

To test for the efficiency of the Treasury bill market, Fama (1975) examined the residuals from a linear regression relating the rate of change in the value of money to a (previous) market forecast of its value. His maintained hypothesis is that if serial correlation of the regression residuals is found, market participants do not use all of the available information in an efficient manner; errors of forecast can be reduced by using the information contained in the persistent deviations of actual values from the forecast values implied by observable, market values.

The simplicity of the test and the intuitive appeal of Fama's interpretation perhaps account for its frequent use and application to other asset markets. Hamburger and Platt (1975) use current values of the forward rates on Treasury bills to forecast future spot rates. They find evidence of positive serial correlation in the residuals from some of their regressions, and they correct for the "inefficiency" using the familiar, first-order, Cochrane-Orcutt procedure.<sup>1/</sup> Frenkel (1977), (1979), (1980) and others used very similar procedures to test for the efficiency of the foreign exchange market. Figlewski and Wachtel (1981) test the rationality of the individual Livingston price expectations by checking whether forecast errors are serially correlated and find those errors to be serially correlated. They conclude that survey respondents did not use all available information.

This paper shows that the tests for market efficiency are valid in finite samples only if the structures generating the residuals are stationary.<sup>2/</sup> In

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<sup>1</sup> Hamburger and Platt (1975) also report evidence of shifting constant term which they (and others) interpret as a variable liquidity premium. Below, we offer an alternative interpretation.

<sup>2</sup> Lucas' (1978) paper also implies that standard tests of market efficiency may fail but for different reasons. Our point is independent of his and applies also to tests of efficiency relating forward and spot prices. Beveridge and Nelson (1981) examined 38 series compiled by the National Bureau. They concluded that 33 of the series are non-stationary.

general, evidence of serial correlation of residuals does not imply that markets are inefficient. For some stochastic structures and time periods, tests for serial correlation will produce evidence of serial correlation even though the relevant market uses all currently available information. Correcting for serial correlation, using all the information in the residuals, does not eliminate the biases caused by non-stationary parameters.

To illustrate the problem, we first restate the tests of market efficiency used by Fama and Frenkel. Next, we introduce a non-stationary stochastic structure that separates permanent from transitory stochastic changes and show that serially correlated disturbances do not necessarily contain information that can be used to improve forecasts.<sup>3/</sup> Then, we use a simple model to demonstrate that attempts to "correct" for serial correlation do not eliminate the bias in estimates of structural parameters. A conclusion completes the paper and comments on the problems of testing for market efficiency and of correcting for serial correlation.

### I. Tests of Asset Market Efficiency

Fama (1975) proposed the following test of market efficiency. Let  $R_t$  be the nominal return earned in month  $t$  on a Treasury bill with one month to maturity at  $t-1$ . Let  $\Delta_t$  and  $r_t$  be, respectively, the random (at  $t-1$ ) rate of change in the purchasing power of money and the real return on a one month bill during month  $t$ . Ignoring second-order terms, we can write<sup>4/</sup>

$$(1) \quad r_t = R_t + \Delta_t \quad .$$

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<sup>3</sup> This is the stochastic structure proposed by Muth (1960) to justify Friedman's (1957) measure of permanent income. Brunner, Cukierman and Meltzer (1980) use this structure to explain stagflation.

<sup>4</sup> See Fama (1975) section I and equation (5). We have simplified the notation.

$R_t$  is known at the end of  $t-1$ , so the relationship between the expected real rate and the expected rate of change in the purchasing power of money, given the information available at the end of  $t-1$ , is (from (1))

$$(2) \quad E r_t = R_t + E \Delta_t$$

where  $E$  is the expected value conditioned on the information available at the end of period  $t-1$ . Following Fama we assume that the expected real rate is constant <sup>5/</sup> ( $E r_t = E r$  for all  $t$ ) and rewrite (2) as

$$(3) \quad \Delta_t = E r - R_t + \epsilon_t .$$

where  $\epsilon_t \equiv \Delta_t - E \Delta_t$  is the error of forecast in the rate of change of the purchasing power of money. The joint hypothesis of market efficiency and constancy of the expected real rate is tested by running the regression,

$$(3a) \quad \Delta_t = \alpha_0 + \alpha_1 R_t + \epsilon_t .$$

The null hypothesis is  $\alpha_1 = -1$  and no serial correlation in  $\epsilon_t$ .

