theta=1.0$	0.224	0.223	0.225
(b) $\bar{E}_t=\sum_{i=1}^{m} a_i P_{t-i}$ Same as for extrapolative b			
(c) Extreme regressive	0.020	0.020	0.020
(4) Synthesized extrapolative + regressive process (generalized linear process)			
$E_t=P_{t-1}+\alpha_1(\bar{E}_t^R-P_{t-1})+\alpha_2(P_{t-1}-\bar{E}_t^E)$			
Length of lag in months			
12 months lag	0.019	0.019	0.019
24 months lag	0.017	0.017	0.017
36 months lag	0.017	0.017	0.016
48 months lag	0.015	0.016	0.015
60 months lag	0.015	0.015	0.015
72 months lag	0.011	0.011	0.011
(5) Rational Expectations See Synthesized Process			
(6) Frenkel's Process			
$E_t-E_{t-1}=\delta(\bar{E}_t^R-P_{t-1})+\beta\,(P_{t-1}-E_{t-1})$			
a. $\bar{E}_t^R-\bar{E}_{t-1}^R=\gamma[P_{t-1}-\bar{E}_{t-1}^R]$			
b. $\bar{E}_t^R=\sum_{i=1}^{\infty} u_i P_{t-i}$			
For both variants see extrapolative expectations b			

[24] *Remarks to Table* 1: (1) The sample period is always from January 1958 to July 1961, then from January, 1964 to December, 1967 excluding the 3 months during and around the 1967 war (May, June, July, 1967). (2) The cases of $\theta=1.1, 1.2, \ldots, 1.5$ for adaptive expectations and regressive expectations with adaptive formation of the normal range were examined, too, but since they had even worse M. S. S. D. than those with $\theta\leq 1$, they are not presented in the table. (3) For the extrapolative regressive and synthesized processes in which polynomial distributed

(*Continued on next page*)

alone may be too long. This view is reenforced by several other considerations regarding the lag structure of the 72 months' lag process and additional tests. First the sum of the lag coefficients is significantly larger than 1 ($\Sigma b_i = 1.88$ with a standard error of 0.16) which is difficult to reconcile either with steady state theoretical restrictions ($\Sigma b_i = 1$) or with a rational expectations interpretation of such a process ($\Sigma b_i < 1$).[25] Secondly, it is difficult to explain the appearance of the negative weights at the end of the lag structure. At earlier lag periods, synthesized expectations admit of negative weights and the first weight of Frenkel's expectations *may* be negative. However, towards the end of the lag structure the weights of Frenkel's process must be non-negative. As far as the synthesized process is concerned, the weights of the extrapolative component are most probably zero, thus leaving only the regressive weights — u_i — which must be non-negative as well. All these undesirable features disappear for shorter lag periods while the general shape of the lag structure remains the same as in the longer lags. Thirdly, since the polynomial approximation does not allow a test of significance between longer lags processes[26] it seems desirable to base some of the tests on ordinary least squares regressions as well. Chow tests comparisons of various lengths of lags from 12 to 30 lag periods against one, two and three lag periods all show a significant contribution of the longer processes to the reduction in the unexplained variance. However, when the test is performed at the margin comparing each distributed lag between 16 and 30 lag periods with a distributed lag with 4 less lag periods it turns out that the addition of lagged inflation terms contributes significantly to the explained variance of expectations until the 25th lag period but not beyond it.[27] Those combined considerations lead me to believe that the optimal number of lag months is somewhere around 30. Because towards its termination the lag structure is believed to converge to zero, the various Almon lags were also reestimated with a zero restriction on the far end of the lag structure. Since this restriction hardly affected the residual error (M. S. S. D. = 0.017), the finally chosen 30 periods lag structure presented in Figure 1 incorporates such a restriction.[28] There are several things to note about this lag structure: 1) excluding two coefficients at the beginning, all the coefficients are positive; 2) the lag coefficients first increase reaching a maximum between the 8th and the 9th

(*Continued*)
lags were used, the degree of the polynomial is always 4. Some experiments with a third degree polynomial were made, but they did not change the fit nor the lag structure appreciably. (4) In all cases in which a polynomial distributed lag was used the polynomial distributed lag was applied starting from P_{t-1}. M. S. S. D. is measured in basis points rather than percentages. (5) q_u and q_L represent expectations derived from the capital market under the alternative forms of assumption 2. That is $q_u(q_L)$ corresponds to the assumption that expectations overestimated (underestimated) actual inflation by 20% over the sample period.

[25] See Sargent **[27]**.

