

Handout 4: Establishing a Reputation for Dependability by Means of Inflation Targets

Cukierman, *Economics of Governance*, (Feb. 2000)

1 The Model

The **common** objective function of policymakers is given by

$$A(\hat{\pi}_1 - \pi_1^e) - \frac{\hat{\pi}_1^2}{2} + \delta \left(A(\hat{\pi}_2 - \pi_2^e) - \frac{\hat{\pi}_2^2}{2} \right). \quad (1)$$

Here $\hat{\pi}_j$ and π_j^e , $j = 1, 2$ are actual and expected inflation in period j respectively. The relation between actual and planned inflation is given by

$$\hat{\pi}_i = \pi_i + \varepsilon_i, \quad i = D, W \quad (2)$$

where ε_i possesses a uniform distribution with support in the range $(-a_i, a_i)$, $i = D, W$, and $a_W > a_D > 0$. The first inequality reflects the presumption that, W is less dependable than D also in the sense that he institutes procedures that lead to a relatively poorer controls of inflation.

2 Equilibrium in the Second Period

In the second and last period the policymaker in office faces a one period problem that is similar to that of the one period problem with perfect control that appears in Cukierman and Liviatan (1991) and in chapter 16 of Cukierman (1992). Provided his identity has not yet been revealed a W type always announces the same inflation target as his dependable counterpart would have since otherwise he is unmasked already at the beginning of period 2. Such a strategy is individually optimal since full revelation would have curtailed his ability to stimulate employment in the second period. Using the superscript "t" to denote an announced target this implies

$$\pi_{w2}^t = \pi_{d2}^t \quad (3)$$

But since he is not really committed to the target, and since this is the last period, the weak policymaker chooses his instrument, π_{w2} , so as to maximize the expected value (over the distribution of ε_{w2}) of the following expression:

$$A(\pi_{w2} + \varepsilon_{w2} - \pi_2^e) - \frac{(\pi_{w2} + \varepsilon_{w2})^2}{2}. \quad (4)$$

The solution to this problem is:

$$\pi_{w2} = A \quad (5)$$

At the beginning of period 2 the public believes there is a probability β_2 that the policymaker in office is dependable. Consequently expected inflation is given by

$$\pi_2^e = \beta_2 \pi_{d2}^t + (1 - \beta_2)A. \quad (6)$$

I turn now to a characterization of the optimal strategy of a dependable policymaker.

The main difference between him and his weak counterpart is that he chooses the target subject to the dependability constraint

$$\pi_{d2} = \pi_{d2}^t. \quad (7)$$

More precisely, D picks π_{d2} so as to maximize the expected value (over the distribution of ε_{d2}) of the following expression

$$A(\pi_{d2} + \varepsilon_{d2} - \pi_2^e) - \frac{(\pi_{d2} + \varepsilon_{d2})^2}{2} \quad (8)$$

subject to the process of expectation formation in (6) and the dependability constraint in (7).

The solution to this problem is

$$\pi_{d2} = \pi_{d2}^t = (1 - \beta_2)A. \quad (9)$$

The expectation of equilibrium values of second period objectives can be calculated by inserting the appropriate equilibrium strategies into equations (4) and (8). The resulting expressions are:

$$V_{w2}(NS) = A^2\beta_2^2 - \frac{1}{2}(A^2 + \sigma_w^2), \quad V_{w2}(S) = -\frac{1}{2}(A^2 + \sigma_w^2) \quad (10)$$

$$V_{d2}(NS) = -\frac{1}{2}(1 - \beta_2^2)A^2 - \frac{\sigma_d^2}{2}, \quad V_{d2}(S) = -\frac{\sigma_d^2}{2}. \quad (11)$$

2.1 The evolution of reputation and of inflationary expectations

When the realizations of external shocks are such that there is full separation $\beta_2 = 0$ if W is in office and $\beta_2 = 1$ if D is in office. When there is no separation reputation is adjusted according to Bayes' rule which states that¹:

