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Phase uncertainty and loss of interference: A general picture

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The problem of quantum interference in the presence of an environment is considered by two approaches. One treats the problem from the point of view of the trace left by the interfering particle on its environment. The other regards the phase accumulation of the interfering waves as a statistical process, and explains the loss of interference in terms of uncertainty in the relative phase. The equivalence of the two approaches is proven for the general case. The two approaches are applied to dephasing of electron interference by photon modes in coherent and thermal states, and to dephasing by electromagnetic fluctuations in metals.

INTRODUCTION

The suppression of quantum interference due to the coupling of the interfering degree of freedom to many other degrees of freedom is a central subject in the understanding of quantum phenomena.[1,2] Recent developments in the physics of mesoscopic systems emphasize the importance of this subject. The most general approach to deal with this subject is the influence functional method, due to Feynman and Vernon.[1] In this approach the influence of the environment on the interference of two Feynman paths is expressed in terms of a functional of the two paths, and this functional is given in terms of a sum over paths of the environment degrees of freedom. Physically, this influence is commonly explained either by changes the interfering particle induces in the environment, or by the randomization of the interfering particle's phase. Due to the latter, the process is sometimes called "dephasing."

In a previous work,[3] we examined the reduction of the interference term due to the interaction with the environment. Using two thought experiments, we showed that the loss of interference can be explained either by the lack of overlap between the two environment's states coupled to the two interfering partial waves, or by the width of the probability distribution of the particle's phase being comparable to 2π. We claimed that the two explanations are equivalent, and used the two thought experiments to support our claim.

In the present work we discuss the details of the two explanations and prove their equivalence for the general case of two interfering partial waves interacting with their environment. Our work yields a new interpretation of the influence functional, expressed by the statistical properties of the relative phase of two interfering paths. The outline of the paper is as follows. In Sec. I we present the two explanations. In Sec. II we demonstrate their equivalence for a simple two-states environment. This section is a brief review of part of our previous work. In Sec. III we prove the equivalence for all environments that do not have their own independent dynamics. Those will be referred to as nondynamical. In Sec. IV we include the dynamics of the environments in the generalized proof. Section V deals with the case of two partial waves interacting with the *same* environment. We analyze in this section the possibility of different phase-breaking times (τ_ϕ's) for h/e and $h/2e$ conductance oscillations. In Secs. VI and VII we present two applications of our approach. In Sec. VI we investigate dephasing caused by photon modes in coherent versus thermal states, and in Sec. VII we discuss dephasing by electromagnetic fluctuations in metals.[4]

I. TWO DESCRIPTIONS FOR THE LOSS OF QUANTUM INTERFERENCE

As mentioned above, we discuss here the interference of two *given* paths (we will later comment on the way these two paths are chosen). As a guiding example, one may consider an Aharonov-Bohm interference experiment on a ring (see Fig. 1). This experiment starts by a construction of two electron wave packets $l(x)$ and $r(x)$, (l,r stand for left, right) crossing the ring along two opposite sides. The interference is examined after each of the two wave packets traverses half of the ring's circumference. Therefore the initial wave function of the electron

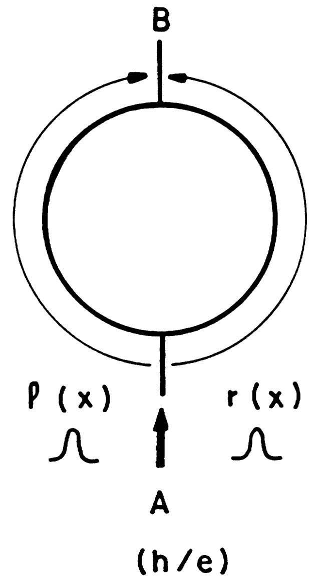

FIG. 1. Schematics of interference experiments in A-B rings. Each partial wave traverses half the ring, and the interference is examined at the point B. This kind of interference gives rise to h/e oscillations of the conductance.