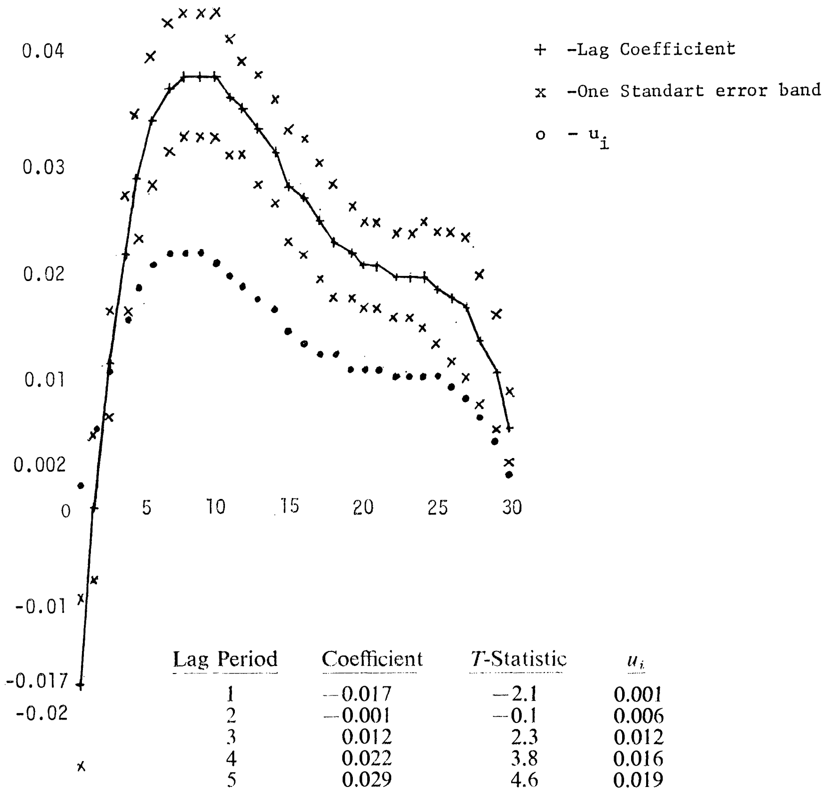
[26] When polynomials of identical degrees are used for all lags, the difference between the degrees of freedom of processes with various lags is identically zero.

[27] The lag periods compared are 16 against 12, 20 against 16, 25 against 20 and 30 against 25.

[28] The constant term is dropped since it is not statistically significant.

FIGURE 1

THE BEST LAG STRUCTURE AND ITS DECOMPOSITION INTO REGRESSIVE WEIGHTS — u_i.



Lag Period	Coefficient	T-Statistic	u_i
1	−0.017	−2.1	0.001
2	−0.001	−0.1	0.006
3	0.012	2.3	0.012
4	0.022	3.8	0.016
5	0.029	4.6	0.019
6	0.034	5.2	0.021
7	0.037	5.8	0.022
8	0.038	6.3	0.022
9	0.038	6.7	0.022
10	0.038	6.9	0.021
11	0.036	6.8	0.020
12	0.035	6.6	0.019
13	0.033	6.1	0.018
14	0.031	5.7	0.017
15	0.028	5.3	0.015
16	0.027	5.0	0.014
17	0.025	4.8	0.013
18	0.023	4.8	0.013
19	0.022	5.0	0.012
20	0.021	5.2	0.012
21	0.021	5.3	0.012
22	0.020	5.1	0.011
23	0.020	4.7	0.011
24	0.020	4.1	0.011
25	0.019	3.5	0.011
26	0.018	3.1	0.010
27	0.017	2.8	0.009
28	0.014	2.6	0.007
29	0.011	2.4	0.005
30	0.006	2.3	0.002

lag month and then decrease steadily to zero; 3) the sum of the lag coefficients is 0.68 and is significantly smaller than 1 (the standard error of the sum is 0.027). The qualitative nature of these results is preserved for lag periods in the vicinity of 30 as well.[29]

Taking the process in Figure 1 as best representing the expectations derived from the capital market there are still several possible interpretations of this process. The possible interpretations are as: extrapolative (b), regressive, synthesized Frenkel's or rational expectations. The predominantly positive structure of the lag coefficients rules out extrapolative (b) so that either version of the extrapolative process is ruled out. (This result contrasts with Turnovsky' s [**33**] who found, using the Livingston survey data, that the extrapolative hypothesis turns out to be best.) The best process would have been consistent with a regressive only interpretation if the two first coefficients of the lag structure were not negative.[30] Hence this interpretation is ruled out as well. It would seem that the synthesized process whose final weight structure is a difference between two positive weight structures may accommodate the negative weights at the beginning of the lag structure is Figure 1. However, the coefficients estimated in Figure 1 do not allow such an interpretation; since C is most probably nonzero[31] the constant of the regression in Figure 1 will be zero if $V_R = V_E = 1$ from which it follows

