1 A simple illustration of the fact that there is an inflation bias even under potential output targeting in the presence of RAP and uncertainty about the state of the economy

1.1 Economic structure

\[ Y = Y_n + \alpha(\pi - \pi^e), \]  
\[ Y_n = Y_p + \epsilon \]  
\[ \pi = m - \gamma\epsilon, \]  

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1.2 CB preferences

\[ L = \frac{A}{2} (Y_p - Y)^2 + \frac{\pi^2}{2} \text{ when } Y_p - Y > 0 \]  \hspace{1cm} (4)
\[ L = \frac{\pi^2}{2} \hspace{1cm} \text{ when } Y_p - Y \leq 0. \]

1.3 The game

Figure 1: The Sequence of Events
1. \( \pi^e \) is formed ——> 2. policy, \( m \), chosen ——> 3. \( \epsilon \) realizes

1.4 Assumption: Direct effect of shock dominates indirect effect

\[ q \equiv 1 - \alpha \gamma > 0 \]  \hspace{1cm} (5)

1.5 Equilibrium

Using equations (1) through (3) in equation (4) the objective function of the central bank may be rewritten as:

\[ L = \frac{A}{2} [\epsilon \gamma + \alpha (m - \pi^e)]^2 + \frac{1}{2} (m - \gamma \epsilon)^2 \hspace{1cm} \text{for } \epsilon < \frac{\alpha}{q} (\pi^e - m) \]  \hspace{1cm} (6)
\[ L = \frac{1}{2} (m - \gamma \epsilon)^2 \hspace{1cm} \text{for } \epsilon \geq \frac{\alpha}{q} (\pi^e - m) \]

1.5.1 Stage 2: decision making at the CB

At this stage the CB takes expectations as given and chooses the planned rate of inflation \( m \) so as to minimize the expected value of its loss function. From equation (6) this expected value is
\[
\frac{A}{2} \int_{-\infty}^{b(\pi^e - m)} \left[q\epsilon + \alpha (m - \pi^e)\right]^2 dG(\epsilon) + \frac{1}{2} E (m - \gamma \epsilon)^2 \tag{7}
\]

where \( E \) is the expected value operator and \( b \equiv \frac{\alpha}{q} \). \( \tag{8} \)

Minimization of equation (7) with respect to \( m \) yields the following behavioral rule for the monetary authority

\[
m = \frac{1}{1 + \alpha^2 AG[b(\pi^e - m)\pi^e - \alpha Aq \int_{-\infty}^{b(\pi^e - m)} \epsilon dG(\epsilon)]}.
\tag{9}
\]

\( G[.] \) is the probability that \( \epsilon \) is smaller than a certain threshold whose value depends on the difference between expected inflation and the level that was planned by the monetary authority.

### 1.5.2 Stage 1: Expectation formation

\[
E\pi \equiv \pi^e = m = -\alpha Aq \int_{-\infty}^{b(\pi^e - m)} \epsilon dG(\epsilon).
\tag{10}
\]

### 1.5.3 Equilibrium

In equilibrium both equations (9) and (10) must be satisfied. It follows that \( \pi^e - m = 0 \) so that equation (10) becomes

\[
E\pi \equiv \pi^e = m = -\alpha Aq \int_{-\infty}^{0} \epsilon dG(\epsilon) = -\alpha Aq G(0) E[\epsilon | \epsilon < 0]. \tag{11}
\]

G[0] is the probability of a recession. More precisely it is the probability that the realization of the employment shock, \( \epsilon \), is lower than the mean of this shock which is zero. \( E[\epsilon | \epsilon < 0] \) is
the expected value of $\epsilon$ conditioned on the economy being in a recession ($\epsilon$ negative). Since the probability of a recession is positive and the expected value of $\epsilon$ conditioned on the economy being in a recession is negative both planned and expected inflation are positive. Furthermore, in spite of its attempt to reduce the size of recessions the CB has no influence on output which remains at its natural level. Had the CB precommitted to a zero rate of monetary expansion output would still be at its natural level. Hence there is an ”inflationary bias” on average.

1.6 Symmetric preferences as a benchmark

$$m = \frac{\alpha^2 A}{1 + \alpha^2 A \pi^e}.$$  

Rationality of expectations implies that $E\pi \equiv \pi^e = m$. In conjunction with the CB reaction function this implies that the average inflationary bias and expected inflation are both zero.

2 Empirical tests and other specifications of RAP

2.1 A cross sectional test

Based on the implication that in the presence of RAP average inflation in a country is positively related to the variance of shocks to the real economy. An empirical test of this implication using data from 22 OECD countries over the 1971-2000 period. Details appears in


Available at: http://www.tau.ac.il/~alexcuk/pdf/ACSG-manc_361-Reprint.pdf
2.2 A time series test for a single country using the Linex

RAP is specified by means of the Linex and applied to quarterly US data between 1970 and 1999. The test is based on a regression of quarterly inflation on last period’s rate of unemployment and on a moving average of the variance of the rate of unemployment. Details appear in

Figure 2: Asymmetric Output Gap Objectives and the Precautionary Demand

For Expansions (Second Parameterization)

Note: $f(x) \equiv \exp(-\gamma x + \gamma x - 1) / (\gamma^2)$. The figure is for $\gamma = 2$. 