(whose coordinate is x) and the environement (whose set of coordinates is denoted by η) is

$$\psi(t=0)=[l(x)+r(x)]\otimes\chi_0(\eta)\ . \tag{1.1}$$

At time τ_0, when the interference is examined, the wave function is, in general,

$$\psi(\tau_0)=l(x,\tau_0)\otimes\chi_l(\eta)+r(x,\tau_0)\otimes\chi_r(\eta)\ , \tag{1.2}$$

and the interference term is

$$2\,\mathrm{Re}\left[l^*(x,\tau_0)r(x,\tau_0)\int d\eta\,\chi_l^*(\eta)\chi_r(\eta)\right]\ . \tag{1.3}$$

Had there been no environment present in the experiment, the interference term would have been just $2\,\mathrm{Re}[l^*(x,\tau_0)r(x,\tau_0)]$. So the effect of the interaction is to multiply the interference term by $\int d\eta\,\chi_l^*(\eta)\chi_r(\eta)$. The first way to understand this effect is seen directly from this expression, which is the scalar product of the two environment's states coupled to the two partial waves. At $t=0$ these two states are identical. During the time of the experiment, each partial wave has its own interaction with the environment, and therefore the two states become different. Since the environment is not observed in the interference experiment, its coordinate is integrated upon, i.e., the scalar product of the two states is taken. When the two states do not overlap at all, the final state of the environment identifies the path the electron took. Quantum interference, which is the result of an uncertainty in this path, is then lost. Thus interference is lost when the two interfering partial waves shift the environment into states orthogonal to each other.

The second explanation for the loss of quantum interference regards it from the point of view of how the environment affects the partial waves, rather than how the waves affect the environment. It is well known that when a static potential $V(x)$ is exerted on one of the partial waves, this wave accumulates a phase (a system of units where $\hbar=1$ is applied)

$$\phi=-\int V(x(t))dt\ , \tag{1.4}$$

and the interference term is multiplied by $e^{i\phi}$. "A static potential" here, and throughout this paper, is a potential that is a function of the particle's coordinate and momentum only, and does not involve any other degrees of freedom. For a given particle's path, the value of a static potential is well defined. When V is not static, but created by environment degree(s) of freedom, its value is not well defined any more. The uncertainty in its value results from the quantum uncertainty in the state of the environment. Therefore ϕ is not definite, too. In fact, ϕ becomes a statistical variable, described by a distribution function $P(\phi)$. (The details of this descriptioin will be given in the following sections.) The effect of the environment on the interference is then to multiply the interference term by the average value of $e^{i\phi}$, i.e.,

$$\langle e^{i\phi}\rangle=\int d\phi\,P(\phi)e^{i\phi}\ . \tag{1.5}$$

$\langle e^{i\phi}\rangle$ is the influence functional, a functional of the two interfering paths. Since $e^{i\phi}$ is periodic in ϕ, $\langle e^{i\phi}\rangle$ tends to zero when $P(\phi)$ is slowly varying over a region much larger than one period, i.e., 2π. This is, then, the second explanation for the loss of quantum interference.

Our statement of equivalence between the two explanations is then put into an equation,

$$\langle e^{i\phi}\rangle=\int d\eta\,\chi_l^*(\eta)\chi_r(\eta)\ . \tag{1.6}$$

This relationship is a central result of this paper, since it relates a description in terms of the environment to a description in terms of the interfering waves. We will demonstrate now this equivalence on several particular examples, ending with the general proof. Each of these examples will be used to emphasize a different feature of this equivalence.

II. NONDYNAMICAL TWO STATES ENVIRONMENT

Our first example considers an environment composed of one scatterer, a spin $\frac{1}{2}$. This spin is localized somewhere along the right path, so that it interacts only with the right partial wave. The interaction takes place along a short interaction range l. The spin-electron interaction is (σ is the scatterer spin) $V_0\sigma_z$ when the electron is in the region of interaction and zero outside this region. (This interaction may be thought of as an Ising-like interaction, where the electron's spin is in an eigenstate of its z component.)