¹A statement of Bayes' theorem can be found in most texts on statistical theory. See for example pages 55-58 of DeGroot (1975).

$$\Pr \left[D \mid \hat{\pi}_1 \right] = \frac{\Pr \left[\hat{\pi}_1 \mid D \right] \Pr [D]}{\Pr \left[\hat{\pi}_1 \mid D \right] \Pr [D] + \Pr \left[\hat{\pi}_1 \mid W \right] \Pr [W]} \quad (12)$$

where $\Pr \left[J \mid \hat{\pi}_1 \right]$, $J = D, W$, is the probability that type J is in office conditional on the realization of first period inflation, $\hat{\pi}_1$, $\Pr \left[\hat{\pi}_1 \mid J \right]$, is the probability that the rate of inflation $\hat{\pi}_1$ has been produced by a policymaker of type J , and $\Pr [J]$ is the initial probability that type J is in office.² Noting that $\Pr [D] = 1 - \Pr [W] = \beta_1$, $\Pr \left[\hat{\pi}_1 \mid D \right] = \frac{1}{2a_d}$, $\Pr \left[\hat{\pi}_1 \mid W \right] = \frac{1}{2a_w}$ and inserting those relations into equation (12) yields:

$$\beta_2 = \frac{\beta_1}{\beta_1 + \frac{a_d}{a_w}(1 - \beta_1)}. \quad (13)$$

Since $a_d < a_w$, reputation in the second period is higher than in the first one. The speed with which reputation goes up, when there is no full separation, is inversely related to the ratio $\frac{a_d}{a_w}$.

Inserting the equilibrium value of π_{d2} into equation (6) inflationary expectations in period 2 can be expressed as

$$\pi_2^e = (1 - \beta_2^2)A. \quad (14)$$

3 Equilibrium in the First Period

Equilibrium choices in the first period take into consideration the effects of those choices both on the values of objectives in the first period as well as on the probability of full separation at the beginning of the second period and through this probability on the value of second period objectives. The weak policymaker mimics the dependable one in the announcement of targets

²The more statistically inclined reader should replace the term "probability" everywhere in this sentence by the term: "probability density".

in the first period as well and for the same reason. But he picks his first period instrument, π_{w1} , so as to maximize the expected value (over the distributions of ε_{w1} and of ε_{w2}) of the following expression:

$$V_w(\cdot) \equiv A(\pi_{w1} + \varepsilon_{w1} - \pi_1^e) - \frac{(\pi_{w1} + \varepsilon_{w1})^2}{2} + \delta \left[-\frac{1}{2}(A^2 + \sigma_w^2) + \Pr(NS/W)\beta_2^2 A^2 \right] \quad (15)$$

where $\Pr(NS/W)$ is the probability of no separation under a weak policymaker. This probability is given by

$$\Pr(NS/W) = \frac{1}{2a_w} [\pi_{d1} - \pi_{w1} + a_d + a_w]. \quad (16)$$

Maximizing equation (15) with respect to π_{w1} and rearranging the optimal strategy of a weak policymaker in the first period is given by:

$$\pi_{w1} = A - \delta \frac{(\beta_2 A)^2}{2a_w}. \quad (17)$$

Since initial reputation is β_1 inflationary expectations in the first period are a weighted average, with weights β_1 and $(1 - \beta_1)$, of the first period target and of the rate of inflation planned by a W type. More precisely

$$\pi_1^e = \beta_1 \pi_{d1}^t + (1 - \beta_1) \pi_{w1}. \quad (18)$$

I turn now to the decision problem of a dependable policymaker in the first period. Unlike his weak counterpart he is bound by the preannounced target. He therefore weights already at the announcement stage the relative impact of the announcement on expectations and on actual inflation. Inserting the dependability constraint, $\pi_{d1} = \pi_{d1}^t$ and equation (18) into equation (1) D's problem is to pick π_{d1} so as to maximize the expected value (over the distributions of ε_{d1}

