DOES A RISE IN MAXIMAL FINES INCREASE OR DECREASE THE OPTIMAL LEVEL OF DETERRENCE?

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The economic literature on crime and law enforcement shows that the optimal level of deterrence always increases when maximal fines rise. This paper shows that this view may be incorrect. In particular, if the gains from crime can be disgorged, as is usually the case in reality, then increasing maximal fines may reduce the optimal level of deterrence. This may happen if offenders' wealth is less than the monetary value of the harm that offenders cause.

1. INTRODUCTION

Many private and business activities generate external harm. For example, oil refineries generate air pollution, overloaded trucks damage highways, messenger services obstruct traffic as they double park, and individuals sometimes litter the streets or speed while driving. To control these harms, the government usually expands resources to apprehend offenders and impose monetary sanctions on them. Since the pioneering work of Becker (1968), there has been an extensive literature exploring the features of optimal law enforcement, in particular of the optimal probability and magnitude of fines and the optimal level of deterrence.[1] This literature gives rise to several basic results. It shows that, under certain conditions, fines should be set to their maximum level, traditionally interpreted as offenders' wealth, and that the probability of punishment should be set so that under-deterrence prevails, meaning that some offenders commit the harmful act even though the harm from doing so exceeds the gains. The explanations of these results are now well-known. If fines, which are assumed to be a socially costless transfer, were not

[1] See, for example, Garoupa (1997), Polinsky and Shavell (1999, 2007).

maximal, any level of deterrence, including the optimal level, could be achieved at lower costs by increasing the fine to its maximum level and reducing the probability of punishment proportionally. This is Becker's argument. In this system, under-deterrence prevails, because at first-best deterrence, the marginal offenders derive gains that are approximately equal to the harm. Therefore, enforcement costs can be saved by lowering the probability of punishment without affecting much social harm.[2] This is Polinsky and Shavell's point. These results have been qualified in the literature on various grounds, including risk aversion, marginal deterrence, avoidance efforts, and more (see, for example, Garoupa, 1997, Polinsky and Shavell, 1999, 2007).

The literature on crime and law enforcement also explores the consequences of relaxing the fine constraint, for example, imagining that the wealth of potential offenders has been increased. This literature demonstrates that as maximal fines rise, optimal fines should rise as well, but the optimal probability of punishment may either fall or rise depending on the degree of under-deterrence (Garoupa, 2001). This latter, less familiar result can be explained as follows. If the level of under-deterrence is substantial, greater deterrence can and should be achieved not only by increasing the optimal fine, but also by increasing the optimal probability of punishment. The additional gains from increasing the probability of punishment can now be cost-justified, because as optimal fines increase, the deterrent value of enforcement efforts increases as well. To illustrate, when optimal fines equal 400, any 1% increase in the probability of punishment has an impact on deterrence of 4

[2] Another way to explain this is by noting that at the optimum, the marginal costs and benefits from increasing the probability of punishment should be equal. Since marginal costs are clearly positive, the marginal benefits must be positive as well. But the marginal benefits stem from increased deterrence, which means that at the optimum some inefficient crime should go unpunished.

(1% times 400), while if optimal fines equal 500, any such 1% increase has an impact of 5 (1% times 500).[3]

Finally, the literature on crime and law enforcement shows that as maximal fines increase, the optimal level of deterrence always increases and approaches first-best deterrence (Garoupa 2001). The explanation is this: as maximal fines rise, the pre-fine-increase optimal level of deterrence can be achieved by increasing the fine and reducing the probability of punishment proportionally. However, since the deterrent value of enforcement efforts is greater, it is never socially desirable to reduce the probability of punishment all the way down so as to maintain the pre-fine-increase optimal level of deterrence. Instead, since under-deterrence prevails, it is socially desirable to achieve greater deterrence. This result is so intuitive that it is sometimes regarded as a "folk theorem" (Garoupa 1997).

This paper shows that this "folk theorem" is not generally correct. In particular, if the gains from crime can be disgorged, as is the case in many real situations, then increasing maximal fines may paradoxically *decrease* the optimal level of deterrence and consequently *increase* the optimal level of crime. This may happen if maximal fines, which for clarity will be understood not to include the disgorgement of gains, are less than the monetary value of the harm caused by the crime.

The explanation for this surprising result, which is discussed in greater detail in Section 3, lies with the impact of an increase in maximal fines on the deterrent value of enforcement efforts after fines are increased and the probability of punishment reduced to maintain the pre-fine-increase optimal level of deterrence. As noted, in the standard law

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[3] If the level of under-deterrence is not substantial, then the optimal probability of punishment should decrease. The reason is that much deterrence is gained by the mere fact that the optimal fines are increased. See further discussion in Section 3.2.

enforcement model, in which offenders' gains are not disgorged, the deterrent value of any 1% increase in the probability of punishment is 1% times the level of the optimal fine. In contrast, if offenders' gains from harmful acts can be disgorged, the deterrent value of enforcement effort is markedly different. As will be explained in Section 3, the deterrent value of 1% increase in the probability of punishment is not only greater than 1% times the optimal fine (since the probability of punishment applies not only to fines but also to the disgorgement of gains) but it is actually also an increasing convex function of the probability of punishment itself. So, for example, if the optimal fine is 400 and the probability of punishment is 33%, it can be shown that the deterrent value is 9; if the probability of punishment is 50%, the deterrent value is 16; and if the probability of punishment is 67% then the deterrent value is 36. Compare these deterrent values with the deterrent value in the standard model which is 4.[4]

Now, as maximal fines increase, the pre-fine-increase optimal level of deterrence can be achieved by increasing the optimal fines to the new maximum level and reducing the probability of punishment appropriately. The increase in optimal fines increases the deterrent value of enforcement efforts. In contrast, the decrease in the probability of punishment reduces the deterrent value of enforcement efforts. If the latter effect is greater than the former effect, then the deterrent value of enforcement efforts at the pre-fine-increase optimal level of deterrence will be reduced. This implies that it would be socially desirable to lower the enforcement efforts, so as to achieve a lower level of deterrence.

[4] The formula, which is derived and discussed in Section 3.1, is $\frac{f_{\max}}{(1-p)^2}$.

It should be observed that an increase in maximal fines unequivocally increases social welfare or, equivalently, reduces the social costs of crime and law enforcement. The reason is that as maximal fines rise, any level of deterrence can be obtained at lower costs. However, as this paper claims, it might nevertheless be socially desirable to achieve less deterrence, because the possible savings in enforcement costs may outweigh the additional social costs associated with less deterrence and more crime.

This paper does not argue that it is common that the optimal level of deterrence falls as maximal fines rise. However, the possible positive relationship between maximal fines and optimal crime levels should not be dismissed for several reasons. First, in many situations, offenders' gains from harmful acts take a monetary or monetary-like form and therefore can be confiscated or disgorged. For example, the gains from all business activities generating external harm are monetary in nature and therefore can be subject to disgorgement. This is in contrast to the standard law enforcement model which assumes that the benefits from harmful acts are non-monetary in nature and therefore cannot be disgorged. Second, as discussed extensively by Bowels etc (2000, 2005), legislation enabling courts to confiscate or remove illegal gains has grown rapidly across a wide range of countries within both civil and common law systems; thus, the possibility to disgorge illegal gains is now a standard feature of the law. Third, the possibility that a rise in maximal fines would lead to lower levels of optimal deterrence might seem unrealistic because, as pointed out, it requires that maximal fines (the wealth of offenders) would be less than the harm caused by the crime. In reality however, this might be a very common situation. Usually, if offenders' wealth is substantially less than the value of the harm, a substantial degree of under-deterrence will result, unless other forms of punishment such as imprisonment are employed. Indeed, under circumstances of low wealth levels in comparison to harm, the use of imprisonment is generally justified (see Shavell, 1985, Posner, 1985). However, as will be explained in Section 2, if the gains from harmful acts can be disgorged, first-best or approximately first-best behavior can be achieved *even if* offenders' wealth is substantially less than the harm.