Detection of serial correlation in  $\epsilon_t$  is taken to imply that the market is inefficient. <sup>6/</sup>

There is very little reason to assume that the rate of change of the purchasing power of money (or the rate of price change) is drawn from a distribution with constant mean value. <sup>7/</sup> Suppose the mean of the distribution

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<sup>5</sup> Fama later allowed for a fluctuating real rate. Our criticism applies to that case as well.

<sup>6</sup> In Fama's words (1975, p. 273) : "Nonzero autocorrelations imply that the market is inefficient; one can improve on the market's assessment of the expected value of  $\Delta_t$  ..."

<sup>7</sup> As a matter of fact the study by Beveridge and Nelson (1981) suggests that most aggregate economic time series are non-stationary.

of  $\Delta$  is not stationary but changes at discrete intervals. A stochastic process with this property is <sup>8/</sup>

$$(4) \quad \Delta_t = \Delta_t^p + \Delta_t^q \quad \text{where}$$
$$\Delta_t^p - \Delta_{t-1}^p \sim N(0, \sigma_p^2 \Delta)$$
$$\Delta_t^q \sim N(0, \sigma_q^2 \Delta)$$
$$\Delta_t^p - \Delta_{t-1}^p \quad \text{and} \quad \Delta_t^q \quad \text{are statistically independent}$$

Fama's test of efficiency is, then, a joint test of constant  $Er_t$ , and  $\Delta_t^p$  and of market efficiency. If there are unanticipated, permanent changes in the purchasing power of money,  $\Delta_t^p$  changes. Fama's test of efficiency could reject the market efficiency hypothesis (in a finite sample) when it is true.

Frenkel (1977), (1979) and (1980) studied the efficiency of the foreign exchange market. In his model, using a similar technique,  $s_t$  is the logarithm of the spot rate of exchange in period  $t$  and  $f_{t-1}$  is the logarithm of the one period forward exchange rate prevailing in the previous period. Assuming that  $f_{t-1}$  is an unbiased forecast of  $s_t$  that reflects all the information available in  $t-1$ , the forecast error is serially uncorrelated. Formally,

$$(5) \quad s_t = f_{t-1} + \eta_t$$

where  $\eta_t \equiv s_t - f_{t-1}$ . Frenkel tests the efficiency of the foreign exchange market by running the regression

$$(5a) \quad s_t = a + bf_{t-1} + \eta_t$$

and by testing the hypothesis that  $a = 0$ ,  $b = 1$  and  $\eta_t$  is serially uncorrelated.

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<sup>8</sup> Note that the specification in the text implies that the first difference of  $\Delta^p$  is independent of its past values. This is consistent with the notion that truly new information is random.

Suppose now that (as was the case with the rate of change in the purchasing power of money) the logarithm of the rate of exchange is composed of two stochastic components; one permanent ( $s_t^p$ ) and one transitory ( $s_t^q$ ) and that

$$(6) \quad s_t = s_t^p + s_t^q, \quad s_t^p - s_{t-1}^p \sim N(0, \sigma_{ps}^2), \quad s_t^q \sim N(0, \sigma_{qs}^2)$$

$s_t^p - s_{t-1}^p$  and  $s_t^q$  are statistically independent.

When  $\Delta s_t^p$  changes, for example, when there are unanticipated permanent changes in the terms of trade, Frenkel can reject the hypothesis of market efficiency (in a finite sample) when it is true.

## II. A More General Model and the Possibility of Serial Correlation

In this section, we reformulate the two models presented above in a more general model that collapses to either of the two models in particular cases. Let  $y_t$  be a stochastic variable. The optimal forecast of  $y_t$  made in period  $t-1$ , using the information available during  $t-1$ , is  $Ey_t$ . Let

$$(7) \quad X_{t-1} = c_0 + cEy_t$$

be a market variable observed in period  $t$  that reflects the market forecast, as of  $t-1$ , of  $y_t$ . By definition

$$(8) \quad y_t = Ey_t + (y_t - Ey_t)$$

solving for  $Ey_t$  from (7) and substituting for the first  $Ey_t$  in (8)

$$(8a) \quad y_t = -\frac{c_0}{c} + \frac{1}{c}X_{t-1} + (y_t - Ey_t)$$

Efficiency of this general model can be tested by running the regression

$$(9) \quad y_t = \beta_0 + \beta X_{t-1} + u_t$$

When  $y_t = \Delta_t$ ,  $X_{t-1} = R_t$  and  $c = -1$  (9) reduces to equation (3a) used by Fama to test the efficiency of the Treasury Bill market;  $\beta_0 = c_0$  becomes an estimate of the (assumed) constant expected value of the real rate of interest. When  $y_t = s_t$ ,  $X_{t-1} = f_{t-1}$ ,  $c_0 = 0$  and  $c = 1$  equation (9) reduces to equation (5a) used by Frenkel to test the efficiency of foreign exchange markets.

