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# HETEROGENEOUS INFLATIONARY EXPECTATIONS, FISHER'S THEORY OF INTEREST AND THE ALLOCATIVE EFFICIENCY OF THE BOND MARKET *

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This letter considers the consequences of heterogeneous inflationary expectations for Fisher's theory of interest. It is further shown that divergent and erroneous expectations cause welfare losses which increase when either the variance of expectations or of actual inflation increase.

## 1. Introduction

Discussions of bond market equilibrium in an inflationary environment traditionally rely on a version of Fisher's theory of interest in which the inflationary expectation whether correct or not, is the *same* for all the participants in the bond market at a given moment of time. [1] However recent evidence suggests that inflationary expectations are not uniform [2] and that the variance of inflationary expectations across individuals is systematically related to the macroeconomic level of inflation uncertainty. [3] This paper presents an analysis of bond market equilibrium for an inflationary environment with heterogeneous expectations.

In section 2 a bond market equilibrium with heterogeneous expectations and linear demand and supply functions is presented. It is shown that for some suitably defined averages of inflationary expectations and perceived real rates Fisher's theory of interest still holds. In section 3, the welfare losses caused by both divergent and erroneous expectations are quantified. It is shown that the expected value of those welfare losses increases with the variance of the rate of inflation, the variance of inflationary expectations and their covariance. Concluding comments follow.

* A first draft of the paper was written while the author was at New York University.

[1] The assumption of a uniformly held expectation in the context of bond market equilibrium and inflation is almost universal. It is originally due to Fisher (1896). More recent works by Fama (1976), Feldstein (1976), Feldstein and Summers (1978), Friedman (1977, 1978), Gibson (1972), and Sargent (1976) constitute but a small sample of literature in which the uniformity assumption is maintained.

[2] See Carlson (1977), and Juster and Comments (1978).

[3] See Cukierman and Wachtel (1978).

## 2. Loan market equilibrium with heterogeneous expectations

Consider a loan market composed of many borrowers and lenders whose real stock demands and supplies of funds depend on the real rate of interest each of them perceives. The real rate perceived by an individual is equal to the market nominal rate of interest minus the rate of inflation expected by that individual for the period of the loan. Assuming that the loan market clears instantaneously, the equilibrium nominal interest rate is determined by the condition that the excess demand for loans is zero. Formally let $I_B = [a_B - (n - \pi_B)]/b_B$ and $I_L = [-a_L + (n - \pi_L)]/b_L$ be the real stock demand function for funds of borrower B and the real stock supply function of funds of lender L, respectively. [4] Here $I_B$, $I_L$, $n$, $\pi_L$ and $\pi_B$ are the real stock demand for funds by borrower B, the real stock supply of funds by lender L, the nominal interest rate, and the rates of inflation expected by lender L, and borrower B respectively. All other symbols are positive parameters. Equating total demands for and supplies of funds the equilibrium nominal interest rate is $n = \Sigma_s v_s a_s + \Sigma_s v_s \pi_s$ where $s = \text{B, L}$. [5] When expectations are uniform $\pi_s = \pi$ for all $s$ and $n = \Sigma_s v_s a_s + \pi$. That is $R \equiv \Sigma v_s a_s$ is the uniform equilibrium real interest rate [6] and it is equal to the nominal rate minus the uniformly expected rate of inflation as in Fisher's theory. But with heterogeneous expectations,

$$r_i = R + \sum_s v_s \pi_s - \pi_i \equiv R + \bar{\pi} - \pi_i\,, \tag{1}$$

which suggests that individual $i$ will perceive a higher (lower) real interest rate than in the case of uniform expectations, if his expectation is lower (higher) than $\bar{\pi}$. By a suitable weighting of each of the elements of eq. (1) and by use of the expression for equilibrium $n$ it follows that:

*Proposition 1. Fisher's theory of interest will hold exactly in the heterogeneous expectations case for an average of inflationary expectations and an average of perceived real rates across individuals provided those two averages are $\Sigma_s v_s \pi_s$ and $\Sigma_s v_s r_s$, respectively.*

## 3. Some welfare aspects of inflation uncertainty and heterogeneous expectations

I shall take both the actual inflation rate, $\mu$, and the expected rate of inflation to be stochastic variables and investigate the allocative welfare effects of an increase in

[4] It is implicitly assumed without loss of generality, that in the relevant range $I_B$ and $I_L$ are non-negative for all B's and L's.