This paper analyzes a simple model of crime and law enforcement which assumes that the gains from harmful acts are monetary or monetary-like in nature and therefore can be disgorged. The model follows a model proposed by Bowels et la. (2000). However, Bowels et la. (2000) focus on deriving the optimal fines and the optimal level of disgorgement of gains in their analysis. They neither explore the optimal probability of punishment or level of deterrence, nor analyze how they change when maximal fines rise. This paper provides such an analysis and shows that, aside from the ambiguous, counter-intuitive effect a rise in maximal fines has on the optimal level of deterrence, the other canonical results of the standard law enforcement model qualitatively carry over to the present model. For example, even if the gains from crime can be disgorged, optimal law enforcement is characterized by some degree of under-deterrence.

Finally, this paper also compares the optimal law enforcement schemes with and without disgorgement of gains, a task which is not fully conducted by Bowels et la. (2000). It shows, for example, that the possibility of disgorging offenders' gains not only increases social welfare, but also unambiguously increases the optimal level of deterrence, bringing it closer to first-best deterrence. Interestingly then, fines and disgorgement of gains affect optimal deterrence differently: When the level of maximal fines increases, the optimal level of deterrence may either increase or decrease, while if a greater fraction of offenders' illegal gains can be disgorged, the optimal level of deterrence is always higher.

The paper proceeds as follows. Section 2 develops a simple model of crime and law enforcement incorporating the option to disgorge the gains that offenders derive from crime, and depicts the optimal law enforcement scheme. Section 3 then analyzes how optimal law enforcement should change as maximal fines rise and derives the main results of this paper. Section 4 compares the optimal law enforcement schemes with and without the possibility to disgorge offenders' gains. This section shows that as the fraction of disgorging illegal gains increases, the level of optimal deterrence always increases. Section 5 summarizes the analysis and the Appendix provides formal proofs for the results.

2. THE MODEL

Risk-neutral individuals (or firms) contemplate whether to commit a harmful act, causing harm of h. Each individual obtains monetary or monetary-like gains g, which are assumed to be randomly chosen from a continuous distribution function with a density function $b(g)$ and a cumulative distribution function $B(g)$ on the support $[0, \hat{g})$. Assume that $\hat{g} > h$, so that some harmful acts are socially desirable in the sense that the gains for certain individuals exceed the harm. This assumption however is not crucial for our qualitative results.

If an individual does commit the harmful act, he will face some probability of being fined and will also risk losing his illicit gains. For clarity, let us define fines as not including the disgorgement of gains. The maximum feasible fine f, so defined, is constrained to $f_{\max}$. Traditionally, $f_{\max}$ is interpreted as the level of wealth of individuals above a subsistence level. Alternatively, it can be interpreted as the maximal fine which is allowed by law, for example, for constitutional considerations. Under this interpretation, $f_{\max}$ serves as an exogenous legal or constitutional constraint on the level of fines.

In addition to being fined, offenders risk losing the gains they derive from the harmful act. For clarity, let us use the term "punishment" to mean both the fine and the disgorgement of the illicit gains. Assume then that "punishment", so defined, must be uniform for all potential offenders in the sense that it takes the form $f + \eta g$, where $f \leq f_{\max}$ and η [0,1] is the fraction of the illicit gains that can be confiscated. This will allow the existence of a unique solution to the social problem. Denoting different individuals by the index i, the maximum feasible punishment for each individual must be less than $f_{\max} + g_i$, implying that punishment can differ across individuals. As usual, fines are assumed to be a socially costless transfer: they entail no administration costs, and the costs imposed on offenders are completely offset by the revenues obtained by the government. The same, it is assumed, applies to the disgorgement of the illicit gains.[5]

The costs of apprehending offenders with probability p are given by a function $c(p)$, which exhibits the usual characteristics $c'(p) > 0$ and $c''(p) \geq 0$. That is, the costs of apprehension increase with p with increasing or constant rates.

[5] Observe that these two assumptions need not be strictly so. First, while fines are socially costless to administer, disgorgement may nevertheless be socially costly (see the treatment in Bowels ~~etc~~'et la. (2000). Second, sometimes the loss to offenders is not offset by the gains to the government. For example, if the illicit gains are monetary-like in nature and take the form of illegal goods such as drugs, confiscation has no value in the hands of the government (the policy will take steps to destroy the illegal goods). However, in most cases, for example, with respect to stolen goods, the assumptions in the text seem to apply.

Social welfare or, equivalently, the social cost of crime and law enforcement, is the sum of gains obtained by those individuals who commit the harmful act, less the harm done, less enforcement costs. To determine social welfare, observe that individuals will commit the harmful act if and only if the gains they derive from doing so exceed the expected punishment they face, that is, iff

(1) $g \geq p(f + \eta g)$,

which means that the level of deterrence is determined by:

(2) $\tilde{g} \equiv \frac{p}{1-\eta p} f$.

Hence, social welfare can be formulated as:

(3) $SW = \int_{\tilde{g}}^{\hat{g}} [g-h]b(g)dg - c(p)$.

The social problem is to choose the probability of punishment p, the magnitude of fines f, and the fraction of disgorgement of illegal gains η that maximize (3). The solution to this problem is characterized in the following proposition.

Proposition 1: (1) The optimal punishment is maximal, that is, the optimal fine and level of disgorgement are maximal $f^* = f_{\max}$ and $\eta^* = 1$ respectively, so that $f_{\max} + g_i$. (2) The optimal probability of punishment, assuming it is positive, satisfies the first order condition $(h - g^*)b(g^*)\frac{f_{\max}}{(1-p^*)^2} = c'(p^*)$, where g^*, is the optimal level of deterrence, and it is such that there is some degree of under-deterrence, that is, $g^* < h$. This also implies that $p^* < \frac{h}{f_{\max} + h}$.

The proof and explanation of Proposition 1 are as follows. The fine should be set at its maximum level, because otherwise it would be possible to increase the fine and reduce the probability of punishment such that the same level of deterrence, including the optimal level of deterrence, would be obtained at lower enforcement costs. The same argument applies with respect to the level of disgorgement, which should be maximal as well. This is essentially Becker's argument. Since the optimality of maximal fines is well-known, and since its application to the disgorgement of illegal gains has been formally proven by Bowels et la. (2000), we will demonstrate it here again with respect only to the disgorgement of the illicit gains. Suppose to the contrary that $\eta^* < 1$, $f^* = f_{\max}$, and $p^* > 0$. The optimal level of deterrence is thus given by $g^* = \frac{p^*}{1-\eta^* p^*} f_{\max}$ (equation 2). Now increase η to η_1 and change p to $p_1 = \frac{p^*}{1+p^*(\eta_1 - \eta^*)}$, so that the level of deterrence remains the same, $\frac{p_1}{1-\eta_1 p_1} f_{\max} = \frac{p^*}{1-\eta p^*} f_{\max} = g^*$. Since $\eta_1 > \eta^*$, then $p_1 < p^*$, implying that enforcement costs are saved. Therefore, social welfare increases, and $\eta^* < 1$ could not be optimal. Before proceeding, let us note that the reduction in p, which is necessary to maintain the level of deterrence after increasing η, is not proportional to the increase in η. This feature will play a role in the analysis in Section 4.2.