To complete the generalization of the models of section I, we endow  $y_t$  with the same qualitative stochastic structure that  $\Delta_t$  and  $s_t$  have. We therefore assume that

$$(10) \quad y_t = y_t^p + y_t^q, \Delta y_t^p \equiv y_t^p - y_{t-1}^p \sim N(0, \sigma_p^2), y_t^q \sim N(0, \sigma_q^2),$$

$\Delta y_t^p$  and  $y_t^q$  are statistically independent.

$y_t^p$  and  $y_t^q$  are the permanent and transitory components of  $y_t$  respectively.<sup>9/</sup> If individuals in the market can observe the permanent and the transitory components of  $y_t$  separately, the best forecast of  $y_t$  as of  $t-1$  is  $y_{t-1}^p$ , and the forecast error  $\Delta y_t^p + y_t^q$  is serially uncorrelated. Individuals observe  $y_t$  but can never observe the permanent and the transitory components of  $y_t$  separately. Given the information that is available in  $t-1$  (including all past values of  $y$  up to and including period  $t-1$ ) the best, in the mean

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<sup>9</sup>  $y_t = \Delta_t$ ,  $y_t^p = \Delta_t^p$ ,  $y_t^q = \Delta_t^q$  imply  $\sigma_p^2 = \sigma_{p\Delta}^2$  and  $\sigma_q^2 = \sigma_{q\Delta}^2$ .

An analogous set of definitions holds for the case  $y_t = s_t$ .

square sense, forecast of  $y_t$  is given by

$$(11) \quad E y_t = \theta \sum_{i=0}^{\infty} (1-\theta)^i y_{t-1-i} \quad 0 < \theta < 1$$

where  $\theta$  is a known function of the ratio of variances  $\sigma_p^2/\sigma_q^2$ .<sup>10/</sup> Using (11) we can write the forecast error which appears in equation (8a) as

$$(12) \quad y_t - E y_t = y_t^q - \theta \sum_{i=0}^{\infty} (1-\theta)^i y_{t-1-i}^q + \sum_{i=0}^{\infty} (1-\theta)^i \Delta y_{t-i}^p$$

The forecast error is serially uncorrelated in the population and the forecast of  $y_t$  is efficient. People use all of the information available at time  $t-1$ , but permanent shocks are not recognized instantly. The length of time required to recognize that a permanent shock has occurred depends on  $\theta$  which is to say on the ratio  $\sigma_p^2/\sigma_q^2$ . The larger is  $\sigma_p^2$ , the larger is  $\theta$ . A large  $\theta$ , close to unity, implies that permanent changes are recognized promptly. A small value of  $\sigma_p^2$  (relative to  $\sigma_q^2$ ) reduces  $\theta$  and delays the adjustment of forecast values to permanent changes.

The public cannot identify permanent changes when they occur. They learn gradually, but optimally, according to equation (11), by observing that  $y$  maintains a value that is greater than expected. During the learning period, measured forecast errors remain on the same side of zero. If  $\theta$  is low so that adjustment is slow, we may find evidence of serial correlation in a finite sample that includes observations from a period in which a permanent change occurs even though there is no serial correlation in the population.

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<sup>10</sup> The proof of this statement appears in Muth (1960). For this stochastic structure  $E y_t$  is also the best forecast, as of  $t-1$ , of the permanent component of  $y$  in that period. For further details see Brunner, Cukierman & Meltzer (1980).