Now, if at $t=0$ the spin σ is in an eigenstate of σ_z, then the scattering is elastic. It causes the right component of the electron a phase shift, which multiplies the interference term by a phase factor.

But if at $t=0$ the spin is, e.g., in the state $|\sigma_x=+1\rangle$, the scattering is inelastic in the sense that the quantum state of the scatterer is changed. Note that this change in the scatterer's state does not involve any exchange of energy with the electron. This inelastic scattering causes dephasing. Let us analyze such a situation in detail.

At $t=0$ the system's (electron and spin) wave function is

$$\frac{1}{\sqrt{2}}[r(t=0)+l(t=0)]\otimes(|\sigma_z=+1\rangle+|\sigma_z=-1\rangle) . \tag{2.1}$$

Under the semiclassical approximation, to first order in V, the wave function evolves in time to

$$\frac{1}{\sqrt{2}}r(t)[|\sigma_z=+1\rangle\exp(-iV_0\tau)+|\sigma_z=-1\rangle\exp(iV_0\tau)] + \frac{1}{\sqrt{2}}l(t)(|\sigma_z=+1\rangle+|\sigma_z=-1\rangle) , \tag{2.2}$$

where $r(t)$ and $l(t)$ describe the evolution in time of the right and left partial waves in the absence of the scatterer and $\tau=l/(P/m)$ is the time the right partial wave spends in the region of interaction. The probability of finding the electron at the point B will then be

$$|\Psi(B,\tau_0)|^2=|r(B,\tau_0)|^2+|l(B,\tau_0)|^2 +\cos(V_0\tau)2\,\mathrm{Re}[r^*(B,\tau_0)l(B,\tau_0)] , \tag{2.3}$$

i.e., the interference term is reduced by the factor $\cos(V_0\tau)$. This result can be interpreted in two alternative ways.

(a) The right partial wave changed the state of the spin. This can be seen by noting that in (2.2) the right partial wave rotated the spin by an angle $2V_0\tau$, as expected for a spin put for a time τ in a magnetic field V_0. In the case of $V_0\tau=\pi/2$ the wave function is

$$|r(\tau_0)\rangle|\sigma_x=-1\rangle+|l(\tau_0)\rangle|\sigma_x=+1\rangle , \tag{2.4}$$

and the interference is completely destroyed.

(b) The right partial wave accumulated a "phase uncertainty" of $\pm V_0\tau$. In this interpretation, we look *only at the electron* and write its wave function as

$$l(\tau_0)+e^{i\phi}r(\tau_0) , \tag{2.5}$$

where ϕ, the electron's phase, is now a statistical variable whose probability distribution is

$$P(\phi)=\begin{cases}0.5 & \text{for the phase to be} -V_0\tau\\ 0.5 & \text{for the phase to be} +V_0\tau .\end{cases} \tag{2.6}$$

When the interference term is calculated, it is obtained as a function of the phase, and then averaged over the phase distribution function. Due to the periodicity of $e^{i\phi}$, the maximal phase uncertainty is $\pm\pi/2$, and it is exactly this value of $V_0\tau$ which destroys the interference.

Our conclusion is, therefore, that the physical process of dephasing by one scatterer can be described by *either of the two alternative and equivalent descriptions:* (a) by different electron partial waves leaving the environment in states different from each other, or (b) by one (or some) of the electron's partial waves accumulating phase uncertainty. In the following sections we will show that it is the uncertainty in the *relative* phase of the two partial waves that describes the dephasing. The one scatterer example discussed in this section also demonstrates that neither an exchange of energy, nor a continuum of degrees of freedom are a necessary condition for an environment to dephase the interference.[3]

III. NONDYNAMICAL CONTINUOUS ENVIRONMENT

The above gedanken experiment of a one-spin environment is usually an oversimplification of reality—the environment is composed of only one degree of freedom, the state of this degree of freedom is described by a two-dimensional Hilbert space, and, perhaps most importantly, the environment has no dynamics of its own. The latter means, of course, that all states of the environment are degenerate in energy, so that it cannot model a thermal bath. Being aware of these simplifications, we will now modify the above gedanken experiment in order to examine more complicated cases.