The second result (Proposition 1(2)), which is not discussed or proven by Bowels et la. (2000), stems from the equality between the marginal benefits and costs of increasing the probability of punishment at the optimum. The marginal costs are definitely positive for any $p > 0$, so the marginal benefits must be positive as well. But the marginal benefits of increasing the probability of punishment are the benefits resulting from greater deterrence, that is, from the reduced number of harmful acts. If these marginal benefits are positive, then it must be that at the optimum some inefficient harmful acts are committed. Put differently, at the optimum, some offenders must commit the harmful act even though the gains they derive from doing so are less than the harm they cause. This means that the optimal solution is characterized by some level of under-deterrence. Formally, the optimal probability of punishment, assuming it is positive, should satisfy the first order condition

$$(4) \qquad (h-g^*)b(g^*)\frac{f_{\max}}{(1-p^*)^2}=c'(p^*),$$

where

$$(5) \qquad g^*\equiv\frac{p^*}{1-p^*}f_{\max}$$

is the gain-threshold which determines the optimal level of deterrence, obtained after substituting $f^*=f_{\max}$ and $\eta^*=1$ in equation 2.[6] Since $c'(p)>0$ for any $p>0$, then $h-g^*>0$, which implies that:

$$(6) \qquad g^*<h.$$

Precisely, this means that under-deterrence is optimal. Moreover, substituting (5) into (6), one obtains that:

$$(7) \qquad p^*<\frac{h}{f_{\max}+h}.$$

Proposition 1 (2) can be viewed as a generalization of the standard result in the economic literature regarding the optimality of under-deterrence.[7]

[6] It is assumed throughout that the second order conditions are satisfied.
[7] As is well known, standard optimal law enforcement models which disregard the possibility of confiscating the offenders' gains (since they are non-monetary in nature) show that the optimal expected fine is less than harm, $p^*f^*<h$, and also that the optimal probability of punishment is less than the

Before proceeding, let us show how first-best behavior can be optimally achieved in the present model. Punishment needs to be maximal, that is, $f_{\max} + g_i$, for exactly the same reason discussed above. The probability of punishment, in contrast, should be set at $\frac{h}{f_{\max} + h}$, so that expected punishment is given by:

(8) $$\frac{f_{\max} + g_i}{f_{\max} + h} h .$$

This expected punishment would guarantee, at the lowest costs, that individuals for whom $g \le h$ are deterred, while individuals for whom $g > h$ are not deterred.

In contrast to the standard law enforcement model, in the present model the expected punishment increases with the gains that offenders derive from the harmful act. The expected punishment is less than the harm for individuals for whom $g < h$, and more than the harm if the reverse is true. Nevertheless, the expected punishment increases with the gains from the harmful act, but at a lower rate (i.e., less steeply) than the increase of the gains themselves, as illustrated in Figure 1.[8] Therefore, first-best behavior can be obtained if the expected punishment equals harm for individuals for whom $g = h$, which is achieved by setting the expected punishment according to (8). This too is illustrated in Figure 1.

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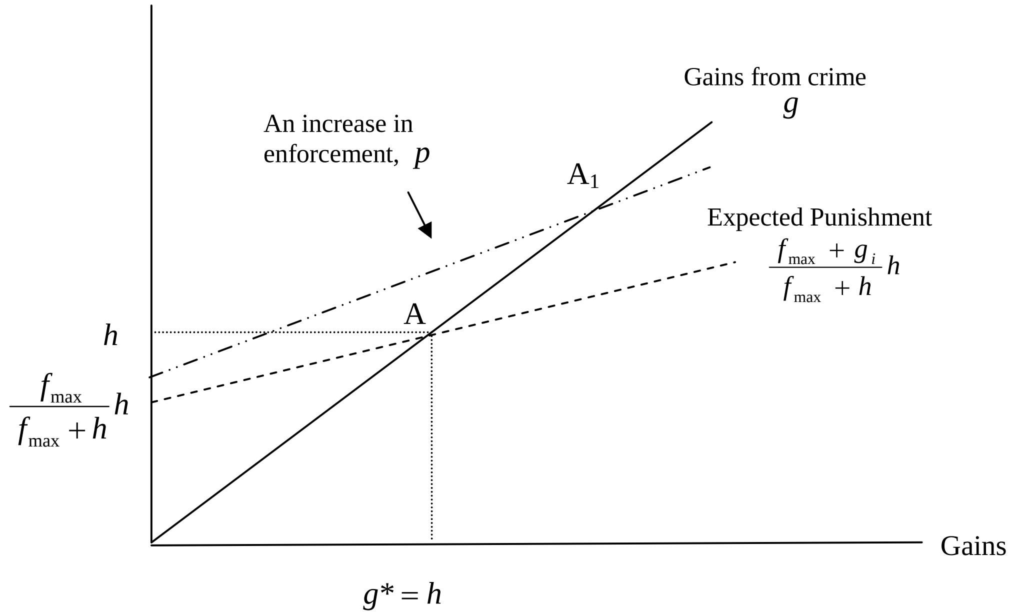
harm divided by offenders' wealth, $p^* < \frac{h}{f_{\max}}$. See, for example, Garoups (1997), Polinsky and Shavell (1999, 2007).

[8] The gains from the harmful act are increased at a rate of 1 with the gains, while the expected punishment is increased at a rate $\frac{h}{f_{\max} + h}$ with the gains, which is less than 1.

Figure 1: How First-Best Deterrence Is Achieved



Observe that in contrast to the standard law enforcement model, in the present model first-best or approximately first-best behavior can be obtained even if $f_{\max}$ is significantly less than the harm caused. For example, if the maximal fine is 400 and the harm is 700, first-best behavior can be induced by setting the probability of punishment at 7/11. The possibility to achieve first-best or approximately first-best behavior even if maximal fines are very low is due precisely to the fact that offenders' gains from the harmful act can be disgorged.

Observe finally that if offenders' wealth is greater than the harm caused, then first-best behavior can be optimally achieved with a probability of punishment which is less than 1/2. Moreover, the optimal probability of punishment in this case will be *necessarily* less than 1/2 (see equation (7)). On the other hand, if offenders' wealth is less than the harm, then first-best behavior can be optimally obtained with a probability of punishment which is greater than 1/2. However, the optimal probability of punishment may be either higher or lower than 1/2 (see again equation 7), depending, among other things, on the costs of enforcement. These observations will play a major role in the following analysis.

3. MAXIMAL FINES RISE

Suppose that maximal fines rise. This can be interpreted as an increase in the wealth of potential offenders, resulting, for example, from an overall increase in the wealth of society. Alternatively, and perhaps more realistically, a rise in maximal fines can be interpreted as relaxing a legal or constitutional constraint on the level of maximal fines. How will optimal law enforcement change?

It needs no explanation that the optimal level of fines should be increased and set equal to the new maximal level. It is also quite obvious that social welfare increases as maximal fines rise, because any level of deterrence can be achieved by increasing the punishment and reducing the probability of punishment appropriately so as to save enforcement costs. The interesting questions are therefore how an increase in maximal fines affects the optimal level of deterrence and how it alters the optimal probability of punishment. Let us analyze these questions in turn.

3.1. The Optimal Level of Deterrence

The effect of increasing the maximal fine on the optimal level of deterrence is obtained by totally differentiating g^* with respect to $f_{\max}$, and is given by:

(9) $$\frac{dg^*}{df_{\max}} = \frac{\partial g^*}{\partial f_{\max}} + \frac{\partial g^*}{\partial p^*}\frac{dp^*}{df_{\max}}$$

Observing that $\frac{\partial g^*}{\partial f_{\max}} = \frac{p^*}{1-p^*}$ and $\frac{\partial g^*}{\partial p^*} = \frac{f_{\max}}{(1-p^*)^2}$, using the Implicit Function Theorem to derive $\frac{dp^*}{df_{\max}}$, and rearranging, we get that (see Appendix A):

(10) $$sign[\frac{dg^*}{df_{\max}}] = sign[(h-g^*)(1-2p^*) + c''(p^*)],$$

To simplify the results, let us assume that the costs of enforcement are proportional to the probability of punishment so that $c''(p) = 0$. Then the following proposition holds.