To show that ex post errors appear to be serially correlated for a time under the circumstances just described, we compute the covariance between two adjacent forecast errors conditioned on the knowledge (to us but not to individuals forming expectations at the time) that a large permanent shock  $\Delta y_t^p$  affected  $y$  in period  $t$ . Using (12) and the fact that  $\Delta y_t^p$  and  $y_t^q$  are uncorrelated with expected values of zero, we use the formula for the summation of an infinite geometric progression to get <sup>11/</sup>

$$(13) \quad E_{p,q} \left[ u_{t+j+1} u_{t+j} / \Delta y_t^p \right] = (1 - \theta)^{2j+1} ((\Delta y_t^p)^2 - \sigma_p^2) + \frac{1}{\theta(2-\theta)} ((1-\theta)\sigma_p^2 - \theta^2\sigma_q^2), \quad j \geq 0$$

The last term in parenthesis on the right hand side of (13) is 0 since  $\sigma_p^2/\sigma_q^2 = \theta^2/(1-\theta)$ . <sup>12</sup> Hence,

$$(14) \quad E_{p,q} \left[ u_{t+j+1} u_{t+j} | \Delta y_t^p \right] = (1-\theta)^{2j+1} [(\Delta y_t^p)^2 - \sigma_p^2], \quad j \geq 0$$

Equation (14) shows that if the change  $\Delta y_t^p$  is large in comparison to  $\sigma_p^2$ , ex post measures of forecast errors appear to be serially correlated for some time after period  $t$ . Measured serial correlation gradually vanishes as the economy gets further away from period  $t$ , since  $1-\theta < 1$ . If  $\theta$  is small, serial correlation of the ex post errors persists. It follows that, if a

<sup>11</sup> For details see appendix. The symbol  $E_{p,q}$  stands for the expected value over the distributions of both  $y_t^q$  and  $\Delta y_t^p$  for every  $t$ . It should be sharply distinguished from the symbol  $E$  which stands for the expected value conditioned on the information available in a particular period.

<sup>12</sup> See Muth (1960) pp. 302-4 and Cukierman (1982) equation (11) and part B of the appendix.

test for serial correlation is computed soon after a period of large, unanticipated permanent changes, the data will not reject serial correlation of forecast errors even if the market is efficient and forecasters use all relevant information optimally.

In tests of the efficiency of the Treasury bill market, the failure to reject serial correlation can be misleading if applied to samples taken shortly after violent permanent changes in the rate of change in the purchasing power of money. <sup>13/</sup> Similarly, the serial correlation test can yield wrong conclusions about the efficiency of the foreign exchange market if it is applied during or shortly after large permanent changes in the exchange rate.

### III. Correcting for Serial Correlation

Many investigators correct for serial correlation by using the information contained in the estimated error structure. The statistical efficiency of sample estimates increases if this information is used correctly. It is well known that standard procedures used to correct for serial correlation do not require the investigator to trade bias for efficiency. If the sample estimates are unbiased before correction, they are unbiased after as well.

We show in this section that the usual econometric techniques may lead investigators to mistake permanent changes in structure for serial correlation even in the absence of serial correlation. Furthermore the frequently used (in such cases) first order serial correlation correction does not alleviate the biases caused by the change in structure and in some cases may compound them. The moral is that when the structure is non-stationary, mechanical correction for serial correlation does not improve matters and sometimes makes them worse.

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<sup>13</sup> For a recent application of this methodology see Hafer and Hein (1982).

To illustrate the point consider the simple linear model

$$(15) \quad \begin{aligned} (a) \quad Y_t &= \alpha + \beta_1 X_t + \epsilon_t & t = 0, 1, \dots, T_1 \\ (b) \quad Y_t &= \alpha + \beta_2 X_t + \epsilon_t & t = T_1 + 1, \dots, T \end{aligned}$$

in which the slope coefficient is equal to  $\beta_1$  until period  $T_1$  and to  $\beta_2$  thereafter. <sup>14/</sup>  $\epsilon_t$  is an identically distributed, serially uncorrelated disturbance term with mean zero and variance  $\sigma^2$ .  $X_t$  and  $\epsilon_t$  are statistically independent so that in the absence of changes in structure ordinary least squares (OLS) would yield unbiased estimators of all parameters. Figure 1 describes the change in the structure of the model from line M1 to line M2 under the assumption that the variable  $X_t$  grows secularly over time. <sup>15/</sup> Sometime after the change in structure an investigator, still unaware of the change, estimates the relationship between  $Y$  and  $X$  using ordinary least squares over the period 0 to  $T$  and obtains the estimators  $a$  and  $b$  for the intercept and slope coefficients respectively. The broken line labelled  $a + bX$  represents the estimated model. Since  $\alpha$  did not change between the subperiods,  $a$  is an unbiased estimator of  $\alpha$ . However  $b$  is an unbiased estimator of:

$$(16) \quad Eb = W_1\beta_1 + W_2\beta_2, \quad W_1 + W_2 = 1,$$

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<sup>14</sup> For simplicity we consider only a one time change in the slope for a given value of the intercept. When both  $\alpha$  and  $\beta$  are specified as non stationary stochastic processes the qualitative features of the analysis in the text still obtain following large permanent changes in the values of either  $\alpha$ ,  $\beta$  or both.