First, we consider an environment composed of one heavy free particle (replacing the spin in the former example). The initial state of this particle is localized at the same point the spin used to be (i.e., somewhere along the right arm of the ring), and its average momentum is zero. The coordinate and momentum of this particle are denoted by η and p_η, respectively (see Fig. 2). The particle's mass M is large enough so that $(\Delta p_\eta/M)\tau_0 \ll \Delta\eta$, where τ_0 is the duration of the experiment. Consequently, the particle's wave function does not change appreciably during the experiment and its kinetic energy plays no role in its time evolution. So, our environment is composed now of one degree of freedom, with no significant dynamics of its own, but with an infinite-dimensional Hilbert space. The interaction between the interfering electron and the environment is assumed to have the form

$$H_{\text{int}}=V(x,\eta) , \tag{3.1}$$

where V is nonzero within some short range of interaction. x here is the position of the interfering particle on the ring, while η is, e.g., the position of the environment particle on the plane of the ring (see Fig. 2).

As before, the experiment starts with the initial state

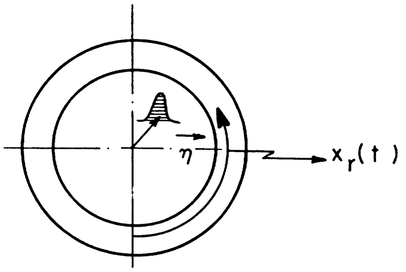

FIG. 2. The right path $x_r(t)$ interacts with an environment composed of one heavy free particle whose coordinate is η.

$$[l(x)+r(x)]\otimes\chi_0(\eta)\ . \tag{3.2}$$

The left partial wave does not interact with the environment, so that its propagation does not affect the latter. The right partial wave interacts with the environment, and the wave function at time τ_0 is

$$\begin{aligned}\psi(\tau_0)=&|l(\tau_0)\rangle\otimes\chi_0(\eta)\\&+|r(\tau_0)\rangle\otimes\exp\left[-i\int_0^{\tau_0}dt V(x_r(t),\eta)\right]\chi_0(\eta)\ .\end{aligned} \tag{3.3}$$

The interference term then becomes

$$\begin{aligned}2\,\mathrm{Re}\Big[&l^*(x,\tau_0)r(x,\tau_0)\\&\times\langle\chi_0(\eta)|\exp\left[-i\int_0^{\tau_0}dt V(x_r,(t),\eta)\right]|\chi_0(\eta)\rangle\Big]\ .\end{aligned} \tag{3.4}$$

Hence the whole effect of the environment on the interference term shows up in the expression

$$\langle\chi_0(\eta)|\exp\left[-i\int_0^{\tau_0}dt V(x_r(t),\eta)\right]|\chi_0(\eta)\rangle\ . \tag{3.5}$$

As before, this expression can be interpreted in terms of the effect of the electron on the environment, or in terms of the effect of the environment on the electron. The first follows from looking at Eq. (3.5) as a scalar product of two environment states, one coupled to the left partial wave, and the other coupled to the right partial wave. The second is a bit more subtle. For a given η, $\phi(\eta)\equiv-\int_0^{\tau_0}dt V(x_r(t),\eta)$ is the phase shift the electron would have experienced had the environment particle been in a position η. Equation (3.5) is then just

$$\int d\eta\,\chi_0^*(\eta)e^{i\phi(\eta)}\chi_0(\eta)=\int d\phi\,|\chi_0(\eta(\phi))|^2e^{i\phi}\frac{d\eta}{d\phi}\ . \tag{3.6}$$