Proposition 2: *As maximal fines rise, the optimal level of deterrence increases (decreases) if the optimal probability of punishment is less (greater) than 1/2.*

To understand this surprising result, consider the consequences of increasing the maximal fine. As maximal fines rise, the pre-fine-increase optimal level of deterrence, g^*, can be achieved by increasing the optimal fines and reducing the probability of punishment appropriately. Recall that prior to the increase in maximal fines the optimal level of deterrence was determined by the equality between the marginal costs and benefits of enforcement efforts (i.e., the probability of punishment);

(4) $$(h-g^*)b(g^*)\frac{f_{\max}}{(1-p^*)^2} = c'(p^*)$$

At the optimal level of deterrence, the costs of increasing the probability of punishment slightly, say, by 1%, were equal to the benefits from so doing in terms of greater deterrence and, accordingly, reduced net harm. The critical question is how this point is characterized after increasing optimal fines and reducing the probability of punishment appropriately. If the marginal benefits are now greater than the marginal costs, then greater deterrence would be socially desirable. If, however, the reverse is true, then less deterrence would be socially optimal.

Observe that the marginal costs of increasing the probability of punishment are unaffected ($c'(p)$) by the above factors. This, of course, is due to our simplified assumption that the costs of enforcement are proportional to the probability of punishment.[9] In addition, the gains from greater deterrence, for example, from a 1% increase in the level of deterrence, in terms of reduced net social harm, are also the same ($(h-g^*)b(g^*)$). The critical question is therefore whether a slight increase in the probability of punishment now has a greater or a lower impact on the level of deterrence. If it has a greater impact on the level of deterrence, then the marginal benefits will be greater than the marginal costs, and accordingly the optimal level of deterrence should be higher; otherwise, the marginal benefits will be less than the marginal costs, and the optimal level of deterrence should be lower. The impact of a slight increase in the probability of punishment on the level of deterrence, which will be termed *the deterrent value of enforcement efforts*,[10] is formally given by:

$$(11) \qquad \frac{\partial \tilde{g}}{\partial p} = \frac{f}{(1-p)^2}$$

As is evident in equation (11), an increase in the level of maximal fines and, consequently, optimal fines, increases the deterrent value of enforcement efforts for *any* given probability of punishment. This should be very intuitive. However, as equation (11) also reveals, the deterrent value of enforcement efforts also depends on the level of p

[9] If marginal costs of enforcement efforts are increasing, then the reduction in the probability of punishment implies that the marginal costs are now lower.
[10] This term is borrowed from Garoupa (2001).

itself. Indeed, the deterrent value of enforcement efforts increases as the probability of punishment increases, and does so in increasing rates. To illustrate, if $f_{\max} = 400$, then the deterrent value of a 1% increase in the probability of punishment is only 4.04 for $p = 0.01$; it becomes 9 for $p = 1/3$; it rises to 16 for $p = 1/2$; and it reaches 36 for $p = 2/3$. The reason for this lies with the fact that the probability of punishment affects not only the fine, but also the gains offenders derive from the harmful act (see equation (1)). As p increases, a greater fraction of gains are eventually disgorged, and this has a greater impact on the level of deterrence. Graphically, this greater impact can be seen by observing that an increase in the probability of punishment increases the slope of the expected punishment curve and also shifts it up. Therefore, the new level of deterrence (point A_1 at which the expected punishment and the gains curves cross) increases in greater proportions than the increase in p (see Figure 1).

Since the probability of punishment that is needed to maintain the level of deterrence decreases as maximal fines rise, the deterrent value of enforcement efforts, on this account, is decreased. Therefore, we have identified two opposing effects: On the one hand, the deterrent value of enforcement efforts is increased because optimal fines are higher (the fine effect); on the other hand, the deterrent value of enforcement efforts tends to be lower because the probability of punishment which is necessary to achieve the pre-fine-increase optimal level of deterrence is lower (the probability of punishment effect). The total effect depends on the relative magnitudes of these opposing effects.

Proposition 2 implies that these opposing effects cancel out exactly if $p^* = 1/2$, while the fine effect dominates the probability of punishment effect if $p^* < 1/2$, and the reverse is true if $p^* > 1/2$.

To illustrate this numerically, suppose that the optimal fine is 400 and the optimal probability of punishment is 1/2. The optimal level of deterrence is therefore 400 ($\frac{1/2}{1-1/2}400$, equation 5), and the deterrent value of a 1% increase in the probability of punishment is 16 ($\frac{400}{(1-1/2)^2}$X1%, equation 11). Suppose now that the maximal fine increases from 400 to 500, that is, by 25%. In such a case, to achieve the same level of deterrence, the probability of punishment should be decreased to 0.44 $(\frac{p_1}{1-p_1}500 = 400)$. It can be easily verified that the deterrent value of a 1% increase in the probability of punishment remains 16 ($\frac{500}{(1-0.44)^2}$X1%). Suppose however that the optimal probability of punishment were only 1/3. The optimal level of deterrence therefore would be 200 $(\frac{1/3}{1-1/3}400)$ and the deterrent value of a 1% increase in the probability of punishment would be 9 ($\frac{400}{(1-1/3)^2}$X1%). To maintain the same level of deterrence, after optimal fines increase from 400 to 500, the probability of punishment should be reduced to 0.286 **(** $\frac{p_1}{1-p_1}500 = 200$ **)**. Therefore the deterrent value of a 1% increase in p is increased to 9.8 ($\frac{500}{(1-0.286)^2}$X1%). Finally, suppose that the optimal probability of punishment were 2/3. The optimal level of deterrence and the deterrent value of enforcement efforts would be 800 $(\frac{2/3}{1-2/3}400)$ and 36 ($\frac{400}{(1-2/3)^2}$X1%), respectively. An increase in optimal fines from 400 to 500 accompanied by a decrease in the probability of punishment to 0.615

($\frac{p_1}{1-p_1}500 = 800$) to maintain the same level of deterrence, would *reduce* the deterrent value of a 1% increase in the probability of punishment to 33.7 ($\frac{500}{(1-0.615)^2}$ X1%).

Let us examine the conditions under which the two parts of Proposition 2 hold. These, of course, are the same conditions for the optimal probability of punishment to be less or greater than 1/2.[11] As pointed out in Section 2, if the level of the maximal fine is greater than the harm created by the harmful act, the optimal probability of punishment is definitely less than 1/2, implying that in those cases the optimal level of punishment will increase with maximal fines, as is commonly expected. Therefore, a necessary but not sufficient condition for the optimal level of deterrence to *decrease* with maximal fines is that $f_{\max} < h$. This condition, which can be interpreted, for example, as a situation in which an offenders' wealth is less than the harm, is not necessarily uncommon in reality. Indeed, in many situations, potential offenders cause great harm and possess in comparison very little wealth, so they are effectively judgment-proof. In standard law enforcement models which disregard the possibility of disgorging offenders' gains, it is observed that if offenders' wealth is substantially less than the harm, substantial under-deterrence will result, unless other forms of punishment such as imprisonment are utilized. Indeed, a severe problem of under-deterrence justifies the use of expensive forms of punishment such as imprisonment (see Shavell, 1985, Posner, 1985). However, if offenders' gains can be disgorged, a substantial level of under-deterrence does not necessarily result even if offenders' wealth is substantially less than the harm (as

[11] The assumption that enforcement costs are proportional to the probability of punishment is maintained. If the costs of enforcement are convex with the probability of punishment, then there is an additional force to increase the optimal level of deterrence as maximal fines increase. The reason is that at the pre-fine-increase level of deterrence, the marginal costs of increasing the probability of punishment are lower.

demonstrated in Section 2). Indeed, if offenders' wealth is substantially less than the harm, then in order to achieve a significant level of deterrence, the probability of punishment should be relatively high. For example, if offenders' wealth is 400 and the harm is 700, then first-best deterrence requires that the probability of punishment would be 7/11 (63.3%), while if offenders' wealth is only 200, the probability of punishment should be 7/9 (77.7%).

