

Partial cross ownership and tacit collusion*

David Gilo, Yossi Moshe, and Yossi Spiegel[†]

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Abstract

This paper examines the effects that passive investments in rival firms have on the incentives of firms to engage in tacit collusion. In general, these incentives depend in a complex way on the entire partial cross ownership (PCO) structure in the industry. We establish necessary and sufficient conditions for PCO arrangements to facilitate tacit collusion and also examine how tacit collusion is affected when firms' controllers make direct passive investments in rival firms.

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[†]Gilo: The Buchman Faculty of Law, Tel-Aviv University, email: gilod@post.tau.ac.il. Moshe: Department of Mathematics, Ben-Gurion University of the Negev, email: <moshey@cs.bgu.ac.il>. Spiegel: Recanati Graduate School of Business Administration, Tel Aviv University, email: spiegel@post.tau.ac.il, <http://www.tau.ac.il/~spiegel>

1 Introduction

There are many cases in which firms acquire their rivals' stock as passive investments that give them a share in the rivals' profits but not in the rivals' decision making. For example, Microsoft acquired in August 1997 approximately 7% of the nonvoting stock of Apple, its historic rival in the PC market, and in June 1999 it took a 10% stake in Inprise/Borland Corp. which is one of its main competitors in the software applications market.¹ Gillette, the international and U.S. leader in the wet shaving razor blade market acquired 22.9% of the nonvoting stock and approximately 13.6% of the debt of Wilkinson Sword, one of its largest rivals.² Investments in rivals are often multilateral; examples of industries that feature complex webs of partial cross ownerships are the Japanese and the U.S. automobile industries (Alley, 1997), the global airline industry (Airline Business, 1998), the Dutch Financial Sector (Dietzenbacher, Smid, and Volkerink, 2000), the Nordic power market (Amundsen and Bergman, 2002), and the global steel industry (Gilo and Spiegel, 2003). There are also many cases in which a controller (majority or dominant shareholder) makes a passive investment in rivals. For instance, during the first half of the 90's, National Car Rental's controller, GM, passively held a 25% stake in Avis, National's rival in the car rental industry, while Hertz's controller, Ford, had acquired 100% of the preferred nonvoting stock of Budget Rent a Car (Purohit and Staelin, 1994 and Talley, 1990).³

While horizontal mergers are subject to substantial antitrust scrutiny and are often opposed by antitrust authorities, passive investments in rivals were either granted a de facto exemption from antitrust liability or have gone unchallenged by antitrust agencies in recent cases (Gilo, 2000).⁴ This lenient approach towards passive investments in rivals stems from the

¹See "Microsoft Investments Draw Federal Scrutiny," *Pittsburgh Post-Gazette*, August 10, 1997, B-11, and "Corel Again Buys a "Victim" of Microsoft Juggernaut," *The Ottawa Citizen*, February 8, 2000, C1.

²*United States v. Gillette Co.*, 55 FR 28312 (1990).

³See also "Will Ford Become The New Repo Man?; Financial Powerhouse Takes Aim at Bad Credit Risks," *N.Y Times*, December 15, 1996, Section 3, p. 1. For additional examples of investments by firms and their controllers in rivals, see Gilo (2000).

⁴To the best of our knowledge, Microsoft's investments in the nonvoting stocks of Apple and Inprise/Borland Corp. were not challenged by antitrust agencies while Gillette's 22.9% stake in Wilkinson Sword was approved by the DOJ after the DOJ was assured that this stake would be passive (see *United States v. Gillette Co.* 55 Fed. Reg. at 28,312). The FTC approved TCI's 9% stake in Time Warner (TCI's main rival in the cable TV industry at the time) and even allowed TCI to raise its stake in Time Warner to 14.99% in the future, after being assured that TCI's stake would be completely passive (see *Re Time Warner Inc.*, 61 FR 50301, 1996). The FTC also agreed to a consent decree approving Medtronic Inc.'s almost 10% passive stake in SurVivaLink, one of the only two rivals of Medtronic's subsidiary in the automated External Defibrillators market (see *Re Medtronic*,

courts' interpretation of the exemption for stock acquisitions "solely for investment" included in Section 7 of the Clayton Act.

In this paper we wish to examine whether this lenient approach of courts and antitrust agencies towards passive investments in rivals is justified. Like other horizontal practices (e.g., horizontal mergers), (passive) partial cross ownership (PCO) arrangements raise two main antitrust concerns: concerns about unilateral competitive effects and concerns about coordinated competitive effects. We focus on the latter and consider an infinitely repeated Bertrand oligopoly model in which firms and/or their controllers acquire some of their rivals' (nonvoting) shares. This simple setting allows us to deal with the complexity generated by the chain-effects of multi-lateral PCO. This complexity arises since in general, the profit of each firm, both under collusion as well as under deviation from collusion, depends on the whole set of PCO in the industry and not only on the firm's own stake in rivals. Another advantage of this model is that PCO does not affect the equilibrium in the one shot case and therefore does not have any unilateral competitive effects. This allows us to focus on the effect of PCO on the ability of firms to engage in tacit collusion. We say that PCO arrangements facilitate tacit collusion if they expand the range of discount factors for which tacit collusion can be sustained.

It might be thought that since PCO allows firms to internalize part of the harm they impose on rivals when deviating from a collusive scheme, any increase in the level of PCO in the industry will necessarily facilitate tacit collusion. This intuition, however, ignores the fact that PCO arrangements create an infinite recursion between the profits of firms who hold each other's shares, both under collusion as well as following a deviation from collusion. Consequently, PCO arrangements affect the incentive of each firm to collude in a complex and subtle way.

Despite this complexity, we are able to prove that an increase in the stake of firm r in a rival firm s never hinders collusion. Moreover, we show that such an increase will surely facilitate tacit collusion provided that (i) each firm in the industry holds a stake in at least one rival, (ii) the *maverick firm* in the industry (the firm with the strongest incentive to deviate

Inc., FTC File No. 981-0324, 1998).

from a collusive agreement)⁵ has a direct or an indirect stake in firm r ,⁶ and (iii) firm s is not the industry maverick. If either one of these conditions fail, firm r 's increased stake in firm s has no effect on tacit collusion. In addition, we show that a controlling shareholder (whether a person or a parent corporation) can facilitate tacit collusion further by making a direct passive investment in rival firms. Such investment particularly facilitates collusion if the controller has a relatively small stake in his own firm.

The unilateral competitive effects of PCO have been already studied in the context of static oligopoly models by Reynolds and Snapp (1986), Bolle and Güth (1992), Flath (1991, 1992), Reitman (1994), and Dietzenbacher, Smid, and Volkerink (2000).⁷ Our paper by contrast, focuses on the coordinated competitive effects of PCO and examines a repeated Bertrand model. The distinction between the unilateral and coordinated competitive effects of PCO is important. In particular, PCO arrangements that may be unprofitable in static oligopoly models are shown to be profitable in our model once their coordinated effects are taken into account. For example, in a perfectly competitive capital market, the price of the rival's shares reflects their post-acquisition value. Hence, the investing firm gains only if its own shares increase in value, which, as Flath (1991) shows, is the case only when product market competition involves strategic complements.⁸ By contrast, our results show that once repeated interaction is taken into account, firms may benefit from investing in rivals even if such investments have no effect in one shot interactions. Reitman (1994) shows that symmetric firms may not wish to invest in rivals because such investments benefit noninvesting firms more than they benefit the investing firms. In our model, there is no such free-rider problem since when firms are symmetric, all of them need to

⁵The Horizontal Merger Guidelines of the US Department of Justice and FTC define maverick firms as "firms that have a greater economic incentive to deviate from the terms of coordination than do most of their rivals," see www.usdoj.gov/atr/public/guidelines/horiz_book/hmg1.html. For an excellent discussion of the role that the concept of maverick firms plays in the analysis of coordinated competitive effects, see Baker (2002).

⁶Firm i has an indirect stake in firm r if it either has a stake in a firm that has a stake in firm r , or if it has a stake in a firm that has a stake in a firm that has a stake in firm r , and so on.

⁷See also Bresnahan and Salop (1986) and Kwoka (1992) for a related analysis of static models of horizontal joint ventures. Alley (1997) and Parker and Röller (1997) provide empirical evidence on the effect of PCO on collusion. Alley (1997) finds that failure to account for PCO leads to misleading estimates of the price-cost margins in the Japanese and U.S. automobile industries. Parker and Röller (1997) find that cellular telephone companies in the U.S. tend to collude more in one market if they have a joint venture in another market.

⁸Charl  ty, Fagart, and Souam (2002) study a related model but consider PCO by controllers rather than by firms. They show that although a controller's investments in rivals lower the profit of the controller's firm, they may increase the rival's profit by a larger amount and thereby benefit the controller at the expense of the minority shareholders in his own firm.

invest in rivals to sustain tacit collusion (i.e., each firm is "pivotal").

We are aware of only one other paper, Malueg (1992), that studies the coordinated effects of PCO. His paper differs from ours in at least three important respects. First, Malueg considers a repeated Cournot game and finds that in general, PCO has an ambiguous effect on collusion. The ambiguity arises because in the Cournot model PCO has two conflicting effects. On the one hand, PCO imply that firms internalize part of the losses that they inflict on rivals when they deviate. On the other hand, PCO also soften product market competition following a breakdown of the collusive scheme and hence strengthen the incentives of firms to collude. We believe that in practice, the first effect is likely to dominate the second effect, otherwise firms would have no incentive to invest in rivals. The Bertrand framework that we use allows us to neutralize the negative effect of PCO on collusion and focus attention on the first positive effect. Second, Malueg considers a symmetric duopoly in which the firms hold identical stakes in one another, while we consider an n firm oligopoly in which firms need not have similar stakes in one another. Third, Malueg effectively considers passive investments in rivals by controllers rather than by firms; consequently, his analysis does not feature the complex chain-effect interaction between the profits of rival firms which is a main focus of our paper.

The rest of the paper is organized as follows: Section 2 examines the effect of PCO on the ability of firms to achieve the fully collusive outcome in the context of an infinitely repeated Bertrand model with symmetric firms. Section 3 shows that PCO by firms' controllers may further facilitate collusion. We conclude in Section 4. Technical proofs are in the Appendix.

2 Partial cross ownership (PCO) by firms

In this section we examine the coordinated competitive effects of PCO in the context of the familiar infinitely repeated Bertrand oligopoly model with $n \geq 2$ identical firms. Specifically, we assume that the n firms produce a homogenous product at a constant marginal cost c and that in every period they simultaneously choose prices and the lowest price firm captures the entire market. In case of a tie, the set of lowest price firms get equal shares of the total sales. Let $Q(p)$ be the downward sloping demand function in the industry. Then the monopoly price

is defined by

$$p^m \equiv \arg \max_p Q(p)(p - c).$$

and the monopoly profit is

$$\pi^m \equiv Q(p^m)(p^m - c).$$

As is well-known (e.g., Tirole, 1988, Ch. 6.3.2.1), the fully collusive outcome in which all firms charge p^m and each firm gets an equal share in the monopoly profit, π^m , can be sustained as a subgame perfect equilibrium of the infinitely repeated game provided that the intertemporal discount factor, δ , is such that

$$\delta \geq \widehat{\delta} \equiv 1 - \frac{1}{n}. \tag{1}$$

That is, the fully collusive outcome can be sustained provided that the firms are sufficiently patient (i.e., care sufficiently about their long run profits).