The limits of the last integral depend on the function $\phi(\eta)$, and are not relevant for our present discussion. (In cases where several values of η lead to the same ϕ, the right-hand side of this equation has to include a summation over these values of η.) The last integral leads us to the second interpretation: this integral is the average value of $e^{i\phi}$, averaged over a probability for the phase shift ϕ. This distribution function, as seen in (3.6), is

$$P(\phi)\equiv|\chi_0(\eta(\phi))|^2\frac{d\eta}{d\phi}\ . \tag{3.7}$$

The effect of the environment on the electron, which would have been a well determined phase shift for a "classical" environment, becomes the average value of all possible phase shifts when the quantum nature of the environment is taken into account.

When does the environment destroy the interference pattern? Again, there are two ways to answer this question, corresponding to the two interpretations discussed above. The interference term reduces to zero when the environment's state coupled to the right partial wave is orthogonal to the one coupled to the left partial wave, or, alternatively, the interference term reduces to zero when its average value, averaged over all possible states of the environment, is zero. By further inspection of (3.6), the second condition can be rephrased in terms of the phase distribution function $P(\phi)$. The environment multiplies the interference term by the factor given by (3.6), and this factor is

$$\langle e^{i\phi}\rangle=\int d\phi\,P(\phi)e^{i\phi}=e^{i\langle\phi\rangle}\int d\phi\,P(\phi)e^{i(\phi-\langle\phi\rangle)}\ , \tag{3.8}$$

where $\langle\phi\rangle\equiv\int\phi P(\phi)d\phi$ is the average phase shift. If $P(\phi)$ is a narrow distribution function, (3.8) can be approximated by

$$\begin{aligned}&e^{i\langle\phi\rangle}\int d\phi\,P(\phi)[1+i(\phi-\langle\phi\rangle)-\tfrac{1}{2}(\phi-\langle\phi\rangle)^2]\\&\quad=e^{i\langle\phi\rangle}(1-\tfrac{1}{2}\langle\delta\phi^2\rangle)(\text{valid for }\langle\delta\phi^2\rangle\ll1)\ .\end{aligned} \tag{3.9}$$

Thus, in the limit of a narrow distribution function, the main effect of the interaction with the environment is to induce a phase shift $\langle\phi\rangle$ in the interference pattern, together with a small reduction in its intensity. It is worth noting that, by (3.6),

$$\begin{aligned}\langle\phi\rangle&=\int\int_0^{\tau_0}d\eta\,dt\,|\chi_0(\eta)|^2V(x_r(t),\eta)\\&=\int_0^{\tau_0}\langle V(x_r(t))\rangle dt\ ,\end{aligned} \tag{3.10}$$

i.e., the average phase shift is the phase shift induced by the expectation value of the potential. Hence, as far as the interfering particle is concerned, the environment potential can, in this limit, be replaced by a static potential $V(x)$.

On the other hand, when $P(\phi)$ is very broad, i.e., $\langle\delta\phi^2\rangle\gg1$, there is an appreciable probability for various values of phase shifts, extending over a range much larger than 2π. Then, excluding pathological cases of extremely nonsmooth distribution functions, the intensity of the interference pattern tends to zero. In this limit, the environment's potential cannot be approximated by a static potential, and the analysis of the experiment has to be done in terms of the system plus environment. The experiment involves then a transition of the particle from a pure state to a mixed state.