As noted, $f_{\max} < h$ is a necessary but not a sufficient condition for the optimal level of deterrence to decrease as maximal fines rise. In addition, it should be the case that the optimal level of deterrence is rather high, which requires that the costs of enforcement are relatively low. Otherwise, if the level of under-deterrence is sufficiently moderate, the optimal probability of punishment would be relatively low. Indeed, as will be demonstrated in the following section, if under-deterrence is substantial, it would be socially desirable not only to obtain a higher optimal level of deterrence, but to do so by increasing the optimal probability of punishment.

3.2. Optimal Probability of Punishment

Examine how an increase in maximal fines affects the optimal probability of punishment. This is given by (see Appendix B):

(12) $$sign[\frac{dp^*}{df_{\max}}] = sign[h - 2g^* - \varepsilon(g^*)(h - g^*)],$$

where $\varepsilon(g^*) = -\frac{b'(g^*)}{b(g^*)} g^*$ is the distribution elasticity of the gains offenders derive from the harmful act.

Proposition 3: *As maximal fines rise, the optimal probability of punishment increases if* $\varepsilon(g^*) < 1$ *and* $g^* < h\dfrac{\varepsilon(g^*)-1}{\varepsilon(g^*)-2}$*; otherwise it decreases.*

Proposition 3 and its explanation parallel those demonstrated by Garoupa (2001) in a model without disgorgement of gains, and therefore can be viewed as a generalization of his result. To understand Proposition 3, consider the consequences of increasing maximal fines, and accordingly, optimal fines, on the marginal costs and benefits from enforcement efforts evaluated at p^*. Observe first that the marginal costs are, of course, unaffected. Therefore, the critical question is whether the marginal benefits of enforcement efforts increase or decrease. As optimal fines rise, the deterrent value of enforcement efforts is increased. On the other hand, the increase in optimal fines increases the level of deterrence, which means that the gains from further deterrence are reduced. The reason for this is that, as the level of deterrence increases, the marginal offenders impose less net harm, because the gains they derive from committing the harmful act are greater. To illustrate, if the gains threshold that determines the level of deterrence is 600 and the harm is 700, then the marginal offender imposes net harm of only 100 (700 – 600). As the level of deterrence is increased, say, to 650, the marginal offender imposes net harm of only 50 (700 – 650). Therefore, whether the optimal probability of punishment should increase or decrease as a result of an increase in maximal fines depends on the relative magnitude of these two opposing effects. If the level of under-deterrence is low, then the reduction in the gains from more deterrence dominates the increased deterrent effect of enforcement efforts, and calls for reducing the optimal probability of punishment. This possibility is most vivid if the level of deterrence is very high, so that an increase in the level of fines will lead to over-deterrence. In such a case, there are actually losses from further deterrence, so the optimal probability of punishment should definitely decrease. On the other hand, if the level of under-deterrence is substantial, then the increase in the deterrent value of enforcement efforts outweighs the reduction in the gains from more deterrence. Therefore, the social planner should achieve a higher level of deterrence not merely by increasing the optimal fines, but also by increasing the optimal probability of punishment.

Let us finally note that the complementarity between maximal fines and the optimal probability of punishment is less likely to arise in the present model than in the standard model in which offenders' gains cannot be disgorged, even though the effect itself takes the same form in the two models. The reason for this, as the next section shows, is that the optimal level of deterrence is always higher with disgorgement than without it.

4. DISGORGEMENT VERSUS NO-DISGOREGMENT

The previous section analyzes how an increase in maximal fines affects optimal law enforcement if offenders' gains from the harmful act can be disgorged. A natural, analogous question is how the possibility to disgorge offenders' gains affects optimal law enforcement. This question can be interpreted in two ways: one, as a question concerning the comparison between optimal law enforcement with and without disgorgement of offenders' gains, and second, as an inquiry of how increasing the fraction of gains that can be physically (or legally) disgorged affects optimal law enforcement.[12]

[12] The latter view assumes that there is some physical or legal constraint on the fraction of illegal gains that can be disgorged and examines the consequences of relaxing such a constraint; for example, imagine that law enforcement gets better at recovering a larger fraction of offenders' gains. Nevertheless, this paper does

Bowels et la. (2000) provide an important analysis of the removal of illegal gains. However, as pointed out in the Introduction, they focus in their analysis on deriving the optimal fines and the optimal level of disgorgement of gains, assuming alternatively that disgorgement of gains is socially costless or socially costly. Bowels et la. (2000) also suggest that disgorgement of gains is socially desirable and that it allows for the achievement of greater deterrence, but they do not provide a formal proof or a satisfactory explanation with respect to the latter claim.

It should be clear that the possibility of disgorging offenders' illegal gains increases social welfare, because this option allows the social planner to achieve any level of deterrence at lower enforcement costs (see also Bowels et la., 2000). It is also clear that offenders' gains should be disgorged to the maximum extent possible, assuming that disgorgement, like levying fines, is a socially costless transfer (see Bowels et la. (2000) and Proposition 1(1)). Again, the interesting questions which are not formally discussed by Bowels et la. (2000), concern how the possibility of disgorging offenders' gains or a larger fraction of these gains affects the optimal level of deterrence and the probability of punishment.

To facilitate the analysis, observe that if the fraction of gains that can be disgorged is constrained to $\eta_{\max} < 1$, the optimal law enforcement scheme will be characterized by an analogous proposition to Proposition 1: The optimal punishment should be maximal, meaning that the optimal level of fines is maximal $f^* = f_{\max}$ and the optimal fraction of disgorgement of gains is maximal $\eta^* = \eta_{\max}$; The optimal probability of punishment

not examine the costs of disgorging offenders' gains or possible measures of offenders to dispose of their wealth or of their gains.

should satisfy $(h-g^*)b(g^*)\dfrac{f_{\max}}{1-\eta^* p^*}=c'(p^*)$, where $g^*=\dfrac{p^*}{1-\eta^* p^*}f_{\max}$, and under-deterrence prevails, $g^*<h$.

4.1 Optimal Level of Deterrence

Instead of directly comparing the optimal levels of deterrence with and without the possibility of disgorging offenders' gains, let us show that the mere possibility of disgorging a tiny fraction of offenders' gains increases the optimal level of deterrence, or generally, that the optimal level of deterrence increases as the fraction of gains that can be disgorged is increased.

Formally, we differentiate the optimal level of deterrence, $g^*=\dfrac{p^*}{1-\eta^* p^*}f_{\max}$, with respect to η^* and obtain:

(13) $\dfrac{dg^*}{d\eta^*}=\dfrac{\partial g^*}{\partial\eta^*}+\dfrac{\partial g^*}{\partial p^*}\dfrac{dp^*}{d\eta^*}$.

Observing that $\dfrac{\partial g^*}{\partial\eta^*}=\dfrac{p^{*2}\ f_{\max}}{(1-\eta^* p^*)^2}$, and $\dfrac{\partial g^*}{\partial p^*}=\dfrac{f_{\max}}{(1-\eta^* p^*)^2}$, and using the Implicit Function Theorem to derive $\dfrac{dp^*}{d\eta^*}$, we have (see Appendix C):

(14) $sign[\dfrac{dg^*}{d\eta^*}]=sign[(h-g^*)(1-\eta^* p^*)+p^{*2}\ c''(p^*)]$.