Taking condition (1) as a benchmark, we shall examine the competitive effects of PCO by looking at its effect on the critical discount factor, $\widehat{\delta}$, above which the fully collusive outcome can be sustained. In other words, $\widehat{\delta}$ will be our measure of the ease of collusion.⁹ We will say that PCO arrangements facilitate tacit collusion if they lower $\widehat{\delta}$ and thereby widen the set of discount factors for which the fully collusive scheme can be sustained. Conversely, we will say that PCO hinder tacit collusion if they raise $\widehat{\delta}$.

2.1 The accounting profits under PCO

To examine the impact of PCO on $\widehat{\delta}$, let α_i^j be firm i 's ownership stake in firm j . We assume that the pricing decisions of each firm are effectively made by its controller (i.e., a controlling shareholder) whose ownership stake is β_i . Now, suppose that all controllers adopt the same trigger strategy whereby they set the monopoly price, p^m , in every period unless at least one

⁹Of course, the repeated game admits multiple equilibria. We focus on the fully collusive outcome and on $\widehat{\delta}$ because this is a standard way to measure the notion of "ease of collusion."

firm has charged a different price in any previous period; then all firms set a price equal to c forever after. To write the condition that ensures that this trigger strategy can support the fully collusive scheme as a subgame perfect equilibrium, we first need to express the profit of each firm under collusion and following a deviation from the fully collusive scheme.

If all firms charge the monopoly price, then each firm earns $\frac{\pi^m}{n}$, and on top of that it also gets a share in its rivals' profits due to its ownership stake in these firms. Hence, the vector of collusive profits in the industry, $\pi = (\pi_1, \pi_2, \dots, \pi_n)'$, is given by the solution to the following system of n equations:

$$\pi = k + A\pi, \tag{2}$$

where $k = (\frac{\pi^m}{n}, \dots, \frac{\pi^m}{n})'$ is an $n \times 1$ vector and

$$A = \begin{pmatrix} 0 & \alpha_1^2 & \cdots & \alpha_1^n \\ \alpha_2^1 & 0 & \cdots & \alpha_2^n \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_n^1 & \alpha_n^2 & \cdots & 0 \end{pmatrix},$$

is an $n \times n$ PCO matrix.

On the other hand, if firm i deviates from the fully collusive scheme and slightly undercuts the rivals' prices, then all firms but i make a profit of 0 while firm i 's profit is arbitrarily close to π^m ; to simplify matters, we simply write it as π^m . Consequently, the vector of firms' profits in the period in which firm i 's controller deviates, $\pi^{d_i} = (\pi_1^{d_i}, \pi_2^{d_i}, \dots, \pi_n^{d_i})'$, is defined by the solution to the following system:

$$\pi^{d_i} = k^{d_i} + A\pi^{d_i}, \tag{3}$$

where $k^{d_i} = (0, \dots, 0, \pi^m, 0, \dots, 0)'$ is an $n \times 1$ vector with π^m in the i -th entry and 0's in all other entries. In all subsequent periods following a deviation from the fully collusive scheme, all firms use marginal cost pricing and make 0 profits. It should be noted from (3) that, unlike the usual model without PCO, nondeviant firms can still make positive profits in the period in which the

deviation from the fully collusive scheme occurs provided that they have direct or indirect stakes in the deviant firm.

Systems (2) and (3) reveal that in general, the profit of each firm depends on the profits of *all* other firms and on the structure of PCO in the industry. For instance, firm 1 may get a share α_1^2 of firm 2's profit which may reflect firm 2's share, α_2^5 , in the profit of firm 5, which in turn may reflect firm 5's share, α_5^1 , in the profit of firm 1. The fact that each firm's profit depends on the whole PCO matrix is striking. It implies for instance that a firm's profit and incentive to collude may be affected by a change in PCO levels among rivals even if this change does not affect the firm directly (i.e., even if the firm's PCO levels in rivals or the rivals' PCO in that firm remain unchanged).

Before proceeding, it is worth noting that with PCO, the accounting profits will in general overstate the firms' cash flows. In particular, the aggregate (accounting) profits of all firms will exceed the monopoly profit, π^m .¹⁰ Nonetheless, if we sum up the n equations in system (2) and rearrange terms, we get

$$\left(1 - \sum_{j \neq 1} \alpha_j^1\right) \pi_1 + \left(1 - \sum_{j \neq 2} \alpha_j^2\right) \pi_2 + \cdots + \left(1 - \sum_{j \neq n} \alpha_j^n\right) \pi_n = \pi^m, \quad (4)$$

where $\left(1 - \sum_{j \neq i} \alpha_j^i\right)$ is the aggregate ownership stake held by firm i 's controller and the firm's outside equityholders. Thus, the collusive payoffs of the controllers and outside investors (i.e., equityholders that are not rival firms) do sum up to π^m and are therefore not overstated. A similar computation shows that this is also the case when one of the controllers deviates from the fully collusive scheme.¹¹

¹⁰See Dietzenbacher, Smid, and Volkerink (2000) and Ritzberger and Shorish (2003) for additional discussions of this effect of PCO.

¹¹To illustrate, suppose that there are only 2 firms that hold 25% stakes in each other; the rest of the 75% ownership stakes in firms 1 and 2 are held by controllers 1 and 2, respectively. Assuming further that $\pi^m = 100$, the collusive profits are $\pi_1 = \frac{100}{2} + 0.25\pi_2$ and $\pi_2 = \frac{100}{2} + 0.25\pi_1$. Solving this system, we get $\pi_1 = \pi_2 = 66.66$, implying that the collusive payoff of each controller is $66.66 \times 0.75 = 50$. Consequently, the controllers' payoffs sum up to 100 (the real cash flow) despite the fact that the accounting profits sum up to 133.33. If firm 1's controller, say, deviates, the profits become $\pi_1 = 100 + 0.25\pi_2$ and $\pi_2 = 0 + 0.25\pi_1$, so $\pi_1 = 106.66$ and $\pi_2 = 26.66$. Now, the controllers' payoffs are 80 and 20, respectively. Again, these payoffs sum up to 100 despite the fact that the firms' profits sum up to 133.33. It is worth noting that the fact that firm 1 receives part of its cash flow back from firm 2, implies that the payoff of firm 1's controller equals 80% of the industry profit despite the fact that the controller only holds a 75% stake in firm 1.

To solve systems (2) and (3), note that the PCO matrix, A , is nonnegative and the sum of each of its columns is strictly less than 1.¹² Consequently, systems (2) and (3) are Leontief systems and have unique solutions $\pi(A) \geq 0$ and $\pi^{d_i}(A) \geq 0$ (see Berck and Sydsæter, Ch. 21.1 - 21.22, p. 111) defined by

$$\pi(A) = Bk, \quad \pi^{d_i}(A) = Bk^{d_i}, \quad (5)$$

where $B \equiv (I - A)^{-1}$ is the inverse Leontief matrix. Using $b_{i,j}$ to denote the entry in the i -th row and j -th column of the inverse Leontief matrix B , (5) implies that the accounting collusive profit of firm i is $\pi_i(A) = \frac{\pi^m}{n} \sum_{j=1}^n b_{i,j}$, its one time profit in the period in which it deviates from the fully collusive scheme is $\pi_i^{d_i}(A) = b_{i,i}\pi^m$, and its one time profit when firm j deviates is $\pi_i^{d_j}(A) = b_{i,j}\pi^m$.

Since the inverse Leontief matrix, B , plays an important role in what follows, we establish some important properties of this matrix in the following result.

Lemma 1: *The inverse Leontief matrix B has the following properties:*

- (i) B is invertible, $b_{i,i} \geq 1$ for all i , and $0 \leq b_{i,j} \leq b_{i,i}$ for all $i = 1, \dots, n$ and all $j \neq i$.
- (ii) Let i and j be two distinct firms. Then, $b_{i,j} = 0$ if and only if there is a partition (X, Y) of the set of firms $\{1, 2, \dots, n\}$ (i.e., $X \cap Y = \emptyset$, $X \cup Y = \{1, 2, \dots, n\}$, $X, Y \neq \emptyset$) such that $i \in X$, $j \in Y$ and $\alpha_i^j = 0$ for each $i \in X$, $j \in Y$ (no firm in the subset X has a direct or an indirect stake in firms in the subset Y).¹³

Proof: *See the Appendix.*

Since $b_{i,i} \geq b_{i,j}$ for all i and all $j \neq i$, part (i) of Lemma 1 implies that the profit that each firm i earns when it deviates from the fully collusive scheme, $\pi_i^{d_i}(A) = b_{i,i}\pi^m$, exceeds its collusive profit, $\pi_i(A) = \frac{\pi^m}{n} \sum_{j=1}^n b_{i,j}$, and its profit when firm j deviates, $\pi_i^{d_j}(A) = b_{i,j}\pi^m$.

¹²The sum of column i represents the aggregate ownership stake that rival firms hold in firm i . This sum must be strictly less than 1 because of the ownership stakes held by firm i 's controller and firm i 's outside shareholders.

¹³That is, no firm in X has a stake in a firm that belongs to Y , nor does it have a stake in a firm that has a stake in a firm that belongs to Y , nor does it have a stake in a firm that has a stake in a firm that has a stake in a firm that belongs to Y , and so on.

Moreover, given that $b_{i,i} \geq 1$, it is possible that the accounting profit of the deviant firm exceeds the monopoly profit, π^m . Part (ii) of Lemma 1 implies that the profit that each firm i earns when firm j deviates from the fully collusive scheme, $b_{i,j}\pi^m$, is strictly positive unless firm i has no direct or indirect stake in firm j . In fact, the profit of firm i when firm j deviates from the fully collusive scheme can exceed the collusive profit of firm i : for instance, if there are n firms in the industry and only firm i has a stake α_i^j in firm j , then the collusive profit of firm i is $(1 + \alpha_i^j) \frac{\pi^m}{n}$, whereas its profit when firm j deviates is $\alpha_i^j \pi^m$. Hence, the latter exceeds the former whenever $\alpha_i^j > \frac{1}{n-1}$.