$$b_1 = -.017 = 1 + \alpha_2(1 - w_1) + \alpha_1(u_1 - 1) \tag{7}$$

α_1, w_1 and u_1 must all be positive and smaller than 1. Otherwise the regressive and extrapolative components in the synthesized process (2) lose their meaning as regressive and extrapolative components respectively. Hence the right hand side of (7) cannot be equal to a negative number as is required by the weight b_1[32] which is significantly different from zero. Hence the interpretation of Figure 1 in terms of a synthesized process is ruled out, too. Now consider the combined adaptive-regressive process proposed by Frenkel. It turns out that Version (a), in which $\bar{E}_t^R$ is formed adaptively, is not consistent with the estimated coefficients in Figure 1. This can be seen as follows: b_2 turned out negative but insigificantly different from 0. If we take it to be zero we obtain from (5) $0 = b_2 = \beta (1-\beta)^2 + \delta\gamma(1-\gamma) + 1 - \beta$. This equality cannot be fulfilled together with the restriction that $1 \geq \delta > \beta > 0$ which is required both if the Frenkel model is to yield its main result,[33] and to accommodate a negative b_1 in Figure 1. Since the ruling out of both the synthesized process and version (a) of Frenkel's process leans

[29] For all lag structures with and without a zero restriction the first coefficient is negative, the lag structure has a reverse U shape and the sum of the lag coefficients for lags which are shorter than 40 is significantly smaller than 1.

[30] The first lag coefficient is significantly different from zero.

[31] Since Israel has lived with a rather high degree of inflation since its creation it is difficult to believe that $C=0$.

[32] A similar problem arises for lengths of lag not too far above and below 30.

[33] See Frenkel [**10**, (407)].

heavily on the negative value of b_1, it is desirable to check to what extent this negative value may be spuriously caused by restricting the lag structure to lie on a fourth degree polynomial. Consequently the various lags were reestimated with P_{t-1} as a separate variable. The coefficient b_1 managed to remain negative and significantly different from zero for all lags.

This leaves only version (b) of Frenkel's process and rational expectations as possible interpretations of the results in Figure 1. As is suggested by Mussa [**22**] these two interpretations are not substitutes but rather complements.

It is obviously possible to find values of β, δ and u_i, $i=1\cdots30$ which will be consistent with the estimated lag structure of Figure 1 and which fulfill the restrictions $1>\delta>\beta>0$ and $u_i\geq0$, $i=1\cdots30$. For example $\delta=0.588$, $\beta=0.570$ and u_i as in Figure 1 are consistent with the estimation results.[34]

What of the interpretation of the process in Figure 1 as rational expectations? A Chow test of the type used by Pesando [**26**] and Turnovsky [**33**] indicates for various lags that there is no significant difference between the lag coefficients of the actual[35] and expected rates of inflation, suggesting that the process in Figure 1 is consistent with a rational expectation.[36]

It should be pointed out that these tests are weak tests of the "rational" interpretation of generalized linear processes in the sense that their fulfillment is a necessary but not a sufficient condition for this interpretation to be true. With this reservation in mind it may be concluded that the generalized linear expectational process is not inconsistent with the concept of rational expectations.

5. CONCLUDING COMMENTS

In this paper a direct measure of inflationary expectations was derived from the linked bond Israeli capital market and used to test which of the various expectational processes which are postulated in the economic and econometric literature best describes the formation of expectations. The empirical results suggest that the best process is a modified version of Frenkel's [**10**] process which is similar to the result obtained by Lahiri [**15**]. Except for a few weights at the beginning of the lag structure, this process is dominated by regressive elements. Its lag structure

[34] Those values may be obtained by solving the system of 30 equations $\beta-\delta+\delta u_1=b_1$ and $u_i=b_i-b_{i-1}(1-\beta)$, $i=2\ldots30$ for the 32 unknowns; u_i, $i=1\ldots30$, δ and β. Obviously two of those unknowns may be picked arbitrarily subject to the restriction in the text.

[35] The actual rate of inflation is measured as the subsequent realization of the rate of inflation over the 18th month starting from the moment the inflationary expectation is formed.

[36] Those tests were done with ordinary least square regressions for both the actual and the expected rates of inflation rather than with polynomial distributed lags. The reason is that the polynomial approximation introduces an additional element of error into the residuals which may seriously bias the Chow test statistic. However, estimation by ordinary least squares shortens, for technical reasons, the number of lags that may be estimated. The test is therefore done only for 12, 16 and 20 lag periods. The corresponding values of the Chow test statistic are 1.0, 1.4 and 1.2 respectively which are all insignificant. It seems to me that this result may be extrapolated to longer lag periods as well.

is longer than those experimented with, by both Turnovsky [33] and Lahiri [15],[37] in their versions of a general linear process. This may be due in part to the fact that the time span of the expectations which are explained here is larger (18 months versus 6 and 12 months) making it more sensible to use information on a more distant past in the formation of those expectations. A test of rationality of the general linear process suggests that it is consistent with rational expectations. Finally it is noteworthy that the empirical evidence of this paper does not support adaptive nor extrapolative expectations in contrast to results obtained by Turnovsky [33] using U. S. survey data.