Since $c''(p^*)\geq 0$, $g^*<h$, and $(1-\eta^* p^*)>0$, it follows that $\dfrac{dg^*}{d\eta^*}>0$. Thus, the following proposition holds.

Proposition 4: *The optimal level of deterrence increases and gets closer to first-best deterrence as the fraction of offenders' gains that can be disgorged increases. The optimal level of deterrence is therefore higher with disgorgement than without it.*

At a first blush, Proposition 4 is not surprising at all. It seems quite obvious that the possibility of disgorging offenders' gains or a larger fraction of such gains should increase the optimal level of deterrence. However, in light of proposition 2, this result deserves explanation.

When the fraction of illegal gains that can be and is disgorged is increased, the level of deterrence is increased as well. In addition, the pre-disgorgement-increase optimal level of deterrence can be achieved at lower enforcement costs by increasing the level of disgorgement and reducing the probability of punishment appropriately. At that point, the marginal costs of increasing the probability of punishment are unchanged, assuming that enforcement costs are proportional to the probability of punishment. In addition, the marginal benefits from increasing the level of deterrence, in terms of reduced social harm, are also unaffected. Again, the question of whether the optimal level of deterrence should increase or decrease boils down to the change in the deterrent value of enforcement efforts after disgorgement is increased and the probability of punishment is appropriately reduced.

When only a fraction of offenders' gains can be disgorged, the deterrent value of enforcement efforts is generally given by:

(15) $\frac{\partial \tilde{g}}{\partial p} = \frac{f}{(1-\eta p)^2}$.

As equation (15) reveals, the deterrent value of enforcement efforts depends on the product of p and η, that is, on ηp. Therefore, an increase in η "compensated" by a proportional decrease in p will leave the product ηp unchanged, and consequently will not affect the deterrent value of enforcement efforts. For example, an increase of 25% in the fraction of gains that are disgorged, accompanied by a 20% decrease of the probability of punishment, will keep the product ηp and therefore the deterrent value of enforcement efforts unchanged.

However, as demonstrated in Section 2, as η increases, the reduction in the probability of punishment which is required to maintain the optimal level of deterrence is less than proportional.[13] The reason is that the probability of punishment affects not only the gains offenders derive from the harmful act, but also the fine imposed on them. Therefore, as η increases and p is reduced to maintain the pre-disgorgement-increase optimal level of deterrence, the product ηp is actually increased, which means that the deterrent value of enforcement efforts is unequivocally increased. Put differently, the increase in the deterrent value of enforcement efforts as a result of an increase in the fraction of gains that are disgorged (the disgorgement effect) always dominates the reduction in the deterrent value of enforcement efforts on account of the lower probability of punishment (the probability of punishment effect) which is necessary to maintain the level of deterrence. Since the deterrent value of enforcement efforts is higher at the pre-

[13] To illustrate, suppose that $p^* = 0.2$, $\eta^* = 0.5$, and $f_{\max} = 400$; then, deterrence is given by approximately 88.8 ($\frac{0.2 * 400}{1 - 0.2 * 0.5}$). Now suppose that η is increased by 20% from 0.5 to 0.6. To maintain the level of deterrence, the probability of punishment should be reduced from 0.2 to 0.196 ($\frac{0.196 * 400}{1 - 0.196 * 0.6} = 88.8)$, that is by only 2%!

disgorgement-increase optimal level of deterrence, the optimal level of deterrence should increase.[14]

4.2. Optimal Probability of Punishment

The comparison between the optimal probabilities of punishment with and without the disgorgement of offenders' gains can be similarly derived by analyzing the effects of changes in the level of disgorgement on the optimal probability of punishment This is given by (see Appendix D):

$$(16)\quad sign[\frac{dp^*}{d\eta^*}] = sign[2h - 3g^* - \varepsilon(g^*)(h - g^*)]\,.$$

Proposition 5: *As the fraction of illegal gains that can be disgorged increases, the optimal probability of punishment increases if* $\varepsilon(g^*) < 2$ *and* $g^* < h\dfrac{\varepsilon(g^*)-2}{\varepsilon(g^*)-3}$*; otherwise it decreases.*

The explanation of Proposition 5 is analogous to that of Proposition 3, and, therefore, is omitted. Let us note, however, that the complementarity between the disgorgement of offenders' gains and the optimal probability of punishment is in a sense more likely to occur than the complementarity between maximal fines and the optimal probability of punishment. This is so because a necessary condition for Proposition 5 is that $\varepsilon(g^*) < 2$, while a necessary condition for Proposition 3 (or for Garoupa's, 2001, result) is that

[14] In fact, there is another reason why the optimal level of deterrence would increase. If enforcement costs are convex with the probability of punishment, then the costs of enforcement will be lower at the pre-disgorgement-increase optimal level of deterrent, which would also justify attaining a higher optimal level of deterrence.

$\varepsilon(g^*) < 1$. The reason for this is that disgorgement of gains, in comparison to maximal fines, has a greater impact on the deterrent value of enforcement efforts.

4. CONCLUSION AND SUMMARY

This paper analyzes the optimal law enforcement scheme if the gains from harmful acts are monetary or monetary-like in nature and therefore can be subject to disgorgement. It also compares this optimal law enforcement scheme to the optimal law enforcement scheme that prevails if offenders' gains cannot be disgorged (for example, because the gains are non-monetary in nature). An important issue which is analyzed concerns the effects of increasing maximal fines. This issue is significant because it can describe situations in which legal constraints to the maximal fine levels are relaxed, or, alternatively, situations in which offenders' resources are increased. The main results of the analysis are summarized in Tables 1 and Table 2.

Table 1: The Characteristics and Comparison of Optimal Law Enforcement with and without Disgorgement of Illegal Gains

	Fines	Enforcement	Deterrence	Social Welfare
No Disgorgement	f_{max}	$p^* < h / f_{max}$,	Under-deterrence	*Lower*
	same	*higher/lower*	*Lower*	
Disgorgement	f_{max}	$p^* < h(f_{max} + h)$,	Under-deterrence	*Higher*
	same	*lower/higher*	*Higher*	

Table 2: The Effects on Optimal Law Enforcement of Increasing Maximal Fines

	Fines	Enforcement	Deterrence	Social Welfare
No Disgorgement	*Higher*	*Higher/Lower*	*Higher*	*Higher*

Disgorgement	*Higher*	*Higher/lower*	*Higher/Lower*	*Higher*

As is evident in Table 1, the possibility of disgorging offenders' gains increases social welfare and the optimal level of deterrence. In addition, it does not affect the main qualitative results associated with the standard model in which disgorgement of gains is not an option. However, as evident from Table 2, an increase in maximal fines may paradoxically *decrease* the optimal level of deterrence and consequently *increase* the optimal level of crime if offenders' gains can be disgorged.