2.2 Collusion with PCO

Given the profits of the n firms under collusion and following a deviation from the fully collusive scheme, the condition that ensures that the fully collusive outcome can be sustained as a subgame perfect equilibrium is

$$\frac{\beta_i \pi_i(A)}{1 - \delta} \geq \beta_i \pi_i^{d_i}(A), \quad i = 1, \dots, n. \quad (6)$$

The left side of (6) is the share of firm i 's controller in the infinite discounted collusive profits of firm i . The right side of (6) is the controller's share in the one time profit that firm i earns in the period in which it undercuts its rivals slightly. If (6) holds, no controller wishes to unilaterally deviate from the fully collusive scheme.¹⁴

Recalling that $\pi_i(A) = \frac{\pi^m}{n} \sum_{j=1}^n b_{i,j}$ and $\pi_i^{d_i}(A) = b_{i,i} \pi^m$, condition (6) gives rise to the following result:

Lemma 2: *With PCO, the fully collusive outcome can be sustained as a subgame perfect equilibrium of the infinitely repeated game provided that*

$$\delta \geq \widehat{\delta}^{po}(A) \equiv \max \left\{ \widehat{\delta}_1(A), \dots, \widehat{\delta}_n(A) \right\}, \quad (7)$$

¹⁴We study here "pure" price fixing: firms fix a price and let consumers randomize their purchases between the n firms. There could be more elaborate collusive schemes in which firms will also divide the market between them, in which case their market shares need not be equal. Such schemes however will require some firms to ration their sales and will therefore be harder for the firms to enforce and easier for antitrust authorities to detect.

where

$$\widehat{\delta}_i(A) \equiv 1 - \frac{\pi_i(A)}{\pi_i^{d_i}(A)} = 1 - \frac{\frac{1}{n} \sum_{j=1}^n b_{i,j}}{b_{i,i}}. \quad (8)$$

The intuition for Lemma 2 is as follows. Although the n firms produce a homogenous product and have the same marginal cost, their incentives to collude are not necessarily identical due to their possibly different levels of ownership stakes in rivals. Lemma 2 shows that whether or not the fully collusive scheme can be sustained depends entirely on the firm with the minimal ratio between the collusive profit, $\pi_i(A)$, and the profit following a deviation, $\pi_i^{d_i}(A)$. In what follows we shall therefore refer to this firm as the *industry maverick*.

Since part (i) of Lemma 1 implies that $b_{i,j} \geq 0$ for all i and all j , it follows immediately from equation (8) that $\widehat{\delta}_i(A) \leq \widehat{\delta} \equiv 1 - \frac{1}{n}$: in the presence of PCO, firms either have the same or stronger incentives to collude than they have absent PCO. The question however is whether, starting from a given PCO structure, an increase in one firm's stake in a rival firm facilitates or hinders collusion. Addressing this question is a formidable task since in general, even a single change in the PCO matrix, A , will affect all entries in the inverse Leontief matrix, B . From an economic standpoint, that means that an increase in, say, firm r 's stake in rival firm s , will affect the profits of all firms in the industry both under the fully collusive outcome and following a deviation from that outcome. That is, the incentives of all firms to collude will be affected. From a purely mathematical standpoint, things are complicated because we are not simply interested in the comparative statics properties of the matrix B . Rather, we wish to know how the lowest ratio between the average value of the entries in row i of B , $\frac{1}{n} \sum_{j=1}^n b_{i,j}$, and the diagonal term in that row, $b_{i,i}$, changes following a change in the PCO matrix A . Nonetheless, in Theorem 1 we are able to show that an increase in firm r 's stake in rival firm s never hinders tacit collusion, and moreover, we establish the precise conditions under which such an increase will surely facilitate tacit collusion. For the purpose of this result, it does not matter whether firm r increases its stake in firm s at the expense of outside shareholders or at the expense of firm s 's controller (as long as the controller retains control).

Theorem 1: *Starting with a PCO matrix A , suppose that firm r increases its stake in firm s*

by some $\Delta > 0$, so that the PCO matrix becomes $A' = A + \Delta e_{r,s}$, where $e_{r,s}$ is an $n \times n$ matrix whose (r, s) -entry is 1 and each other entry is 0. Then, for all $i = 1, \dots, n$,

$$\widehat{\delta}_i(A') \leq \widehat{\delta}_i(A),$$

with equality holding if and only if $b_{i,r} = 0$ or $i = s$. Consequently, $\widehat{\delta}^{po}(A') \leq \widehat{\delta}^{po}(A)$: an increase in firm r 's stake in firm s never hinders tacit collusion.

Proof: See the Appendix.

Theorem 1 may be of independent interest for those interested in the comparative static properties of Leontief systems (these systems play an important role in many areas in economics, e.g., input-output analysis). In our context, Theorem 1 has the important implication that PCO can never hinder tacit collusion. Given this result, one may wonder when PCO will surely facilitate tacit collusion and when it will have no effect on tacit collusion. In the next two corollaries of Theorem 1 we address this question and provide necessary and sufficient conditions for PCO to facilitate tacit collusion.

Corollary 1: *Suppose that at least one firm in the industry does not invest in rivals. Then, $\widehat{\delta}^{po}(A) = \widehat{\delta}$, implying that PCO has no effect on the ability of firms to engage in tacit collusion.*

Proof: Suppose that firm i does not invest in rivals, so $\alpha_i^j = 0$ for each $j \neq i$. Then by Lemma 1, $b_{i,j} = 0$ for all $j \neq i$. Hence, equation (8) implies that $\widehat{\delta}_i(A) = 1 - \frac{1}{n} = \widehat{\delta}$. Since Theorem 1 establishes that PCO never hinder collusion, this means that $\widehat{\delta}^{po}(A) = \widehat{\delta}$. ■

Corollary 1 implies that a necessary condition for PCO to facilitate tacit collusion is that *every* firm in the industry has a stake in at least one rival. In other words, so long as at least one firm does not invest in rivals, all other PCO in the industry have no effect on tacit collusion. From a policy perspective, this implies that in industries with similar firms, antitrust authorities should not be too concerned with unilateral PCO since only multilateral PCO arrangements can facilitate tacit collusion.

While Corollary 1 provides only a necessary condition for PCO to facilitate tacit collusion, the next corollary provides necessary and sufficient conditions for that to be the case.

Corollary 2: $\hat{\delta}^{po}(A') = \hat{\delta}^{po}(A)$ if and only if $\hat{\delta}^{po}(A) = \hat{\delta}_i(A)$ for i with $b_{i,r} = 0$ or $i = s$. That is, an increase in firm r 's stake in firm s will surely facilitate tacit collusion if and only if (i) the maverick, firm i , has a direct or an indirect stake in firm r , and (ii) firm s is not the industry maverick.

Proof: (“If” part) Let i be an industry maverick, i.e., a firm for which $\hat{\delta}^{po}(A) = \hat{\delta}_i(A)$, and assume that $b_{i,r} = 0$ or $i = s$. Then by Theorem 1, $\hat{\delta}_i(A) = \hat{\delta}_i(A')$. Thus,

$$\hat{\delta}^{po}(A) = \hat{\delta}_i(A) = \hat{\delta}_i(A') \leq \hat{\delta}^{po}(A').$$

However, Theorem 1 also shows that $\hat{\delta}^{po}(A') \leq \hat{\delta}^{po}(A)$. Hence, $\hat{\delta}^{po}(A) = \hat{\delta}^{po}(A')$ and $\hat{\delta}^{po}(A') = \hat{\delta}_i(A')$.

(“Only if” part) Assume that $\hat{\delta}^{po}(A') = \hat{\delta}^{po}(A)$. Since by Theorem 1, $\hat{\delta}_i(A') \leq \hat{\delta}_i(A)$ for all i , we must have $\hat{\delta}_i(A') = \hat{\delta}_i(A)$ for some i with $\hat{\delta}^{po}(A) = \hat{\delta}_i(A)$. By Theorem 1 then it must be the case that $b_{i,r} = 0$ or $i = s$. ■

Corollary 2 implies that in the presence of multilateral PCO arrangements, an increase in a firm's passive stake in a rival will in general have anticompetitive coordinated effects and should raise antitrust concerns. The only two exceptions to this conclusion are cases in which the rival firm is either the industry maverick or the industry maverick does not have a direct or an indirect stake in the investing firm. In either of these cases, passive investments in rivals warrants a lenient treatment.

To illustrate Corollary 2, suppose that there are 10 firms in the industry and firms 1 – 4 invest only in each other so that none of them has direct or indirect stakes in firms 5 – 10. Then, any increase in the stakes that firms 5 – 10 hold in rivals, including changes in their stakes in firms 1 – 4, will surely facilitate tacit collusion unless (i) the industry maverick is either firm 1, 2, 3, or 4, or (ii) the increased ownership stake is in the maverick firm. When either (i) or (ii) hold, the increased stake of firms 5 – 10 in rivals will have no effect on tacit collusion and will justify a lenient treatment by antitrust authorities.

Condition (ii) in Corollary 2 says that investment in a maverick firm has no effect on tacit collusion. This result is striking because the Horizontal Merger Guidelines of the US

Department of Justice and FTC state that the "acquisition of a maverick firm is one way in which a merger may make coordinated interaction more likely."¹⁵ This concern indicates that there is a fundamental difference between horizontal mergers in which firms obtain control over their rivals and passive investments in rivals that we study here. In particular, while gaining control over a maverick firm via a horizontal merger particularly raises antitrust concerns, Corollary 2 shows that a mere passive investment in a maverick firm should not raise any antitrust concerns.

2.3 The symmetric PCO case

To obtain further insights about the effect of PCO on tacit collusion, we now consider the symmetric case in which all firms hold exactly the same ownership stakes in each other, i.e., $\alpha_i^j = \bar{\alpha}$ for all $i = 1, \dots, n$ and all $j \neq i$. In the Appendix we show that in this case, the fully collusive outcome can be sustained as a subgame perfect equilibrium of the infinitely repeated game provided that

$$\delta \geq \widehat{\delta}^{po} = \widehat{\delta} - \frac{(n-1)\bar{\alpha}}{n(1-(n-2)\bar{\alpha})}. \quad (9)$$

This expression gives rise to the following result:

Proposition 1: *Suppose that $\alpha_i^j = \bar{\alpha}$ for all $i = 1, \dots, n$ and all $j \neq i$. Then, holding $\bar{\alpha}$ fixed, $\widehat{\delta}^{po}$ is increasing with n if $(n-1)\bar{\alpha} < \frac{1}{2}$, i.e., if the aggregate stake of rivals in each firm is less than 50%, and is decreasing with n if $(n-1)\bar{\alpha} > \frac{1}{2}$.*

As equation (1) shows, absent PCO, an increase in the number of firms hinders collusion. Proposition 1 shows that in the presence of PCO, this is no longer true: when the aggregate stake of rivals in each firm exceeds 50%, an increase in the number of firms facilitates collusion rather than hinders it. The reason for this surprising result is that, holding $\bar{\alpha}$ fixed, an increase in n implies that each firm receives a larger fraction of its profits from rivals. Hence, deviation from the fully collusive scheme which hurts rivals may become unattractive. When n is sufficiently large, this positive effect of n outweighs the usual negative effect and as a result, collusion

¹⁵See www.usdoj.gov/atr/public/guidelines/horiz_book/hmg1.html

becomes easier.¹⁶ To illustrate, suppose that each firm holds a passive stake of 10% in rivals. Then, moving from 6 to 7 firms will facilitate collusion whereas moving from 4 firms to 5 will hinder it.

Next, we ask how a deviation from the symmetric stakes case affects tacit collusion. To this end, suppose that one firm, say firm 1, raises its *aggregate* stake in rivals by Δ so that $\alpha_1^2 + \alpha_1^3 + \dots + \alpha_1^n = (n-1)\bar{\alpha} + \Delta$. To ensure that the stakes that rivals hold in each firm j are less than 1, we will assume that $\Delta < 1 - (n-1)\bar{\alpha}$. All firms other than firm 1 continue to hold an ownership stake $\bar{\alpha}$ in each of their rivals.

