

Central Banks, Credibility and the Financial System-

Exercise 1

RAP and Rogoff's Conservative Central Bank

December 15, 2008

A. Recession avoidance preferences (RAP)

The following two questions refer to Handout 2

1. Derive the reaction function on equation (9).
2. Prove that with symmetric preferences there is no inflation bias.

B. Rogoff's conservative CB

3. The objectives of society and the structure of the economy are given by the following relations.

The loss function of society is:

$$\frac{A}{2}(N^* - N)^2 + \frac{\pi^2}{2}. \quad (1)$$

The short run Phillips curve (PC) is given by:

$$N - N_n = \alpha(\pi - \pi^e) + \varepsilon. \quad (2)$$

The rate of inflation is given by:

$$\pi = m + v - \gamma\varepsilon. \quad (3)$$

All symbols have the same meaning as in class. Society appoints a central bank (CB) that is more conservative than society. That is, the loss function of the CB is:

$$\frac{A}{2}(N^* - N)^2 + (1 + c)\frac{\pi^2}{2}, \quad c > 0 \quad (4)$$

where c is the degree of conservativeness of the CB.

Questions

1. Derive the reaction function of the CB for a given level of inflationary expectations.
2. Show that in a rational expectations equilibrium

$$\pi^e = Em = \frac{\alpha A(N^* - N_n)}{1 + c}. \quad (5)$$

3. Derive expressions for actual inflation and employment in terms of the realizations of the shocks v and ε . (Hint: Substitute the CB reaction function from your answer to question 1 into equations (2) and (3).)

4. Show that the expected level of losses of society when monetary policy is delegated to a central bank with degree of conservativeness, c , is given by:

$$\frac{A}{2} \left\{ \left(1 + \frac{\alpha^2 A}{(1 + c)^2} \right) (N^* - N_n)^2 + \frac{(1 + c)^2 + \alpha^2 A}{(1 + c + \alpha^2 A)^2} \sigma^2 \right\} \equiv \frac{A}{2} \{ K_s s^2 + K_\sigma \sigma^2 \}, \quad s \equiv N^* - N_n \quad (6)$$

where σ^2 is the variance of the supply shock, ε . (Hint: Substitute the expressions from your answer to question 3 into society's loss function in equation (1), take expected values, use the

fact that $Ev = E\varepsilon = 0$, and rearrange.)

5. Explain intuitively why the variance of v does not appear in equation (6).

6. Show that the partial derivative of K_s with respect to c is negative and that the partial derivative of K_σ with respect to c is positive and use those facts to characterize the socially optimal level of conservativeness.

7. Show that the optimal level of conservativeness, c , is larger than zero, but smaller than infinity.

8. Use comparative statics to show that c is an increasing function of N^* and a decreasing function of σ^2 .