<sup>15</sup> This is a realistic assumption for many time series.

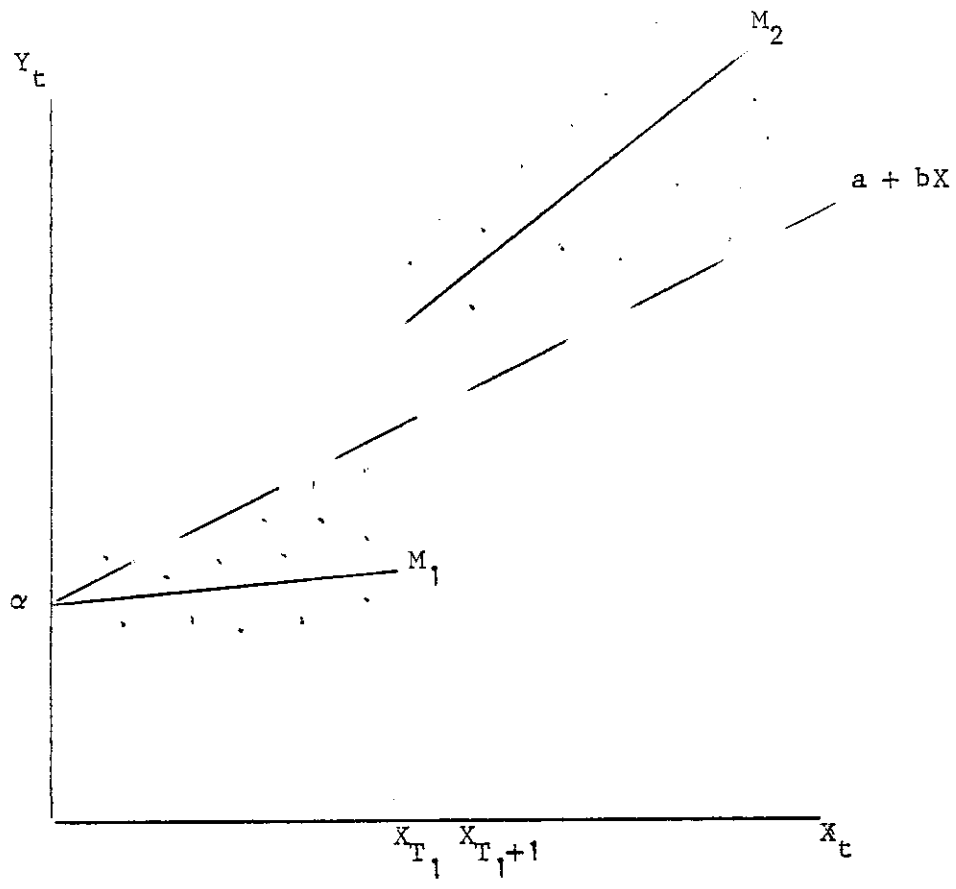


FIGURE 1

where

$$(17) \quad W_1 \equiv \frac{\sum_{t=0}^{T_1} X_t^2 - \bar{X} \sum_{t=0}^{T_1} X_t}{\sum_{t=0}^{T_1} (X_t - \bar{X})^2} ; \quad W_2 \equiv \frac{\sum_{t=T_1+1}^T X_t^2 - \bar{X} \sum_{t=T_1+1}^T X_t}{\sum_{t=0}^T (X_t - \bar{X})^2}$$

and  $\bar{X}$  is the mean of  $X_t$  over the entire sample <sup>16/</sup>. In many cases, as drawn in Figure 1, the slope of the estimated line is at an intermediate value between  $\beta_1$  and  $\beta_2$ . Since the observations before period  $T_1+1$  are generated by model 1, most of them will tend to be below the estimated line. Similarly since the observations from period  $T_1+1$  and on are generated by model 2, they will tend to bunch above the line. Hence the investigator will conclude that there is serial correlation where there is none.