Proposition 2: *Starting from the symmetric case in which $\alpha_i^j = \bar{\alpha}$ for all $i = 1, \dots, n$ and all $j \neq i$, suppose that firm 1 changes its aggregate stake in rivals by $\Delta < 1 - (n-1)\bar{\alpha}$.*

- (i) *If $\Delta > 0$, then $\widehat{\delta}^{po} < \widehat{\delta}$ (tacit collusion is facilitated) provided that Δ is spread over at least two of firm 1's rivals and $\widehat{\delta}^{po}$ is minimized (i.e., the incentives to collude are strongest) when Δ is spread equally among all of firm 1's rivals.*
- (ii) *If $\Delta < 0$, then $\widehat{\delta}^{po} > \widehat{\delta}$ (tacit collusion is hindered) although only the aggregate change in firm 1's stake in rivals matters and not how it is spread among firm 1's rivals.*

Proof: *See the Appendix.*

Proposition 2 indicates that if we start from a symmetric PCO configuration, a unilateral increase in PCO by one firm raises more antitrust concerns the more evenly it is spread among the rival firms. Intuitively, the firm in which firm 1 has invested the most becomes the industry maverick since its controller gains the most from deviation as a larger fraction of its profit from deviation flows back to the firm via its stake in firm 1. Obviously, an even spread of Δ among all rivals minimizes firm 1's stake in the industry maverick and therefore minimizes the incentive of the maverick's controller to deviate from the fully collusive scheme. Interestingly, when Δ is concentrated in only one firm, the change in PCO has no effect on the ease of collusion and

¹⁶Note that since we consider passive investments in rivals, the fact that rival firms have a combined share of more than 50% in the profits of each firm does not prevent the firm's controller from controlling more than 50% of the voting rights.

should raise no antitrust concerns. This result is consistent with Corollary 2: when firm 1 invests in only one rival, this firm becomes the industry maverick, so by Corollary 2 the investment has no effect on tacit collusion.

Using Proposition 2, one may be tempted to conjecture that, starting with any arbitrary PCO structure, if a firm increases its stake in rivals by an aggregate amount of Δ , then tacit collusion is particularly facilitated when Δ is spread evenly among the rivals. The next example shows that this conjecture is false.

Example 1 (an equal spread of a firm’s additional investment in rivals is not necessarily more collusive than an unequal spread): Consider an industry with 3 firms and let the PCO matrix be

$$A = \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{8} & 0 & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{8} & 0 \end{pmatrix}.$$

Now suppose that firm 1 acquires additional stakes Δ_2 and Δ_3 in firms 2 and 3 such that $\Delta_2 + \Delta_3 = \frac{1}{4}$, and consider three scenarios: (1) $\Delta_2 = \Delta_3 = \frac{1}{8}$, (2) $\Delta_2 = \frac{1}{4}$ and $\Delta_3 = 0$, and (3) $\Delta_2 = 0$ and $\Delta_3 = \frac{1}{4}$. The associated PCO matrices under the three scenarios are

$$A_1 = \begin{pmatrix} 0 & \frac{3}{8} & \frac{3}{8} \\ \frac{1}{8} & 0 & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{8} & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{8} & 0 & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{8} & 0 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{8} & 0 & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{8} & 0 \end{pmatrix}.$$

Using (8), it is straightforward to check that $\widehat{\delta}^{po}(A_2) < \widehat{\delta}^{po}(A_1) < \widehat{\delta}^{po}(A_3) < \widehat{\delta}^{po}(A)$: all scenarios facilitate tacit collusion, but scenario 2, in which firm 1 increases its stake only in firm 2, facilitates it more than scenario 1, in which firm 1 increases its stake in firms 2 and 3 by equal amounts, and the latter scenario facilitates collusion more than scenario 3, in which firm 1 increases its stake only in firm 3.

Proposition 2 assumes implicitly that when firm 1 increases its stake in rivals, it buys additional shares from outside investors or from controllers. The next corollary examines what

happens when there is a transfer of ownership from one rival firm to another. A recent example of such a transfer occurred in the steel industry, where Luxembourg based Arcelor, the world's largest steelmaker, increased its stake in Brazilian CST, one of the world's largest steelmakers, from 18.6% to 27.95% by buying shares from Acesita, another Brazilian steelmaker.¹⁷

Proposition 3: *Starting from the symmetric case in which $\alpha_i^j = \bar{\alpha}$ for all $i = 1, \dots, n$ and all $j \neq i$, suppose that firm 1 buys a stake $\Delta \leq \bar{\alpha}$ in firm 3 from firm 2, so after the transaction, firm 1's stake in firm 3 increases to $\bar{\alpha} + \Delta$ while firm 2's stake in firm 3 falls to $\bar{\alpha} - \Delta$. This change in the PCO configuration hinders tacit collusion and more so when Δ increases.*

Proof: *See the Appendix.*

Proposition 3 differs from Proposition 2 in that the increase in firm 1's ownership stake comes at the expense of firm 2's stake. Hence, the aggregate amount of shares held by rival firms in each other does not increase as in Proposition 2 but rather remains constant. The intuition why this change hinders tacit collusion is as follows: following the transfer of ownership, firm 2 becomes the industry maverick since it now has the smallest stake in rivals in the industry. Consequently, firm 2's controller has a stronger incentive to deviate from the fully collusive scheme than he had before and hence tacit collusion is hindered. Together, Propositions 2 and 3 suggest that with identical firms, symmetric PCO configurations are the most conducive to tacit collusion and should therefore raise particular anticompetitive concerns.

3 PCO by controllers

In this section we consider the possibility that controllers will directly acquire (passive) ownership stakes in rival firms. As mentioned in the Introduction, a case in point is the car rental industry in the first half of the 90's where National Car Rental's controller, GM, passively held a 25% stake in Avis, National's rival, while Hertz's controller, Ford, had acquired 100% of the preferred

¹⁷Prior to the sale, Acesita held a 18.7% stake in CST but sold its entire stake in CST to Arcelor and to CVRD, which is a large Brazilian miner of iron and ore. In addition to its stake in CST, Arcelor also owns stakes in Acesita and in Belgo-Mineira which is another Brazilian steelmaker (see "CVRD, Arcelor Team up for CST," *The Daily Deal*, December 28, 2002, M&A; "Minister: Steel Duties Still Under Study - Brazil," *Business News Americas*, April 8, 2002.)

nonvoting stock of Budget Rent a Car. The question we address is what effect, if any, such investments have on tacit collusion above and beyond the effect that we have already identified in the previous section.

To this end, let β_i^j be the stake that firm i 's controller obtains in firm $j \neq i$, in addition to his controlling stake in firm i , β_i (in terms of the new notation, $\beta_i \equiv \beta_i^i$). To avoid triviality, we assume that β_i^j represents a completely passive investment (e.g., non-voting shares) that gives the controller a share β_i^j of firm j 's profit but no control over its actions. Then, the condition that ensures that collusion can be sustained becomes

$$\frac{\beta_i \pi_i(A) + \sum_{j \neq i} \beta_i^j \pi_j(A)}{1 - \delta} \geq \beta_i \pi_i^{d_i}(A) + \sum_{j \neq i} \beta_i^j \pi_j^{d_i}(A), \quad i = 1, \dots, n. \quad (10)$$

Condition (10) generalizes condition (6) to the case where controllers hold direct stakes in rival firms. The left side of (10) is the infinite discounted payoff of firm i 's controller under collusion which consists of his share in the collusive profit of all firms in which he has a stake (including of course firm i). The right side of (10) is the controller's share in the one time profit that all firms he has stakes in earn when the firm he controls, firm i , undercuts its rivals slightly. Notice that by Lemma 1, $\pi_j^{d_i}(A) > 0$ if firm j has a direct or an indirect stakes in firm i .

Using (10) and recalling that $\pi_i(A) = \frac{\pi^m}{n} \sum_{j=1}^n b_{i,j}$ and $\pi_j^{d_i}(A) = b_{j,i} \pi^m$, it follows that with PCO by controllers, the fully collusive scheme can be sustained as a subgame perfect equilibrium of the infinitely repeated game provided that

$$\delta \geq \widehat{\delta}^c(A) \equiv \max \left\{ \widehat{\delta}_1^c(A), \dots, \widehat{\delta}_n^c(A) \right\}, \quad (11)$$

where

$$\delta \geq \widehat{\delta}_i^c(A) \equiv 1 - \frac{\frac{1}{n} \beta_i \sum_{j=1}^n b_{i,j} + \frac{1}{n} \sum_{j \neq i} \beta_i^j \sum_{k=1}^n b_{j,k}}{\beta_i b_{i,i} + \sum_{j \neq i} \beta_i^j b_{j,i}}. \quad (12)$$

Without PCO by firms (i.e., when $A = 0$), $B = I$ so $b_{i,i} = 1$ and $b_{i,j} = 0$ for all i and all $j \neq i$. Hence, (12) implies that PCO by controllers facilitate collusion as $\widehat{\delta}_i^c(0) = 1 - \frac{1}{n} - \frac{\frac{1}{n} \sum_{j \neq i} \beta_i^j}{\beta_i} \leq 1 - \frac{1}{n}$. The following theorem proves that this continues to be the case even

when $A > 0$.

Theorem 2: *PCO by controllers facilitate tacit collusion in the sense that $\widehat{\delta}_i^c(A) \leq \widehat{\delta}_i(A)$ for all $i = 1, \dots, n$ with strict inequality holding whenever $\beta_i^j > 0$ for some $j \neq i$. Moreover, $\widehat{\delta}_i(A) - \widehat{\delta}_i^c(A)$ increases as β_i falls; hence, PCO by firm i 's controller is more effective in strengthening the controller's incentive to collude the smaller is the controller's stake in his own firm.*

Proof: Using equations (12) and (8),

$$\begin{aligned} \widehat{\delta}_i(A) - \widehat{\delta}_i^c(A) &= \frac{\sum_{j \neq i} \beta_i^j \frac{1}{n} \sum B_j - \frac{1}{n} \frac{\sum B_i}{b_{i,i}} \sum_{j \neq i} \beta_i^j b_{j,i}}{\beta_i b_{i,i} + \sum_{j \neq i} \beta_i^j b_{j,i}} \\ &= \frac{\frac{1}{n} \sum_{j \neq i} \beta_i^j \left(\sum B_j - \frac{b_{j,i}}{b_{i,i}} \sum B_i \right)}{\beta_i b_{i,i} + \sum_{j \neq i} \beta_i^j b_{j,i}}, \end{aligned} \quad (13)$$

where $\sum B_j \equiv \sum_{k=1}^n b_{j,k}$. By Lemma A3 in the Appendix, $\sum B_j - \frac{b_{j,i}}{b_{i,i}} \sum B_i > 0$ for every distinct pair of firms, i, j . Hence, $\widehat{\delta}_i^c(A) < \widehat{\delta}_i(A)$ if $\beta_i^j > 0$ for some $j \neq i$ and $\widehat{\delta}_i^c(A) = \widehat{\delta}_i(A)$ otherwise. Finally, note from (14) that $\frac{\partial}{\partial \beta_i} \left(\widehat{\delta}_i(A) - \widehat{\delta}_i^c(A) \right) < 0$. ■

Theorem 2 shows that when firm i 's controller invests in at least one rival firm, the controller is willing to participate in the fully collusive scheme for a wider set of discount factors. Moreover, this set become even wider as the controller's stake in the firm he controls, i.e., firm i , becomes smaller. This implies in turn that firm i 's controller can lower $\widehat{\delta}_i^c(A)$ either by raising his stake in rival firms or by diluting his stake in firm i (subject of course to retaining control over the firm's actions). Such dilution effectively raises the weight that the controller assigns to rivals' profits and therefore weakens the controller's incentive to deviate from the collusive scheme. This implies in turn that even relatively small direct passive investments by controllers in rival firms can raise considerable antitrust concern. It should also be noted that $\widehat{\delta}_i^c(A)$ depends only on the stakes that firm i 's controller has in rival firms but is completely independent of the stakes that other controllers have in rival firms.