[5] $v_s \equiv (1/b_s)/[\Sigma_s(1/b_s)]$ and it measures the sensitivity of the demand for or supply of funds of trader $s$ with respect to the real interest rate relatively to the sum of that same parameter over all market participants.

[6] In particular it is the real rate under price stability.

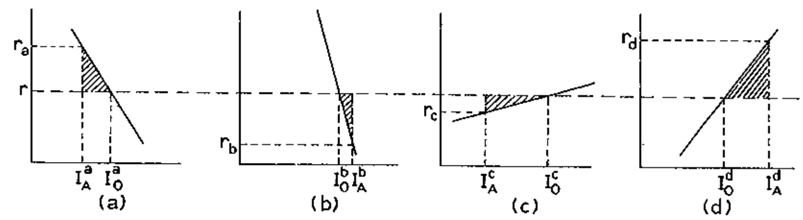


Fig. 1

the spread of either of those distributions. It has been shown elsewhere that the variances of those distributions can be changed, at least within some range, by changing the variance of monetary policy. [7] Here I will take the variance of both the rate of inflation and of inflationary expectations to be the exogenous policy variables and will not bother to model their links to monetary policy explicitly. [8] I further assume that although inflationary expectations may differ across individuals they are all drawn from the same distribution. [9]

Taking the demand for funds to represent the marginal efficiency of capital and the supply of funds the alternative portfolio investment possibilities of accumulated savings, it is intuitively clear that any differences among loan market participants in perceived real rates will cause distortions in the allocation of funds. [10] Those welfare losses are illustrated graphically in fig. 1 which represents the real stock demand and supply curves of loans of two loan demanders (firms a and b) and two loan suppliers (individuals c and d).

Every point on a given supply curve shows what is the real rate of return at which the lender is willing to transfer a given amount of funds from other assets to loan assets. Hence, the area between the rate of return that a lender gets and his supply curve, up to the amount of loans he gives measures his surplus. Similarly, the area between the demand curve for funds and the real interest rate on the market up to his amount of loans, measures the producer's surplus of a typical loan demander. Had traders known in advance that $r \equiv n - \mu$ would be the real rate of interest, they would have chosen the quantities labelled by $I_0^i$, $i$ = a, b, c, d in fig. 1. But firm a borrows and invests only $I_A^a$. It therefore foregoes a producer's surplus measured by the shaded area between $I_0^a$ and $I_A^a$. [11] Since this loss depends on the

[7] See Lucas (1973), Barro (1976), and Cukierman and Wachtel (1977).

[8] For a preliminary version of an investigation of a capital market in which those links are modeled explicitly, see Barro (1978).

[9] This is in the implication of all the models mentioned in the previous footnote.

[10] This is already recognized by Leijonhufvud (1977) as far as investments are concerned.

[11] The usual caveats concerning the use of consumer surplus seem to be less important in the present context since the shaded areas in fig. 1 may be interpreted as losses in physical outputs or returns. This will be strictly so if the supply curves of funds reflect only alternative real portfolio investment opportunities and no subjective time preference. However, once risk aversion is reckoned with, a subjective element is reintroduced into the analysis. The analysis in the text also assumes that investment opportunities are not transferable.

actual as well as the expected rate of inflation both of which are random, it is obviously a stochastic variable too. [12] It can be shown that the expected value of total welfare losses is given by [13]

$$EL = \tfrac{1}{2} \sum_i \frac{E(\pi_i - \mu)^2}{b_i}$$

$$= \tfrac{1}{2} \sum_i \frac{1}{b_i} [E(\pi_i - \pi)^2 + E(\mu - E\mu)^2 + (E\pi_i - E\mu)^2 + 2E(\mu - E\mu)(\pi_i - E\pi_i)] , \qquad (2)$$

where $i$ = B, L, which suggests that the higher is the expected value of the square deviation of the expected rate of inflation from the actual rate of inflation the higher will be the expected value of welfare losses. The extreme right-hand side of (2) suggests that an increase in either the variance of inflationary expectations or the variance of the rate of inflation increases the expected value of welfare losses. Furthermore, those losses increase with any *ceteris paribus* increase in the bias in expectations $(E\pi_i - E\mu)$, or in the covariance between $\mu$ and $\pi_i$. [14]