Suppose now that he uses a first order serial correlation correction to correct for the computed serial correlation. More precisely, he picks a value of the first order serial correlation coefficient,  $\rho$ , and estimates the regression

$$(18) \quad Y_t^* = A + B X_t^* \quad t = 1, \dots, T$$

using OLS where

$$(19) \quad Y_t^* \equiv Y_t - \rho Y_{t-1} ; \quad X_t^* \equiv X_t - \rho X_{t-1}, \quad t = 1, \dots, T$$

It is shown in the appendix that the expected value of the resulting slope estimator B for any given value of  $\rho$  is

$$(20) \quad EB = V_1 \beta_1 + V_2 \beta_2, \quad V_1 + V_2 = 1$$

<sup>16</sup> For the derivation of the equivalent of equation (16) for the general linear model see Friedman (1979).

where

$$(21a) \quad V_1 = \frac{\sum_{t=1}^{T_1} (X_t^*)^2 - T_1 \bar{X}_1^* \bar{X}^* - \rho X_{T_1} (X_{T_1+1}^* - \bar{X}^*)}{\sum_{t=1}^T (X_t^*)^2 - T(\bar{X}^*)^2}$$

$$(21b) \quad V_2 = \frac{\sum_{t=T_1+1}^T (X_t^*)^2 - (T-T_1) \bar{X}_2^* \bar{X}^* + \rho X_{T_1} (X_{T_1+1}^* - \bar{X}^*)}{\sum_{t=1}^T (X_t^*)^2 - T(\bar{X}^*)^2}$$

and  $\bar{X}^*$ ,  $\bar{X}_1^*$ ,  $\bar{X}_2^*$  are respectively the means of  $X^*$  over the subperiods 1 to  $T$ , 1 to  $T_1$  and  $(T_1+1)$  to  $T$ .

When the investigator runs his regressions the true slope coefficient is  $\beta_2$ . Obviously, both  $b$  and  $B$  are in general biased estimators of  $\beta_2$  since the expected values of both coefficients are weighted averages of  $\beta_1$  and  $\beta_2$ . The serial correlation correction does not annihilate the bias caused by the change in structure, and worsens it when  $W_2 < V_2$ .

This example shows that when permanent changes occur in the structure of the economy they may be mistakenly identified as serial correlation of the residuals. Correction for this pseudo-serial correlation can aggravate the bias problems caused by the change in structure and produce less accurate estimates. On the positive side, the example suggests that when there is evidence of serial correlation in a sample, the econometrician should test for non-stationarity of the coefficients before correcting for serial correlation.

As far as tests of market efficiency are concerned, the example of this section suggests again that, in a non-stationary world, evidence of serial correlation in a sample does not necessarily imply that markets are inefficient.

#### Conclusion

Optimal, but slow, learning may invalidate serial correlation as a test of market efficiency in finite samples. Measured forecast errors will show evidence of serial correlation following large, unanticipated, permanent changes even if the market is efficient. Mishkin (1981) tests the rationality of the Livingston data on inflationary expectations during the 1959-69 period and finds that rationality is strongly rejected. However when he performs the same test over the longer 1954-76 period rationality is not rejected. Those results are consistent with the framework proposed here. The period 1959-69 is a relatively short period that includes a large permanent change in the rate of inflation. Standard tests of market efficiency may as we saw reject market efficiency in such periods even if markets are efficient. The fact that rationality is not rejected when this period is embedded within a larger time period supports this view. A similar explanation applies to rejection of rationality for expected rate of exchange changes during the German hyperinflation by Frenkel (1977) and by Papadia (1982) for survey based inflationary expectations in Italy and Denmark. The common denominator of all those cases is that there is a strong likelihood that the samples examined were in all those cases dominated by large permanent changes. Similarly the serial correlation detected by Figlewski and Wachtel (1981) in forecast errors of individual respondents to the Livingston survey may be reflecting the non-stationarity of the inflationary process rather than

inefficient use of information. We illustrate this point using a particular stochastic process in which permanent and transitory shocks occur but cannot be distinguished for some time after their occurrence. Similar qualitative results obtain using other stochastic processes to model the permanent-transitory confusion; for example, the permanent component can be a random walk and the transitory component a first-order Markov process. Another example (presented in section III for somewhat different purposes) is the case of a one time permanent change in the slope coefficient of a linear model.