An important implication of Theorem 2, that to the best of our knowledge has been overlooked in antitrust cases involving PCO by controllers, is that antitrust agencies need to be concerned not only with a controller's stakes in rival firms, but also with the controller's

stake (current or future) in his own firm. This suggests in turn that consent decrees approving passive investment by controllers should stipulate that the controllers will abstain from further diluting their stakes in their own firms.¹⁸ For example, shortly after it acquired a passive stake in Budget, Ford diluted its controlling stake in Hertz from 55% to 49% by selling shares to Volvo.¹⁹ Theorem 2 suggests that such dilution by Ford may have promoted collusion in the car rental industry. Similarly, the FTC has approved TCI's passive 9% stake in Time Warner and even allowed this stake to increase to 14.99% in the future despite the fact that TCI controlled movie networks Starz and Encore (with an 80% stake) while Time Warner wholly owned rival movie networks HBO and Cinemax.²⁰ Theorem 2 suggests that the FTC should have been concerned not only with TCI's stake in Time Warner, but also with its stake in movie networks Starz and Encore. In particular, it suggests that the consent decree approving TCI's stake in Time Warner should have stipulated that TCI should refrain from diluting its stake in Starz and Encore in the future since such dilution may facilitate tacit collusion in markets in which these movie networks compete against each other.²¹

Theorem 2 also has implications for the recent decision of the Brazilian antitrust authorities to allow Telecom Italia (TI) to raise its stake in Telecom Brazil (TB) from 19% to 37.3% provided that TI would be a passive investor as far as TB's cellular and long distance operations are concerned. TI holds a 56% controlling stake in Telecom Italia Mobile (TIM), Brazil's second largest cellular provider while TB had acquired a cellular license and will be competing with TIM in Brazilian cellular markets.²² Theorem 2 suggests that stipulating that TI will be a passive investor in TB was not enough to alleviate anticompetitive concerns in the Brazilian cellular market, and moreover, it implies that the fact TI's controlling stake in TB is merely 56% (rather than 100%) exacerbates these concerns.

¹⁸In firms that are controlled by managers, compensation that is linked to the profits of rivals may play the same role as investments in rivals. This suggests that in these cases, executive compensation should receive similar antitrust scrutiny as investments of controllers in rival firms.

¹⁹See "Chrysler Buying Thrifty Rent-A-Car," *St. Louis Post-Dispatch*, May 19, 1989, Business, 8C.

²⁰See Proposed Consent Agreement, Time Warner, Inc., 61 Fed. Reg. 50,301 (Sept. 25, 1996) for the FTC's decision and Waterman and Weiss (1997) for details about the cable TV industry.

²¹See Gilo (2000) for more details on these and similar examples.

²²See "Spying Becomes an Issue In Brazilian Phone Dispute", *N. Y Times*, July 23, 2004, Section W; Column 3; http://www.company.tim.it/investor/cp_dettaglio/0,,29_94_97,00.html; "Telecom Italia Refocuses in Brazil," *Financial Times*, August 29, 2002, London Edition 1, p. 26, "2 From Europe Make Brazil Phone Deal," *N. Y Times*, January 17, 2003, Section C; Column 4, p. 13.

Interestingly, the ability of firms to collude is greatly diminished when a firm's controller internalizes the interests of the minority shareholders and acts to maximize total firm value rather than only the value of his own stake. This is because such behavior has the exact opposite effect of dilution of the controller's stake: a controller who acts to maximize total firm value acts as if $\beta_i = 1$ in which case $\widehat{\delta}_i^c(A)$ is maximized. In this sense, minority shareholders would prefer the controller to disregard their interests when choosing the firm's pricing decisions. Thus, contrary to conventional wisdom that sees the disregard of minority shareholders as a value decreasing "agency cost," here such disregard is actually beneficial to all shareholders.

One may wonder if Theorem 1 continues to hold when controllers hold stakes directly in rival firms. That is, is it still true that any increase in one firm's stake in a rival firm will never hinder collusion? The following example shows that the answer is no.

Example 2 (an increase in a firm's stake in rivals may hinder collusion): Consider an industry with 2 firms and let the PCO matrix be

$$A = \begin{pmatrix} 0 & \alpha_1^2 \\ \alpha_2^1 & 0 \end{pmatrix}.$$

Moreover, suppose that the controller of firm $i = 1, 2$ has a stake of β_i^1 in firm 1 and β_i^2 in firm 2. It is straightforward to verify that

$$B = \begin{pmatrix} \frac{1}{1-\alpha_1^2\alpha_2^1} & \frac{\alpha_1^2}{1-\alpha_1^2\alpha_2^1} \\ \frac{\alpha_2^1}{1-\alpha_1^2\alpha_2^1} & \frac{1}{1-\alpha_1^2\alpha_2^1} \end{pmatrix}.$$

Using equation (12) we get:

$$\begin{aligned} \widehat{\delta}_1^c(A) &= 1 - \frac{\beta_1^1(1 + \alpha_1^2) + \beta_1^2(1 + \alpha_2^1)}{2(\beta_1^1 + \beta_1^2\alpha_2^1)} \\ &= \frac{1}{2} - \frac{\beta_1^1\alpha_1^2 + \beta_1^2}{2(\beta_1^1 + \beta_1^2\alpha_2^1)}. \end{aligned}$$

It is easy to see that $\widehat{\delta}_1^c(A)$ decreases with α_1^2 : an increase in firm 1's stake in firm 2

strengthens the incentive of firm 1's controller to collude. However, so long as $\beta_1^2 > 0$, $\widehat{\delta}_1^c(A)$ increases with α_2^1 implying that an increase in firm 2's stake in firm 1 weakens the incentive of firm 1's controller to collude. Consequently, whenever $\widehat{\delta}_1^c(A) > \widehat{\delta}_2^c(A)$ (the controller of firm 1 is the industry maverick) and $\beta_1^2 > 0$ (firm 1's controller holds a stake in firm 2), an increase in firm 2's stake in firm 1 will hinder collusion rather than facilitate it. Moreover, this effect becomes stronger as the stake that firm 1's controller holds in firm 2, β_1^2 , increases. Hence, Theorem 1 is no longer true in this case.

Finally, Corollary 2 above implies that absent PCO by controllers, an increase in firm r 's stake in firm s does not affect neither firm s 's incentive to collude nor the incentive of each firm i for which $b_{i,r} = 0$, i.e., each firm i that does not have a direct or an indirect stake in firm r . The following result examines whether this is still true in the presence of PCO by controllers.

Proposition 4: *Starting with a PCO matrix A , suppose that firm r increases its stake in firm s by some $\Delta > 0$, so that the PCO matrix becomes $A' = A + \Delta e_{r,s}$, where $e_{r,s}$ is an $n \times n$ matrix whose (r, s) -entry is 1 and each other entry is 0. Then,*

- (i) $\widehat{\delta}_s^c(A') \geq \widehat{\delta}_s^c(A)$ with equality holding only if either $\beta_s^j = 0$ for all $j \neq s$, or $\beta_s b_{s,r} + \sum_{j \neq s} \beta_s^j b_{j,r} = 0$,
- (ii) $\widehat{\delta}_i^c(A') = \widehat{\delta}_i^c(A)$ for each firm i such that $b_{i,r} = 0$ and $\sum_{j \neq i} \beta_i^j b_{j,r} = 0$.

Proof: *See the Appendix.*

Proposition 4 shows that in the presence of PCO by controllers, an increase in firm r 's stake in firm s will not affect firm s 's incentive to collude only if firm s 's controller either has no stake in any of firm s 's rivals (i.e., $\beta_s^j = 0$ for all $j \neq s$) or has no direct or indirect stake in firm r (i.e., $\beta_s b_{s,r} + \sum_{j \neq s} \beta_s^j b_{j,r} = 0$: firm s has no direct or indirect stake in firm r and its controller has no stake in any firm that has a stake - direct or indirect - in firm r). Otherwise, the increase in firm r 's stake in firm s will weaken firm s 's incentive to collude. In the context of the car rental industry case mentioned above, Proposition 4 implies that had Budget made a passive investment in Hertz, Hertz's incentive to engage in tacit collusion would have become weaker given that Hertz's controller, Ford, already held a passive stake in Budget. Similarly, a

passive investment by Avis in National would have weakened National’s incentive to engage in tacit collusion given that its controller, GM, also held a passive stake in Avis. This suggests in turn that firms have no incentive to acquire stakes in rivals when their controllers hold stakes in those rivals. Indeed, in the cases involving PCO by controllers discussed here and in Gilo (2000), PCO by controllers in rivals was never accompanied by PCO by the firms themselves in rivals.

In addition, Proposition 4 shows that when firm i has no direct or indirect stake in firm r (i.e., $b_{i,r} = 0$), an increase in firm r ’s stake in firm s will not affect firm i ’s incentive to collude only if firm i ’s controller does not hold a stake in any firm that has a stake - direct or indirect - in firm r (i.e., $\sum_{j \neq i} \beta_i^j b_{j,r} = 0$).

To illustrate Proposition 4, consider an industry with 10 firms such that firms 1 – 4 invest only in each other but none of them has a stake in firms 5 – 10, while each of firms 5 – 10 has either direct or indirect stakes in all rivals. Now suppose that firm 5 increases its stake in firm 4. Then, part (i) of Proposition 4 shows that the incentive of firm 4’s controller to collude will remain unchanged if he has no stake in other firms or has stake only in firms 1 – 4. If firm 4’s controller has a stake in at least one of firms 5 – 10 then his incentive to collude would be weakened. Part (ii) of Proposition 4 shows that the increase in firm 5’s stake in firm 4 will not affect the incentives of firms 1 – 3 to collude provided that their controllers do not have stakes in firms 5 – 10.

4 Conclusion

Acquisitions of one firm’s stock by a rival firm have been traditionally treated under Section 7 of the Clayton Act which condemns such acquisitions when their effect “may be substantially to lessen competition.” However, the third paragraph of this section effectively exempts investments made “solely for investment.” As argued in Gilo (2000), antitrust agencies and courts, when applying this exemption, did not conduct full-blown examinations as to whether such passive investments among rivals may substantially lessen competition.²³

²³We are aware of only two cases in which the ability of passive investments to lessen competition was acknowledged: the FTC’s decision in *Golden Grain Macaroni Co.* (78 F.T.C. 63, 1971), and the consent decree

In this paper we showed that although there are cases in which passive investments in rivals have no effect on the ability of firms to engage in tacit collusion, an across the board lenient approach towards such investments may be misguided. This is because passive investments in rivals may well facilitate tacit collusion, especially when these investments are multilateral, are in firms that are not industry mavericks, and are by firms in which mavericks hold either direct or indirect stakes. In addition, we showed that direct investments by firms' controllers in rivals may either substitute investments by the firms themselves or facilitate collusion further, especially when the controllers have small stakes in their own firms. On the other hand, if a firm's controller holds a stake in a rival firm, passive investment by this rival in the controller's firm warrants a lenient antitrust approach. We believe that antitrust courts and agencies should take account of these factors when considering cases involving passive investments among rivals.