and of ε_{d2}) of the following expression

$$V_d(\cdot) \equiv A[\pi_{d1} + \varepsilon_{d1} - (\beta_1\pi_{d1} + (1 - \beta_1)\pi_{w1})] - \frac{(\pi_{d1} + \varepsilon_{d1})^2}{2} - \frac{\delta}{2} [\sigma_d^2 + \Pr(NS/D)(1 - \beta_2^2)A^2]. \quad (19)$$

where

$$\Pr(NS/D) = \frac{1}{2a_d}(\pi_{d1} - \pi_{w1} + a_d + a_w) \quad (20)$$

is the probability of no separation when a dependable policymaker is in office. The solution to this problem is:

$$\pi_{d1} = (1 - \beta_1)A - \frac{\delta}{4a_d}(1 - \beta_2^2)A^2. \quad (21)$$

To sum up we have established that, given the condition in part 1 of the appendix, the first period equilibrium strategies of the two policymaker types, inflationary expectations in the first period and the dynamic evolution of reputation when there is no full separation are given respectively by

$$\begin{aligned} \pi_{w1} &= A - \delta \frac{(\beta_2 A)^2}{2a_w} \\ \pi_{d1} &= (1 - \beta_1)A - \frac{\delta}{4a_d}(1 - \beta_2^2)A^2 \\ \pi_1^e &= \beta_1\pi_{d1}^t + (1 - \beta_1)\pi_{w1} \\ \beta_2 &= \frac{\beta_1}{\beta_1 + \frac{a_d}{a_w}(1 - \beta_1)}. \end{aligned} \quad (22)$$

4 The Effects of Initial Reputation and of Other Parameters on Equilibrium Strategies and on the Evolution of Reputation

Proposition 1 : *In the case of no full separation second period reputation, β_2 , is higher the higher initial reputation, β_1 , and the lower the ratio $\frac{a_d}{a_w}$.*

Proposition 2 : *The higher the precision of inflation control under a dependable policymaker (the lower is a_d) the more conservative are the policy plans of **both** policymaker types in the first period (both π_{d1} and π_{w1} are lower).*

Proposition 3 : *The higher initial reputation, β_1 , the more conservative is the policy plan of the weak policymaker in the first period (π_{w1} is lower).*

5 Determinants of the Probability of Full Separation

Due to imperfect control of inflation full separation, or a shock treatment as this is sometime called in the literature on inflation stabilization, is a random event that may or may not materialize. But policymakers can influence the probability of full separation via the choice of their planned inflation rates in the first period. The further apart the planned rates of inflation of the two types the larger the probability of full separation. From equation (22) the difference between the strategies of the two types is

$$\pi_{w1} - \pi_{d1} = A\beta_1 + \frac{\delta A^2}{2} \left[\frac{1}{2a_d} - \left(\frac{1}{a_w} + \frac{1}{2a_d} \right) \frac{\beta_1^2}{\left(\beta_1 + \frac{a_d}{a_w}(1 - \beta_1) \right)^2} \right]. \quad (23)$$

Proposition 4 : (i) *The probability of a shock treatment increases or decreases with the discount factor, δ , depending on whether initial reputation is lower or higher than a threshold, β_{1c} , that is given by*

$$\beta_{1c} \equiv \frac{\frac{a_d}{a_w} \sqrt{\frac{1}{1+2\frac{a_d}{a_w}}}}{1 - \left(1 - \frac{a_d}{a_w}\right) \sqrt{\frac{1}{1+2\frac{a_d}{a_w}}}}. \quad (24)$$

(ii) *When initial reputation is equal to β_{1c} the probability of a shock treatment does not depend on the discount factor.*

Proposition 5 : *An increase in A raises the probability of a shock treatment if $\beta_1 \leq \beta_{1c}$.*