Although the simple tests of market efficiency are, at most, one way tests, market efficiency is not an empty concept. Tests for market efficiency, or for serial correlation, must distinguish ex post from ex ante serial correlation more carefully than is customary. Market efficiency does not imply that there should be no evidence of serial correlation in finite samples when there are permanent changes in the structure of the economy.

Non-stationarity of the economic structure also has implications for the use of serial correlation corrections in econometric practice. Many investigators have computed coefficients that confirm theoretical implications about magnitude and sign but found evidence of serial correlation of residuals. The evidence is interpreted as an indication that the estimates are statistically inefficient. Correction for serial correlation, using well-known techniques, often changes the coefficients and rejects the hypothesis or produces meaningless results. We have shown that when there are large permanent changes in the structure of the economy standard serial correlation corrections do not alleviate the biases that such changes cause in ordinary least square estimators. The conclusion seems to be that serial

correlation corrections should be used only after tests for changes in the structure have failed to show evidence of structural change.

1. Derivation of Equation (13)

Using (12)

$$\begin{aligned}
 E_{p,q} \left[ u_{t+j+1} u_{t+j} / \Delta y_t^p \right] &= E_{p,q} \left[ (y_{t+j+1}^q - \theta \sum_{i=0}^{\infty} (1-\theta)^i y_{t+j-i}^q + \sum_{i=0}^{\infty} (1-\theta)^i \Delta y_{t+j+1}^p) \cdot \right. \\
 &\left. (y_{t+1}^q - \theta \sum_{i=0}^{\infty} (1-\theta)^i y_{t+j-1-i}^q + \sum_{i=0}^{\infty} (1-\theta)^i \Delta y_{t+j}^p) \right] = \\
 &= - \frac{\theta \sigma_q^2}{2-\theta} + (1-\theta) \sigma_p^2 (1+(1-\theta)^2 + \dots + (1-\theta)^{2j} + \dots) + (\Delta y_t^p)^2 (1-\theta)^{2j+1}
 \end{aligned}$$

Summing the infinite geometric progression in the last expression and rearranging we obtain (13).

2. Derivation of Equations (20) and (21)

The L.S. estimate, B, from equation (18) is

$$(22) \quad B = \frac{\sum_{j=1}^T \frac{x_j^* y_j^*}{\sum_{t=1}^T x_t^*}}{\sum_{j=1}^T \frac{TX_j^* - \sum_{t=1}^T X_t^*}{\sum_{t=1}^T X_t^* - (\sum_{t=1}^T X_t^*)^2}} (Y_j - \rho Y_{j-1}) \equiv \sum_{j=1}^T R_j (Y_j - \rho Y_{j-1})$$

where  $x_j^* \equiv X_j^* - \bar{X}^*$ ,  $\bar{X}^* \equiv \sum_{t=1}^T X_t^* / T$ ,  $y_j^* \equiv Y_j^* - \bar{Y}^*$ ,  $\bar{Y}^* \equiv \sum_{t=1}^T Y_t^* / T$  and  $R_j$  is defined by the extreme right hand side equality in (22).

Substituting (15a) and (15b) into (22) and rearranging

$$(23) \quad B = \alpha(1-\rho) \sum_{j=1}^T R_j + \sum_{j=1}^T R_j (\epsilon_j - \rho \epsilon_{j-1}) + \sum_{j=1}^{T_1} R_j X_j^* \beta_1$$

$$+ \sum_{j=T_1+2}^T R_j X_j^* \beta_2 + R_{T_1+1} \left[ (X_{T_1+1} - \rho X_{T_1}) \beta_2 + \rho X_{T_1} (\beta_2 - \beta_1) \right]$$

Noting that  $\sum_{j=1}^T R_j = 0$ , taking the expected value of (23) and rearranging

$$EB = \frac{\left[ T \sum_{j=1}^{T_1} (X_j^*)^2 - \sum_{j=1}^{T_1} X_j^* \sum_{t=1}^T X_t^* \right] \beta_1 + \left[ T \sum_{j=T_1+1}^T (X_j^*)^2 - \sum_{j=T_1+1}^T X_j^* \sum_{t=1}^T X_t^* \right] \beta_2 + \rho X_{T_1} (T X_{T_1+1}^* - \sum_{t=1}^T X_t^*) (\beta_2 - \beta_1)}{T \sum_{t=1}^T (X_t^*)^2 - \left( \sum_{t=1}^T X_t^* \right)^2}$$

Equations (20) and (21) in the text follow by dividing numerator and denominator in this last expression by T and by rearranging.

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