Throughout the paper we have focused exclusively on the effect of PCO on the ability of firms to engage in (tacit) price fixing. However, if in addition to price fixing firms can also divide the market among themselves, then they would clearly be able to sustain collusion for a larger set of discount factors since they would have more instruments (the collusive price and the market shares). In particular, it would be possible to relax the incentive constraints of maverick firms by increasing their market shares at the expense of firms with nonbinding incentive constraints. This suggests in turn that in the presence of market sharing schemes, firms may have an incentive to become industry mavericks in order to receive a larger share of the market. As our analysis shows, one way to become an industry maverick is to avoid investing in rivals.²⁴ Interestingly, this implies that beside the fact that market sharing schemes are harder to enforce (firms need to commit to ration their sales) and are more susceptible to antitrust scrutiny, they have another drawback, which is that they provide firms with a disincentive to invest in rivals and thereby facilitate tacit collusion.

Finally, throughout the paper we make two simplifying assumptions. The first assumption is that firms produce a homogeneous product and have the same cost functions. In Gilo and

reached with the DOJ regarding US West's acquisition of Continental Cablevision (this decree was approved by the district court in *United States v. US West Inc.*, 1997-1 Trade cases (CCH), ¶71,767, D.C., 1997).

²⁴Indeed, in a previous version of the paper, we showed that under market sharing schemes and cost asymmetries, only the most efficient firm in the industry has an incentive to invest in rivals to sustain collusion while all other firms find it optimal to not invest in rivals.

Spiegel (2003) we began looking at the case where firms have asymmetric costs. We showed that even unilateral PCO by the most efficient firm in its rivals may facilitate tacit collusion and the resulting collusive price is higher than it would be absent PCO. Moreover, we showed that the most efficient firm prefers to first invest in its most efficient rival both because this is the most effective way to promote tacit collusion and because such investment leads to a collusive price that is closer to the most efficient firm's monopoly price. The second simplifying assumption that we make in this paper is that the level of PCO in the industry is exogenously given. In a sense then our analysis is done from the perspective of antitrust authorities: when can you allow a firm to acquire a passive stake in a rival firm and when should you disallow such acquisition. In future research we wish to also look at PCO from the perspective of firms: that is, we wish to endogenize the configuration of PCO in the industry and examine when should a firm try to acquire a passive stake in rivals and when shouldn't it.

5 Appendix

Following are the proofs of Lemma 1, Theorem 1, and Propositions 2-4.

Proof of Lemma 1: Since the PCO matrix, A , is nonnegative and the sum of each of its columns is strictly less than 1, the inverse Leontief matrix is such that

$$B \equiv (I - A)^{-1} = I + A + A^2 + \dots,$$

(see Berck and Sydsæter, Ch. 21.22, p. 111). Hence, $b_{i,j} \geq 0$ for all i and all j and $b_{i,i} \geq 1$ for all i . Moreover, $b_{i,r} = 0$ if and only if the (i,r) -entry of A^k is 0 for each k . Matrices A with this property have been studied by Frobenius (1912) and are called *reducible matrices* (see also Jones, Klin, and Moshe, (2002)). A theorem of Frobenius shows that this property of A is equivalent to the existence of such a partition.

Finally, to prove that $b_{i,j} \leq b_{i,i}$, let C_k denote the k -th column of B , and let e_k denote the k -th column of I . Since $(I - A)B = I$, we obtain

$$(I - A)C_k = e_k, \quad k = 1, \dots, n.$$

Subtracting this equation for column j from that for column i yields

$$(I - A)(C_i - C_j) = e_i - e_j.$$

Noting that the i -th coordinate of $C_i - C_j$ is $b_{i,i} - b_{i,j}$, we obtain by Cramer's rule,

$$b_{i,i} - b_{i,j} = \frac{\det(D)}{\det(I - A)},$$

where the matrix D is obtained by replacing the i -th column of $I - A$ with $e_i - e_j$. Thus, $D = I - A_i$, where the matrix A_i is obtained by replacing the i -th column of A with e_i (hence A_i is also a PCO matrix). We complete the proof by showing that $\det(I - A), \det(D) \geq 0$.

To this end, assume by way of negation that $\det(I - A) < 0$ and consider the function

$$f(c) = \det(I - cA).$$

Since the determinant operation is continuous, f is a continuous function with $f(0) = \det(I) = 1$. Since we assume that $f(1) = \det(I - A) < 0$, there must be a number $0 < c < 1$ such that $f(c) = 0$. But, since the sum of each column in cA is less than 1 (the sum of each column i is the aggregate stake that rival firms hold in firm i) and thus $I - cA$ is an invertible matrix for all c . Hence, $f(c) \neq 0$ for all c , thus contradicting the assumption that $f(1) = \det(I - A) < 0$. Hence, $\det(I - A) \geq 0$. Recalling that A_i is also a PCO matrix, we also have $\det(D) = \det(I - A_i) \geq 0$.

■

In order to prove Theorem 1, we begin with three lemmas. To this end, let $B_k = (b_{k,1}, \dots, b_{k,n})$ denote the k -th row of B and recall that $e_{r,s}$ is an $n \times n$ matrix whose (r, s) -entry is 1 and each other entry is 0.

Lemma A1: *Let A, A' be two PCO matrices such that $A' = A + \Delta e_{r,s}$ for some $\Delta > 0$ (A' differs from A in that firm r 's stake in firm s has increased by Δ). Then $\Delta b_{s,r} < 1$ and*

$$B' \equiv (I - A')^{-1} = \begin{pmatrix} b_{1,1} + \varepsilon_1 b_{s,1}, & b_{1,2} + \varepsilon_1 b_{s,2}, & \dots, & b_{1,n} + \varepsilon_1 b_{s,n} \\ b_{2,1} + \varepsilon_2 b_{s,1}, & b_{2,2} + \varepsilon_2 b_{s,2}, & \dots, & b_{2,n} + \varepsilon_2 b_{s,n} \\ & & \vdots & \\ b_{n,1} + \varepsilon_n b_{s,1}, & b_{n,2} + \varepsilon_n b_{s,2}, & \dots, & b_{n,n} + \varepsilon_n b_{s,n} \end{pmatrix},$$

where $\varepsilon_i = \frac{\Delta b_{i,r}}{1 - \Delta b_{s,r}} \geq 0$ with equality holding for $b_{i,r} = 0$.

Proof: Observe that

$$(B')^{-1}B = (I - A')B = ((I - A) - \Delta e_{r,s})B = (I - A)B - \Delta e_{r,s}B = I - \Delta e_{r,s}B.$$

Since by Lemma 1, $(B')^{-1}$ and B are both invertible matrices, so is $I - \Delta e_{r,s}B$. Hence,

$$B' = B(I - \Delta e_{r,s}B)^{-1}. \tag{A-1}$$

Note that the only nonzero row in the matrix $\Delta e_{r,s}B$ is the r -th row:

$$\Delta e_{r,s}B = \begin{matrix} & \begin{matrix} 1 \\ 2 \\ \vdots \\ r \\ \vdots \\ n \end{matrix} & \begin{pmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \Delta b_{s,1} & \Delta b_{s,2} & \dots & \Delta b_{s,n} \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix} & . \end{matrix} \quad (\text{A-2})$$

If $\Delta b_{s,r} = 1$, then the r -th column of $I - \Delta e_{r,s}B$ is 0, contradicting the fact that $I - \Delta e_{r,s}B$ is invertible. Thus,

$$\Delta b_{s,r} \neq 1. \quad (\text{A-3})$$

A straightforward calculation shows that

$$(I - \Delta e_{r,s}B)\left(I + \frac{1}{1 - \Delta b_{s,r}}\Delta e_{r,s}B\right) = I. \quad (\text{A-4})$$

Using (A-1) and (A-4), we get

$$B' = B(I - \Delta e_{r,s}B)^{-1} = B\left(I + \frac{1}{1 - \Delta b_{s,r}}\Delta e_{r,s}B\right) = B + \frac{1}{1 - \Delta b_{s,r}}B\Delta e_{r,s}B.$$

By (A-2) we easily obtain that the i -th row in $B(\Delta e_{r,s}B)$ is $b_{i,r}\Delta B_s$. Thus, the i -th row of B' is $B_i + \varepsilon_i B_s$, where $\varepsilon_i = \frac{\Delta b_{i,r}}{1 - \Delta b_{s,r}}$.

It remains to prove that $\Delta b_{s,r} < 1$. We already obtained $\Delta b_{s,r} \neq 1$. Assume by way of negation that $\Delta b_{s,r} > 1$. Then $b_{s,r} \neq 0$ and $\Delta > \frac{1}{b_{s,r}}$. Thus the sum of each column in $A + \frac{1}{b_{s,r}}e_{r,s}$ is strictly less than 1. Let

$$\Delta_1 = \frac{1}{b_{s,r}}, \quad A_1 = A + \frac{1}{b_{s,r}}e_{r,s}.$$

Note that the pair A, A_1 satisfies the assumptions of the lemma. Thus, by (A-3) we have

$\Delta_1 b_{s,r} \neq 1$, contradicting the definition of Δ_1 . \blacksquare

Denote by $\sum B_k$ the sum of entries in the k -th row in B (i.e., $\sum B_k = \sum_{j=1}^n b_{k,j}$).

Lemma A2: Let $A' = A + \Delta e_{r,s}$ for some $\Delta > 0$. Then for every i ,

$$\hat{\delta}_i(A) - \hat{\delta}_i(A') = \frac{\varepsilon_i}{n(b_{i,i} + \varepsilon_i b_{s,i})} \left(\sum B_s - \frac{b_{s,i}}{b_{i,i}} \sum B_i \right), \quad (\text{A-5})$$

where $\varepsilon_i = \frac{\Delta b_{i,r}}{1 - \Delta b_{s,r}} \geq 0$.