APPENDIX A

Proof Proposition 2:

To examine how the level of deterrence changes with the maximal fine, we inquire:

(A1) $\frac{dg*}{df_{\max}} = \frac{\partial g*}{\partial f_{\max}} + \frac{\partial g*}{\partial p*}\frac{dp*}{df_{\max}}$

Where $g* = \frac{p*}{1-p*} f_{\max}$, $\frac{\partial g*}{\partial f_{\max}} = \frac{p*}{1-p*}$, $\frac{\partial g*}{\partial p*} = \frac{f_{\max}}{(1-p*)^2}$, $\frac{dp*}{df_{\max}} = -\frac{SW_{pf_{\max}}}{SW_{pp}}$

$$\frac{dg*}{df_{\max}} = \frac{p*}{1-p*} - \frac{f_{\max}}{(1-p*)^2}\frac{SW_{pf_{\max}}}{SW_{PP}}$$

$$= -\frac{1}{(1-p*)^2 SW_{pp}}[f_{\max}SW_{pf_{\max}} - p*(1-p*)SW_{pp}]$$

Since $-\frac{1}{(1-p*)^2 SW_{pp}} > 0$ by the SOCs:

(A2) $sign[\frac{dg*}{df_{\max}}] = sign[f_{\max}SW_{pf_{\max}} - p*(1-p*)SW_{pp}]$

Recalling that:

$SW_p = (h-g^*)\dfrac{\partial B(g^*)}{\partial p^*} - c'(p^*) = (h-g^*)b(g^*)\dfrac{f_{\max}}{(1-p^*)^2} - c'(p^*) = 0$, then, (A3)

$$SW_{pf_{\max}} = \frac{\partial(h-g^*)}{\partial f_{\max}} b(g^*)\frac{f_{\max}}{(1-p^*)^2} + \frac{\partial b(g^*)}{\partial f_{\max}}(h-g^*)\frac{f_{\max}}{(1-p^*)^2} + \frac{1}{(1-p^*)^2}(h-g^*)b(g^*)$$

$$= -\frac{\partial g^*}{\partial f_{\max}} b(g^*)\frac{f_{\max}}{(1-p^*)^2} + \frac{\partial g^*}{\partial f_{\max}} b'(g^*)(h-g^*)\frac{f_{\max}}{(1-p^*)^2} + \frac{1}{(1-p^*)^2}(h-g^*)b(g^*)$$
$$= -\frac{\partial g^*}{\partial f_{\max}} b(g^*)\frac{f_{\max}}{(1-p^*)^2} + \frac{\partial g^*}{\partial f_{\max}} b'(g^*)(h-g^*)\frac{f_{\max}}{(1-p^*)^2} + \frac{1}{(1-p^*)^2}(h-g^*)b(g^*)$$
$$= \frac{b(g^*)}{(1-p^*)^2}[-\frac{p^* f_{\max}}{(1-p^*)} + \frac{p^* f_{\max}}{(1-p^*)}\frac{b'(g^*)}{b(g^*)}(h-g^*) + (h-g^*)]$$

Define $\varepsilon(g^*) = -\dfrac{b'(g^*)}{b(g^*)} g^*$, substitute $g^* = \dfrac{p^* f_{\max}}{1-p^*}$, and rearrange,

(A3) $SW_{pf_{\max}} = \dfrac{b(g^*)}{(1-p^*)^2}[h - 2g^* - \varepsilon(g^*)(h-g^*)]$,

Multiplying (A3) by $f_{\max}$ we have;

(A4) $f_{\max} SW_{pf_{\max}} = \dfrac{f_{\max} b(g^*)}{(1-p^*)^2}[h - 2g^* - \varepsilon(g^*)(h-g^*)]$.

In addition,

$$\text{(A5)}\ SW_{pp} = \frac{\dfrac{\partial[(h-g^*)b(g^*)f_{\max}]}{\partial p^*}(1-p^*)^2 + 2(1-p^*)(h-g^*)b(g^*)f_{\max}}{(1-p^*)^4} - c''(p^*)$$

$$\frac{\partial[(h-g^*)b(g^*)f_{\max}]}{\partial p^*} = -\frac{\partial g^*}{\partial p^*} b(g^*)f_{\max} + \frac{\partial g^*}{\partial p^*} b'(g^*)(h-g^*)f_{\max}$$

$$= -\frac{b(g^*)f_{\max}^2}{(1-p^*)^2} + \frac{b'(g^*)}{(1-p^*)^2}(h-g^*)f_{\max}^2$$

(A5) $SW_{pp} = \dfrac{f_{\max} b(g^*)}{(1-p^*)^3}[-\dfrac{f_{\max}}{1-p^*} + \dfrac{b'(g^*)f_{\max}}{b(g^*)(1-p^*)}(h-g^*) + 2(h-g^*)] - c''(p^*)$

Multiplying by $p^*(1-p^*)$, we get (A6)

$$p^*(1-p^*)SW_{pp} = \frac{f_{\max} b(g^*)}{(1-p^*)^2}[(h-g^*)\frac{p^* f_{\max}}{1-p^*}\frac{b'(g^*)}{b(g^*)} - \frac{p^* f_{\max}}{1-p^*} + 2p^*(h-g^*)] - p^*(1-p^*)c''(p^*)$$

Recalling that $g^* = \frac{p^* f_{\max}}{1-p^*}$ and $\varepsilon(g^*) = -\frac{b'(g^*)}{b(g^*)} g^*$, we have that:

(A6) $p^*(1-p^*)SW_{pp} = \frac{f_{\max} b(g^*)}{(1-p^*)^2}[-\varepsilon(g^*)(h-g^*) - g^* + 2(h-g^*)] - p^*(1-p^*)c''(p^*)$

Subtracting (A6) from (A4), we get:

(A7) $[f_{\max} SW_{pf_{\max}} - p^*(1-p^*)SW_{pp}] = \frac{f_{\max} b(g^*)}{(1-p^*)^2}[h - 2g^* - \varepsilon(g^*)(h-g^*) +$

$$+\varepsilon(g^*)(h-g^*) + g^* - 2p^*(h-g^*)] + p^*(1-p^*)c''(p^*)$$

(A7) $[f_{\max} SW_{pf_{\max}} - p^*(1-p^*)SW_{pp}] = \frac{f_{\max} b(g^*)}{(1-p^*)^2}(h-g^*)(1-2p^*) + p^*(1-p^*)c''(p^*)$

Since $c''(p^*) \geq 0$, it is clear that if $p^* < \frac{1}{2}$, then sign (A2) is positive, that is, $\frac{dg^*}{df_{\max}} > 0$.

If, however, $p^* > \frac{1}{2}$ (and assuming that $c''(p) = 0)$, then sign of (A2) is negative, and

$\frac{dg^*}{df_{\max}} < 0$. Thus, Proposition 2 follows.

APPENDIX B

Proof of Proposition 3:

Let us examine how the optimal probability of punishment p^* changes with the maximal

fine; $sign[\frac{dp^*}{df_{\max}}]$.

By the Implicit Function Theorem:

(B1) $\dfrac{dp^*}{df_{\max}} = -\dfrac{SW_{pf_{\max}}}{SW_{pp}}$.

Since $-SW_{pp} > 0$ by the SOCs:

(B2) $sign[\dfrac{dp^*}{df_{\max}}] = sign[SW_{pf_{\max}}]$.

From (A3) we have that:

$$SW_{pf_{\max}} = \frac{b(g^*)}{(1-p^*)^2}[h - 2g^* - \varepsilon(g^*)(h-g^*)],$$

and since $\dfrac{b(g^*)}{(1-p^*)^2} > 0$,

(B3) $sign[SW_{pf_{\max}}] = sign[h - 2g^* - \varepsilon(g^*)(h-g^*)]$.

Now, $h - 2g^* - \varepsilon(g^*)h + \varepsilon(g^*)g^* >< 0$;

(B4) $g^*[\varepsilon(g^*) - 2] >< h[\varepsilon(g^*) - 1]$;

If, $\varepsilon(g^*) \geq 1$, then $g^* < h\dfrac{[\varepsilon(g^*)-1]}{[\varepsilon(g^*)-2]}$, since $g^* < h$, (Proposition 1(2), Eq. 6), which implies that $sign[\dfrac{dp^*}{df_{\max}}] < 0$.

If $\varepsilon(g^*) < 1$, then $sign[\dfrac{dp^*}{df_{\max}}] > 0$, if $g^* < h\dfrac{[\varepsilon(g^*)-1]}{[\varepsilon(g^*)-2]}$. Otherwise, $sign[\dfrac{dp^*}{df_{\max}}] < 0$.

Thus, proposition 3 follows.