## 4. Concluding comments

The framework developed here gives further support to the view that on welfare grounds the variances of both the anticipated and the actual rates of inflation should be made as small as possible. [15] While the particular functional form for the welfare losses derived here depends on the assumed linearity of the model, [16] I believe it conveys a germ of truth that applies to other functional forms as well. The framework presented here has additional implications for unexpected redistribution, for the volume of trade in existing obligations and for empirical tests of Fisher's hypothesis. [17]

## References

Barro, R.J., 1976, Rational expectations and the role of monetary policy, Journal of Monetary Economics 2, no. 1, Jan., 1–32.

[12] Similar considerations apply to the other traders. Note that each trader acts on the basis of a different real rate since $r_a \neq r_b \neq r_c \neq r_d$.

[13] Proof available upon request.

[14] In existing multiple expectations models like Barro (1976) or Cukierman and Wachtel (1978), the bias in expectations is 0 and the covariance between $\mu$ and $\pi_i$ is positive.

[15] This is in the spirit of conclusions reached by Barro (1976) and Cukierman (1978).

[16] However it would apply approximately for non-linear functions as well, provided the distributions of $\mu$ and $\pi_i$ are not too spread out.

[17] Like those of Fama (1975, 1976) for example.

Barro, R.J., 1978, A capital market in an equilibrium business cycle model, Unpublished.
Carlson, J.A., 1977, A study of price forecasts, Annals of Economic and Social Measurement 6, no. 1.
Cukierman, A. and P. Wachtel, 1978, Differential inflationary expectations and the variability of the rate of inflation: Some theory and evidence, Working paper no. 77–22 (Center for Applied Economics, New York University); forthcoming in American Economic Review.
Cukierman, A., 1978, Variable information structures and rational expectations or rational expectations and the role of monetary policy: A generalization, Working paper no. 78–03 (Center for Applied Economics, New York University); forthcoming in Journal of Monetary Economics.
Fama, E.C., 1975, Short term interest rates as predictors of inflation, American Economic Review 65, June, 269–282.
Fama, E.C., 1976, Inflation uncertainty and expected returns on treasury bills, Journal of Political Economy 84, June, 427–448.
Feldstein, M., 1976, Inflation, income taxes and the rate of interest: A theoretical analysis, American Economic Review 66, Dec., 809–820.
Feldstein, M. and L. Summers, 1978, Inflation, tax rules, and the long term interest rate, Discussion paper no. 606, March (Harvard Institute of Economic Research, Cambridge, MA).
Fisher, I., 1896, Appreciation and interest, Publications of the American Economic Association, 3rd series II, 331–442.
Friedman, B.M., 1977, Who puts the inflation premium into nominal interest rates?, Paper presented at December 1977 meeting of the American Finance Association.
Friedman, B.M., 1978, Price inflation, portfolio choice, and nominal interest rates, Discussion paper no. 603 (Harvard Institute of Economic Research).
Gibson, W., 1972, Interest rates and inflationary expectations, American Economic Review 62, Dec., 854–865.
Juster, F.T. and R. Comment, 1978, A note on the measurement of price expectations, Unpublished.
Leijonhufvud, A., 1977, Tests and consequences of inflation, in: G. Harcourt, ed., The microeconomic foundations of macroeconomics, Proceedings of an I.E.A. conference (London).
Lucas, R., 1973, Some international evidence on output inflation tradeoffs, American Economic Review 63, June, 326–335.
Sargent, T.J., 1976, Interest rates and expected inflation: A selective summary of recent research, Explorations in Economic Research 3, Summer, 303–325.