Proof: By Lemma A1, the i -th row of B' is $B_i + \varepsilon_i B_s$. Thus,

$$\begin{aligned} \hat{\delta}_i(A) - \hat{\delta}_i(A') &= \frac{\frac{1}{n}(\sum B_i + \varepsilon_i \sum B_s)}{b_{i,i} + \varepsilon_i b_{s,i}} - \frac{\frac{1}{n} \sum B_i}{b_{i,i}} \\ &= \frac{\varepsilon_i b_{i,i} \sum B_s - \varepsilon_i b_{s,i} \sum B_i}{n(b_{i,i} + \varepsilon_i b_{s,i}) b_{i,i}} \\ &= \frac{\varepsilon_i}{n(b_{i,i} + \varepsilon_i b_{s,i})} \left(\sum B_s - \frac{b_{s,i}}{b_{i,i}} \sum B_i \right). \end{aligned}$$

\blacksquare

Lemma A3: For every distinct pair of firms, i, s we have

$$\sum B_s - \frac{b_{s,i}}{b_{i,i}} \sum B_i > 0.$$

Proof: Assume by way of negation that

$$\sum B_s - \frac{b_{s,i}}{b_{i,i}} \sum B_i \leq 0. \quad (\text{A-6})$$

Let $M \equiv BE$, where E is a diagonal $n \times n$ matrix with $E_{i,i} = 1 - \frac{\sum B_i}{b_{i,i}}$ and $E_{k,k} = 1$ for $k \neq i$. That is, M is obtained by multiplying the i -th column of B by $1 - \frac{\sum B_i}{b_{i,i}}$. Now let M_k denote

the k -th row in M and define

$$\sum M \equiv M \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} \sum M_1 \\ \sum M_2 \\ \vdots \\ \sum M_n \end{pmatrix}. \quad (\text{A-7})$$

That is, $\sum M$ is a column vector that specifies the sum of the rows of M . Note that by construction,

$$\sum M_i = 0, \quad \sum M_s \leq 0,$$

where the last inequality follows from (A-6).

Our next task is to prove that $\sum M_k \geq 0$ for each $k \neq s$. Multiplying both sides of (A-7) by $I - A$ and recalling that $M \equiv BE$ and $B \equiv (I - A)^{-1}$, we get

$$u \equiv (I - A) \sum M = (I - A)BE \begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} = E \begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ \vdots \\ 1 - \frac{\sum B_i}{b_{i,i}} \\ \vdots \\ 1 \end{pmatrix}, \quad (\text{A-8})$$

where $1 - \frac{\sum B_i}{b_{i,i}}$ is the i -th entry of u (and each other entry of u equals 1).

Let $\overline{A}, \overline{I - A}$ denote the $(n - 1) \times (n - 1)$ matrices obtained from $A, I - A$ by omitting the i -th row and i -th column, and let $\overline{\sum M}, \overline{u}$ be the column vectors obtained from $\sum M, u$ by omitting the i -th entry (note that \overline{u} is an $n - 1$ unit vector). Since $u = (I - A) \sum M$ and $\sum M_i = 0$, we obtain

$$\overline{u} = (\overline{I - A}) \overline{\sum M}. \quad (\text{A-9})$$

Denote $\overline{B} = (\overline{I - A})^{-1}$ and observe that $\overline{B} = I + \overline{A} + \overline{A}^2 \dots \geq 0$. Multiplying (A-9) by \overline{B} ,

$$\overline{B}\overline{u} = \overline{\sum M}.$$

Recalling that \overline{u} is a unit vector, we get $\overline{\sum M} \geq 0$; since $\sum M_i = 0$, it follows that $\sum M \geq 0$.

In order to obtain a contradiction, we compute the s -th entry of $u \equiv (I - A) \sum M$. Since $\sum M \geq 0$ and $\sum M_s \leq 0$, we must have $\sum M_s = 0$. Observe that the only positive entry in the s -th row of $I - A$ is the diagonal term $1 - \alpha_s^s$. Thus, the s -th entry of $u \equiv (I - A) \sum M$ is nonpositive. This contradicts (A-8). ■

Proof of Theorem 1: First, note that if $b_{i,r} = 0$, then $\varepsilon_i = \frac{\Delta b_{i,r}}{1 - \Delta b_{s,r}} = 0$. Second, note that if $i = s$, then $\sum B_s = \frac{b_{s,i}}{b_{i,i}} \sum B_i$. In both cases, equation (A-5) above implies that $\hat{\delta}_i(A) = \hat{\delta}_i(A')$.

Next, assume that $i \neq s$ and $b_{i,r} \neq 0$. In Lemma A1 we show that $\Delta b_{s,r} < 1$ and hence get $\varepsilon_i > 0$. By Lemma A3, $B_s - \frac{b_{s,i}}{b_{i,i}} \sum B_i > 0$, so $\hat{\delta}_i(A) > \hat{\delta}_i(A')$. This immediately implies that $\hat{\delta}^{po}(A') < \hat{\delta}^{po}(A)$. ■

The fully symmetric case: Suppose that $\alpha_i^j = \overline{\alpha}$ for all $i = 1, \dots, n$ and all $j \neq i$. Then, system (2) has a symmetric solution

$$\pi_i = \frac{\pi^m}{n(1 - (n-1)\overline{\alpha})}, \quad i = 1, \dots, n. \quad (\text{A-9})$$

If firm i 's controller deviates from the fully collusive scheme, then system (3) can be written as

$$\begin{aligned} \pi_i^{d_i} &= \pi^m + (n-1)\overline{\alpha}\pi_j^{d_i}, \\ \pi_j^{d_i} &= \overline{\alpha}\pi_i^{d_i} + (n-2)\overline{\alpha}\pi_j^{d_i}, \quad j = 1, \dots, n, \quad j \neq i. \end{aligned}$$

Solving this system for $\pi_i^{d_i}$ yields,

$$\pi_i^{d_i} = \frac{(1 - (n-2)\overline{\alpha})\pi^m}{(1 - (n-1)\overline{\alpha})(1 + \overline{\alpha})}. \quad (\text{A-10})$$

Substituting from (A-9) and (A-10) into equation (8) in the text yields equation (9) in the text. ■

Proof of Proposition 2: Given that $\alpha_i^j = \bar{\alpha}$ for all $i \neq 1$ and all $j \neq i$, system (2) can be written as

$$\begin{aligned}\pi_1 &= \frac{\pi^m}{n} + \alpha_1^2 \pi_2 + \alpha_1^3 \pi_3 + \cdots + \alpha_1^n \pi_n, \\ \pi_j &= \frac{\pi^m}{n} + \bar{\alpha} \sum_{k \neq j} \pi_k,\end{aligned}$$

where $\sum_{j \neq 1} \alpha_1^j = (n-1)\bar{\alpha} + \Delta$. By symmetry, $\pi_2 = \dots = \pi_n$; hence, the solution of the system is

$$\begin{aligned}\pi_1 &= \frac{(1 + \bar{\alpha} + \Delta) \frac{\pi^m}{n}}{H - \bar{\alpha}\Delta}, \\ \pi_j &= \frac{(1 + \bar{\alpha}) \frac{\pi^m}{n}}{H - \bar{\alpha}\Delta}, \quad j = 2, \dots, n,\end{aligned}\tag{A-11}$$

where $H \equiv (1 - (n-1)\bar{\alpha})(1 + \bar{\alpha})$.

We now need to compute the profit that each firm obtains when its controller deviates from the fully collusive scheme. If firm 1's controller deviates, then system (3) becomes

$$\begin{aligned}\pi_1^{d_1} &= \pi^m + ((n-1)\bar{\alpha} + \Delta) \pi_j^{d_1}, \\ \pi_j^{d_1} &= \bar{\alpha} \pi_1^{d_1} + (n-1)\bar{\alpha} \pi_j^{d_1}, \quad j = 2, \dots, n.\end{aligned}$$

Solving for $\pi_1^{d_1}$ yields,

$$\pi_1^{d_1} = \frac{(1 - (n-2)\bar{\alpha}) \pi^m}{H - \bar{\alpha}\Delta}.\tag{A-12}$$

From (A-11) and (A-12) it follows that

$$\hat{\delta}_1 \equiv 1 - \frac{\pi_1}{\pi_1^{d_1}} = \hat{\delta} - \frac{(n-1)\bar{\alpha} + \Delta}{n(1 - (n-2)\bar{\alpha})}.\tag{A-13}$$

If the controller of some firm $i \neq 1$ deviates from the fully collusive scheme, then system (3) can

be written as

$$\begin{aligned}\pi_1^{d_i} &= \alpha_1^i \pi_i^{d_i} + (\bar{\alpha}(n-1) + \Delta - \alpha_1^i) \pi_j^{d_i}, \\ \pi_i^{d_i} &= \pi^m + \bar{\alpha} \pi_1^{d_i} + (n-2) \bar{\alpha} \pi_j^{d_i}, \\ \pi_j^{d_i} &= \bar{\alpha} \pi_1^{d_i} + \bar{\alpha} \pi_i^{d_i} + (n-3) \bar{\alpha} \pi_j^{d_i}, \quad j = 2, \dots, n, \quad j \neq i.\end{aligned}$$

Solving this system for $\pi_i^{d_i}$ yields,

$$\pi_i^{d_i} = \frac{(H - \bar{\alpha}\Delta + \bar{\alpha}(1 + \alpha_1^i)) \pi^m}{(1 + \bar{\alpha})(H - \bar{\alpha}\Delta)}, \quad i \neq 1. \quad (\text{A-14})$$

From (A-11) and (A-14) it follows that

$$\widehat{\delta}_i \equiv 1 - \frac{\pi_i}{\pi_i^{d_i}} = \widehat{\delta} - \frac{\bar{\alpha}((n-1)(1 + \bar{\alpha}) + \bar{\alpha} - \alpha_1^i + \Delta)}{n(H + \bar{\alpha}(1 + \alpha_1^i - \Delta))}. \quad (\text{A-15})$$

To compare (A-13) and (A-15), note that holding Δ constant, $\widehat{\delta}_i$ is increasing with α_1^i and hence is minimized at $\alpha_1^i = \bar{\alpha}$, i.e., when the increase in firm 1's PCOs is in firms other than i . Now, for all $i \neq 1$,

$$\widehat{\delta}_i \Big|_{\alpha_1^i = \bar{\alpha}} - \widehat{\delta}_1 = \frac{\Delta(H - \bar{\alpha}\Delta)}{n(1 - (n-2)\bar{\alpha})(H - \bar{\alpha}\Delta + \bar{\alpha}(1 + \bar{\alpha}))}. \quad (\text{A-16})$$

If $\Delta \geq 0$, then $\widehat{\delta}_i > \widehat{\delta}_1$ for all values of α_1^i and all $i \neq 1$. Now suppose that firm 1's largest PCO is in firm i so that $\alpha_1^i \geq \alpha_1^j$ for all $j \neq 1$. Since $\widehat{\delta}_i$ is increasing with α_1^i , $\max\{\widehat{\delta}_2, \widehat{\delta}_3, \dots, \widehat{\delta}_n\} = \widehat{\delta}_i$. That is, firm i is the industry maverick and $\widehat{\delta}^{po} = \widehat{\delta}_i$. When either $\Delta = 0$ (firm 1 does not increase its stake in rivals so that $\alpha_1^i = \bar{\alpha}$) or $\alpha_1^i = \bar{\alpha} + \Delta$ (firm 1 increases its ownership stake only in firm j), $\widehat{\delta}_i$ coincides with the expression in equation (9). Otherwise, since $\widehat{\delta}_i$ decreases with Δ , tacit collusion is facilitated when firm 1 increases its aggregate stake in rivals. Since $\widehat{\delta}_i$ increases with α_1^i , tacit collusion is particularly facilitated when Δ is spread equally among all of its rivals in which case, for every Δ , α_1^i is minimal and equal to $\bar{\alpha} + \frac{\Delta}{n-1}$.