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APPENDIX

1. *The Derivation of Expectations from the Capital Market.* n_t is measured as the interest rate on bill brokerage transactions which are short term loans (about 18 months on average) extended by surplus units to deficit units, through the banking system. This rate is the closest approximation to the free market rate in the period under investigation. From observations made in Section 2 of the text it follows

$$(A1) \qquad 1 + n_t = (1 + r_t)(1 + \delta_t)(1 + q_t) \qquad t = 1, \ldots, T$$

All the elements of the differential $n_t - r_t$ which are not caused by the expected rate of inflation(denoted by δ_t) include 1) a risk premium for the uncertainty of the real return on the nominal instrument; 2) a risk of default premium to compensate for the fact that the risk of default on bill brokerage is larger than on indexed government bonds; 3) n_t is the rate paid by the borrower while r_t is the yield obtained by the lender causing an additional differential; 4) a term structure differential caused by the fact that the average maturity of linked bonds is longer than the average maturity of Bill brokerage. However, evidence from previous work [3] suggests that the real yield curve is rather flat during the period under investigation. Therefore the term structure differential, if it exists at all, is negligible. Of the other two elements the two risk premiums seem to be the most important components of δ_t. (The difference between the lending and the borrowing rates in the bill brokerage market is accounted for by the intermediating bank commission, which is overwhelmingly a payment for the risk of default which the bank takes upon itself). δ_t can therefore be viewed as representing mostly the two risk premiums.

Two assumptions are made in order to make the derivation of q_t possible; 1) $(1+\delta)/(1+r_t) = C_0$ for $t = 1, \ldots, T$. Namely there is a constant relationship between the real risk premiums and the real rate of interest. 2) $\prod_t (1+q_{mt}) = \prod_t (1+P_t)$ where q_{mt} is the expected rate of inflation derived from the capital market,

[37] 18 months (6 quarters) in Turnovsky's work and even less than that in Lahiri's paper.

measured on a monthly basis, and P_t the actual monthly rate of inflation. This assumption claims that whereas expectations may overestimate actual inflation in some periods and underestimate them in others, expectations are fulfilled over long enough periods of time.

The first assumption is mostly a matter of convenience. It was preferred to the assumption that δ_t is constant because the differential between the lending and the borrowing rate on bill brokerage tended to move together with the general level of interest rates. Replication of the tests with the alternative assumption that δ_t/r_t is constant did not change the qualitative nature of the results in the text. Since the test is performed over a finite sample period a certain divergence between actual and expected inflation over all the sample period may still persist. Therefore some sensitivity analysis with assumption 2 was performed as well. The alternative assumptions $\prod_t(1+q_{mt})=(1+a)\prod_t(1+P_t)\,(a>-1)$ were tried for a 20% overestimate and a 20% underestimate of actual inflation. The values of a which correspond to a 20% overestimate (a_u) and a 20% underestimate (a_L) of the overall actual rate of inflation, $\bar{P}$ over the sample period are obtained from the conditions $1+1.2\bar{P}=(1+a_u)(1+\bar{P})$ and $1+0.8\bar{P}=(1+a_L)(1+\bar{P})$ respectively. The corresponding values of expectations are denoted by q_u and q_L respectively. As is evident from Section 3 the choice of the best process is invariant to the values of the parameter a tried.

Equation (A1) together with the two assumptions and the relation $1+q_t=(1+q_{mt})^{12}$ for $t=1,\dots,T$, form a system of $3T+1$ equations with the $3T+1$ unknowns q_t, q_{mt}, δ_t (for $t=1,\dots,T$) and C_0. Several straightforward substitutions yield

$$q_t=\left[(1+a)\prod_t(1+P_t)^{12}/\prod_t\frac{1+n_t}{(1+r_t)^2}\right]^{\frac{1}{T}}\frac{1+n_t}{(1+r_t)^2}$$

Sources of data: n_t is from Menzli [**17**]; r_t is the average yield to maturity in escalated bonds from Ben-Shahar and Cukierman [**3**]; P_t from the Statistical Abstracts of Israel [**32**]. Observations on n_t were available only for the periods January 1958 to July 1961 and January 1964–December, 1967. Consequently, q_t is not available for 1962–1963.

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