APPENDIX C

Proof of Proposition 4:

To examine how the optimal level of deterrence changes with changes in the fraction of the gains that can be disgorged, let us assume that $f^* = f_{\max}$, $\eta^* = \eta$, and p^* satisfies the first order condition:

(C1) $SW_p = (h - g^*)b(g^*)\dfrac{f_{\max}}{(1-\eta^* p^*)^2} - c'(p^*) = 0$,

Where

(C2) $g^* = \dfrac{p^* f_{\max}}{1-\eta^* p^*}$

The effect is then given by:

(C3) $$\frac{dg^*}{d\eta^*} = \frac{\partial g^*}{\partial \eta^*} + \frac{\partial g^*}{\partial p^*}\frac{dp^*}{d\eta^*}$$

$$= \frac{p^{*2} f_{\max}}{(1-\eta^* p^*)^2} - \frac{f_{\max}}{(1-\eta^* p^*)^2}\frac{SW_{p\eta}}{SW_{PP}}$$

$$= -\frac{f_{\max}}{(1-\eta^* p^*)^2 SW_{PP}}(SW_{p\eta} - p^{*2} SW_{pp})$$

Since $= -\dfrac{f_{\max}}{(1-\eta^* p^*)^2 SW_{PP}} > 0$, then:

(C4) $sign[\dfrac{dg^*}{d\eta^*}] = sign[SW_{p\eta} - p^{*2} SW_{pp}]$

Now:

(C5) $$SW_{p\eta} = \frac{\dfrac{\partial[(h-g^*)b(g^*) f_{\max}]}{\partial \eta^*}(1-\eta^* p^*)^2 + 2p^*(1-\eta^* p^*)(h-g^*)b(g^*) f_{\max}}{(1-\eta^* p^*)^4},$$

$$\frac{\partial[(h-g^*)b(g^*) f_{\max}]}{\partial \eta^*} = -\frac{\partial g^*}{\partial \eta^*}b(g^*) f_{\max} + \frac{\partial g^*}{\partial \eta^*}b'(g)(h-g^*) f_{\max}$$

$$= -\frac{p^{*2} f_{\max}^2}{(1-\eta^* p^*)^2}b(g^*) + \frac{p^{*2} f_{\max}^2}{(1-\eta^* p^*)^2}b'(g)(h-g^*)$$

(C5) $SW_{p\eta} = \frac{p^* f_{\max} b(g^*)}{(1-\eta^* p^*)^3}[-\frac{p^* f_{\max}}{(1-\eta^* p^*)} + \frac{p^* f_{\max}}{(1-\eta^* p^*)}\frac{b'(g^*)}{b(g^*)}(h-g^*) + 2(h-g^*)]$

(C5) $SW_{p\eta} = \frac{p^* f_{\max} b(g^*)}{(1-\eta^* p^*)^3}[2h - 3g^* - \varepsilon(g^*)(h-g^*)]$

In addition,

(C6)

$$SW_{pp} = \frac{\frac{\partial[(h-g^*)b(g^*) f_{\max}]}{\partial p^*}(1-\eta^* p^*)^2 + 2\eta^*(1-\eta^* p^*)(h-g^*)b(g^*) f_{\max}}{(1-\eta^* p^*)^4} - c''(p^*)$$

$$\frac{\partial[(h-g^*)b(g^*) f_{\max}]}{\partial p^*} = -\frac{\partial g^*}{\partial p^*} b(g^*) f_{\max} + \frac{\partial g^*}{\partial p^*} b'(g^*)(h-g^*) f_{\max}$$

$$= -\frac{b(g^*) f_{\max}^2}{(1-\eta^* p^*)^2} + \frac{b'(g^*)}{(1-\eta^* p^*)^2}(h-g^*) f_{\max}^2$$

(C6) $SW_{pp} = \frac{f_{\max} b(g^*)}{(1-\eta^* p^*)^3}[-\frac{f_{\max}}{1-\eta^* p^*} + \frac{b'(g^*) f_{\max}}{b(g^*)(1-\eta^* p^*)}(h-g^*) + 2\eta(h-g^*)] - c''(p^*)$

Multiplying by p^{*2}, we get,

(C7)

$$p^{*2}\, SW_{pp} = \frac{p^* f_{\max} b(g^*)}{(1-\eta^* p^*)^3}[-\frac{p^* f_{\max}}{1-\eta^* p^*} + \frac{b'(g^*) p^* f_{\max}}{b(g^*)(1-\eta^* p^*)}(h-g^*) + 2\eta^* p^*(h-g^*)] - p^{*2}\, c''(p^*)$$

Recalling that $g^* = \frac{p^* f_{\max}}{1-\eta^* p^*}$ and $\varepsilon(g^*) = -\frac{b'(g^*)}{b(g^*)} g^*$, we have that:

(C7) $p^{*2}\, SW_{pp} = \frac{p^* f_{\max} b(g^*)}{(1-\eta^* p^*)^3}[-g^* - \varepsilon(g^*)(h-g^*) + 2\eta^* p^*(h-g^*)] - p^{*2}\, c''(p^*)$

Subtracting (C7) from (C5), we get:

(C8) $[SW_{p\eta} - p^{*2}\, SW_{pp}] = \frac{p^* f_{\max} b(g^*)}{(1-\eta^* p^*)^3}[2h - 3g^* - \varepsilon(g^*)(h-g^*) +$

$$+g^* + \varepsilon(g^*)(h - g^*) - 2\eta^* p^*(h - g^*)] + p^{*2} c''(p^*)$$

(C8) $[SW_{p\eta} - p^{*2} SW_{pp}] = \frac{2p^* f_{\max} b(g^*)}{(1-\eta^* p^*)^3}(h - g^*)(1 - \eta^* p^*) + p^{*2} c''(p^*)$

Now since $c''(p^*) \geq 0$, $h - g^* > 0$, and $(1 - \eta^* p^*) > 0$, it follows that

$sign[SW_{p\eta} - p^{*2} SW_{pp}]$ is positive, and that $\frac{dg^*}{d\eta^*} > 0$. Thus, Proposition 4 follows.

APPENDIX D

Proof of Proposition 5:

Let us examine how the optimal probability of punishment p^* changes with the maximal

level of disgorgement. $sign[\frac{dp^*}{d\eta^*}]$.

By the Implicit Function Theorem, we have:

(D1) $\frac{dp^*}{d\eta^*} = -\frac{SW_{p\eta}}{SW_{pp}}$.

Since, $-SW_{pp} > 0$ (SOCs), we have that,

(D2) $sign[\frac{dp^*}{d\eta^*}] = sign[SW_{p\eta}]$.

From (C5) we have:

$SW_{p\eta} = \frac{p^* f_{\max} b(g^*)}{(1-\eta^* p^*)^3}[2h - 3g^* - \varepsilon(g^*)(h - g^*)]$, therefore,

(D3) $sign[SW_{p\eta}] = sign[2h - 3g^* - \varepsilon(g^*)(h - g^*)]$.

Now, $2h - 3g^* - \varepsilon(g^*)h + \varepsilon(g^*)g^* >< 0$;

(D4) $g^*[\varepsilon(g^*) - 3] >< h[\varepsilon(g^*) - 2]$;

If, $\varepsilon(g^*) \geq 2$, then $g^* < h\frac{[\varepsilon(g^*)-1]}{[\varepsilon(g^*)-2]}$, since $g^* < h$, (Proposition 1(2), Eq. 6), which implies that $sign[\frac{dp^*}{d\eta}] < 0$.

If $\varepsilon(g^*) < 2$, then $sign[\frac{dp^*}{d\eta}] > 0$ if $g^* < h\frac{[\varepsilon(g^*)-2]}{[\varepsilon(g^*)-3]}$. Otherwise, $sign[\frac{dp^*}{d\eta}] < 0$.

Thus, proposition 5 follows.

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