By contrast, if $\Delta < 0$, then (A-15) implies that $\widehat{\delta}_i$ is maximized at $\alpha_1^i = \bar{\alpha}$, i.e., whenever firm 1 lowers its ownership stake in firms other than firm i . Moreover, (A-16) shows that $\widehat{\delta}_i < \widehat{\delta}_1$ for all $i \neq 1$. Consequently, $\widehat{\delta}^{po} = \widehat{\delta}_1$. From (A-12) it is easy to see that $\widehat{\delta}_1$ is increasing as Δ

falls, implying that tacit collusion is hindered. ■

Proof of Proposition 3: Given the transfer of ownership stake in firm 3 from firm 2 to firm 1, system (2) becomes

$$\begin{aligned}
\pi_1 &= \frac{\pi^m}{n} + \bar{\alpha}\pi_2 + (\bar{\alpha} + \Delta)\pi_3 + \dots + \bar{\alpha}\pi_n, \\
\pi_2 &= \frac{\pi^m}{n} + \bar{\alpha}\pi_1 + (\bar{\alpha} - \Delta)\pi_3 + \dots + \bar{\alpha}\pi_n, \\
&\vdots \\
\pi_n &= \frac{\pi^m}{n} + \bar{\alpha}\pi_1 + \bar{\alpha}\pi_2 + \dots + \bar{\alpha}\pi_{n-1}.
\end{aligned} \tag{A-17}$$

By symmetry, $\pi_3 = \dots = \pi_n$; hence, the solution of the system is given by

$$\begin{aligned}
\pi_1 &= \frac{(1 + \bar{\alpha} + \Delta)\pi^m}{nH}, & \pi_2 &= \frac{(1 + \bar{\alpha} - \Delta)\pi^m}{nH}, \\
\pi_i &= \frac{\pi^m}{n(1 - (n-1)\bar{\alpha})}, & i &= 3, \dots, n.
\end{aligned} \tag{A-18}$$

If the controller of firm 1 deviates from the fully collusive scheme, then system (A-17) needs to be modified by replacing $\frac{\pi^m}{n}$ with π^m in the first line of the system and replacing $\frac{\pi^m}{n}$ with 0 in all other lines. Solving the modified system for firm 1's profit yields,

$$\pi_1^{d_1} = \frac{((1 - (n-2)\bar{\alpha})(1 + \bar{\alpha}) + \bar{\alpha}\Delta)\pi^m}{H(1 + \bar{\alpha})}. \tag{A-19}$$

Using (A-18) and (A-19) yields

$$\hat{\delta}_1(\Delta) \equiv 1 - \frac{\pi_1}{\pi_1^{d_1}} = \hat{\delta} - \frac{\bar{\alpha}(1 + \bar{\alpha})(n-1) + \Delta}{n((1 - (n-2)\bar{\alpha})(1 + \bar{\alpha}) + \bar{\alpha}\Delta)}. \tag{A-20}$$

Likewise, if firm 2's controller deviates, the solution to the modified system (A-17) is such that

$$\pi_2^{d_2} = \frac{((1 - (n-2)\bar{\alpha})(1 + \bar{\alpha}) - \bar{\alpha}\Delta)\pi^m}{H(1 + \bar{\alpha})}. \tag{A-21}$$

Using (A-18) and (A-21) yields

$$\widehat{\delta}_2(\Delta) \equiv 1 - \frac{\pi_2}{\pi_2^{d_2}} = \widehat{\delta} - \frac{\bar{\alpha}(1 + \bar{\alpha})(n - 1) - \Delta}{((1 - (n - 2)\bar{\alpha})(1 + \bar{\alpha}) - \bar{\alpha}\Delta)}.$$

And, if the controller of some firm $i \neq 1, 2$ deviates, the solution to the modified system (2) shows that its profit, $\pi_i^{d_i}$, is equal to the right-hand side of (11). Since the collusive profit of firm $i \neq 1, 2$ in (A-20) is equal to the right-hand side of (A-9), it follows that $\widehat{\delta}_i(\Delta) = \widehat{\delta}^{po}$ for all $i \neq 1, 2$, where $\widehat{\delta}^{po}$ is given by (9).

Now note that (i) $\widehat{\delta}_1(\Delta) = \widehat{\delta}_2(-\Delta)$, (ii) $\widehat{\delta}_1(0) = \widehat{\delta}_i(\Delta)$, and (iii) $\widehat{\delta}'_1(\Delta) < 0$. Since $\Delta > 0$, it follows that $\widehat{\delta}_2(\Delta) > \widehat{\delta}_i(\Delta) > \widehat{\delta}_1(\Delta)$. Hence, the critical discount factor above which the fully collusive outcome can be sustained as a subgame perfect equilibrium of the infinitely repeated game is $\widehat{\delta}_2(\Delta)$. Since $\widehat{\delta}_2(\Delta) > \widehat{\delta}_i(\Delta) = \widehat{\delta}^{po}$, it follows that tacit collusion is hindered. ■

Proof of Proposition 4: Let $B' = (I - A')^{-1}$ and suppose that $i = s$ or i is such that $b_{i,r} = 0$. Using equation (13) and recalling from Theorem 1 that $\widehat{\delta}_i(A') = \widehat{\delta}_i(A)$ for all i such that $b_{i,r} = 0$ or $i = s$ yields

$$\begin{aligned} \widehat{\delta}_i^c(A') - \widehat{\delta}_i^c(A) &= \left(\widehat{\delta}_i(A) - \widehat{\delta}_i^c(A) \right) - \left(\widehat{\delta}_i(A') - \widehat{\delta}_i^c(A') \right) \\ &= \frac{\frac{1}{n} \sum_{j \neq i} \beta_i^j \left(\sum B_j - \frac{b_{j,i}}{b_{i,i}} \sum B_i \right)}{\beta_i b_{i,i} + \sum_{j \neq i} \beta_i^j b_{j,i}} - \frac{\frac{1}{n} \sum_{j \neq i} \beta_i^j \left(\sum B'_j - \frac{b'_{j,i}}{b'_{i,i}} \sum B'_i \right)}{\beta_i b'_{i,i} + \sum_{j \neq i} \beta_i^j b'_{j,i}}. \end{aligned} \quad (\text{A-22})$$

Notice that since $\widehat{\delta}_i(A') = \widehat{\delta}_i(A)$, equation (8) implies that $\frac{\sum B'_i}{b'_{i,i}} = \frac{\sum B_i}{b_{i,i}}$ (recall that $\sum_{j=1}^n b_{i,j} \equiv \sum B_i$). Hence,

$$\begin{aligned} \sum B'_j - \frac{b'_{j,i}}{b'_{i,i}} \sum B'_i &= \sum B'_j - \frac{b'_{j,i}}{b_{i,i}} \sum B_i \\ &= \sum B_j + \varepsilon_j \sum B_s - \frac{b_{j,i} + \varepsilon_j b_{s,i}}{b_{i,i}} \sum B_i \\ &= \sum B_j - \frac{b_{j,i}}{b_{i,i}} \sum B_i + \frac{\Delta b_{j,r}}{1 - \Delta b_{s,r}} \left(\sum B_s - \frac{b_{s,i}}{b_{i,i}} \sum B_i \right), \end{aligned} \quad (\text{A-23})$$

where the second equality follows since by Lemma A1 in the Appendix, $b'_{j,i} = b_{j,i} + \varepsilon_j b_{s,i}$, and

the third equality follows since $\varepsilon_j = \frac{\Delta b_{j,r}}{1 - \Delta b_{s,r}}$. Moreover, given that $b'_{j,i} = b_{j,i} + \frac{\Delta b_{j,r}}{1 - \Delta b_{s,r}} b_{s,i}$,

$$\begin{aligned}
\beta_i b'_{i,i} + \sum_{j \neq i} \beta_i^j b'_{j,i} &= \beta_i \left(b_{i,i} + \frac{\Delta b_{i,r}}{1 - \Delta b_{s,r}} b_{s,i} \right) + \sum_{j \neq i} \beta_i^j \left(b_{j,i} + \frac{\Delta b_{j,r}}{1 - \Delta b_{s,r}} b_{s,i} \right) \\
&= \left(\beta_i b_{i,i} + \sum_{j \neq i} \beta_i^j b_{j,i} \right) + \left(\beta_i \frac{\Delta b_{i,r}}{1 - \Delta b_{s,r}} + \sum_{j \neq i} \beta_i^j \frac{\Delta b_{j,r}}{1 - \Delta b_{s,r}} \right) b_{s,i} \quad (\text{A-24}) \\
&= \left(\beta_i b_{i,i} + \sum_{j \neq i} \beta_i^j b_{j,i} \right) + \frac{\Delta b_{s,i}}{1 - \Delta b_{i,r}} \left(\beta_i b_{i,r} + \sum_{j \neq i} \beta_i^j b_{j,r} \right).
\end{aligned}$$

Now, suppose that $i = s$. Then equation (A-23) implies that $\sum B'_j - \frac{b'_{j,s}}{b'_{s,s}} \sum B'_s = \sum B_j - \frac{b_{j,s}}{b_{s,s}} \sum B_s$, while equation (A-24) implies that $\beta_s b_{s,s} + \sum_{j \neq s} \beta_s^j b_{j,s} \geq \beta_s b'_{s,s} + \sum_{j \neq s} \beta_s^j b'_{j,s}$ with strict inequality whenever $\beta_s b_{s,r} + \sum_{j \neq s} \beta_s^j b_{j,r} > 0$. Together with (A-22), it follows that $\hat{\delta}_s^c(A') \geq \hat{\delta}_s^c(A)$. Clearly, if $\beta_s^j = 0$ for all $j \neq s$ (firm s 's controller does not invest in any of firm s 's rivals), then by (A-22), $\hat{\delta}_s^c(A') = \hat{\delta}_s^c(A)$. If $\beta_s^j > 0$ for some $j \neq s$, then the inequality is strict unless $\beta_s b_{s,r} + \sum_{j \neq s} \beta_s^j b_{j,r} = 0$.

Next, suppose that $i \neq s$ but i is such that $b_{i,r} = 0$. If in addition $\sum_{j \neq i} \beta_i^j b_{j,r} = 0$, then by (A-24), $\beta_i b'_{i,i} + \sum_{j \neq i} \beta_i^j b'_{j,i} = \beta_i b_{i,i} + \sum_{j \neq i} \beta_i^j b_{j,i}$. Moreover, using (A-23),

$$\begin{aligned}
\sum_{j \neq i} \beta_i^j \left(\sum B'_j - \frac{b'_{j,i}}{b'_{i,i}} \sum B'_i \right) &= \sum_{j \neq i} \beta_i^j \left(\sum B_j - \frac{b_{j,i}}{b_{i,i}} \sum B_i \right) \\
&\quad + \frac{\Delta}{1 - \Delta b_{s,r}} \sum_{j \neq i} \beta_i^j b_{j,r} \left(\sum B_s - \frac{b_{s,i}}{b_{i,i}} \sum B_i \right) \quad (\text{A-25}) \\
&= \sum_{j \neq i} \beta_i^j \left(\sum B_j - \frac{b_{j,i}}{b_{i,i}} \sum B_i \right).
\end{aligned}$$

Hence, it follows from (A-22) that $\hat{\delta}_i^c(A') = \hat{\delta}_i^c(A)$. \blacksquare

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