The equivalence between the two descriptions discussed above can also be understood in terms of the uncertainty principle. Suppose that the free particle's state is initially a minimal wave packet, i.e., $\Delta\eta\Delta p_\eta\sim1$. Suppose also that the interaction between the free particle and the interfering particle is smooth enough so that it can be expanded as

$$V(x_r(t),\eta)\approx V(x_r(t),\eta=0)+\nabla_\eta V(x_r(t))\cdot\eta\ , \tag{3.11}$$

where $\eta=0$ is chosen to be the center of the minimal wave packet. The zeroth-order contribution to the expansion is just a static potential, which induces a well-defined phase shift. By (3.3), the state of the environment (i.e., the free particle) after the interaction is

$$\exp\left[-i\int_0^{\tau_0}dt\cdot\eta\nabla_\eta V(x_r(t))\right]\chi_0(\eta)\ , \tag{3.12}$$

so that the interaction shifts the free particle's momentum by $\int_0^{\tau_0} dt\, \nabla_\eta V(x_r(t))$. When this momentum shift is larger than the uncertainty in the free particle's momentum Δp_η, the environment is shifted into a state orthogonal to its original one and the interference is destroyed. The condition for this destruction is, therefore,

$$\int_0^{\tau_0} dt \nabla_\eta V(x_r(t)) > \Delta p_\eta \ . \tag{3.13}$$

On the other hand, by looking at this loss of interference from the point of view of phase uncertainty, we find that the condition for this loss is

$$\langle \delta\phi^2 \rangle^{1/2} = \int_0^{\tau_0} dt \nabla_\eta V(x_r(t)) \Delta\eta > 1 \ . \tag{3.14}$$

The assertion $\Delta\eta \Delta p_\eta \sim 1$ yields the agreement of the two conditions. When the uncertainty in momentum is large, it takes a large momentum shift, i.e., a long interaction time, in order to shift the environment to a state orthogonal to its initial one. On the other hand, large momentum uncertainty yields small uncertainty in position. This, in turn, yields a well-defined potential, and again, a long interaction time is needed for the phase to become uncertain. Thus the equivalence between the two descriptions is a consequence of the momentum-position uncertainty in the minimal wave packet. The case of a nonminimal wave packet will be discussed later in this work.

IV. DYNAMICAL ENVIRONMENT

The second modification in our spin thought experiment involves an environment with a nontrivial independent dynamics of its own. The Hamiltonian of the environment will be denoted by $H_{\text{env}}(\eta, p_\eta)$, while the interaction term is still $V(x_r(t), \eta)$. As before, the left partial wave does not interact with the environment.

Starting with the initial wave function (3.2) the wave function at time τ_0 is

$$\begin{aligned} \psi(\tau_0) = {} & l(\tau_0) e^{-iH_{\text{env}}\tau_0} \chi_0(\eta) \\ & + r(\tau_0) \hat{T} \exp\left[-i \int_0^{\tau_0} dt (H_{\text{env}} + V) \right] \chi_0(\eta) \ , \end{aligned} \tag{4.1}$$

where $\hat{T}$ is the time ordering operator. It is useful at this point to write $\psi(\tau_0)$ in terms of $V_I(t) \equiv e^{iH_{\text{env}}t} V(x_r(t), \eta) e^{-iH_{\text{env}}t}$, i.e., the potential V in the interaction picture. Using V_I, $\psi(\tau_0)$ can be written as

$$\begin{aligned} \psi(\tau_0) = {} & l(\tau_0) \otimes e^{-iH_{\text{env}}\tau_0} \chi_0(\eta) \\ & + r(\tau_0) \otimes e^{-iH_{\text{env}}\tau_0} \hat{T} \\ & \times \exp\left[-i \int_0^{\tau_0} dt V_I(x_r(t), t) \right] \chi_0(\eta) \ . \end{aligned} \tag{4.2}$$

Hence the interference term is multiplied by

$$\begin{aligned} & \langle \chi_0 | e^{iH_{\text{env}}\tau_0} \hat{T} \exp\left[-i \int_0^{\tau_0} dt (H_{\text{env}} + V) | \chi_0 \rangle \right. \\ & \quad = \langle \chi_0 | \hat{T} \exp\left[-i \int_0^{\tau_0} dt V_I(x_r(t), t) \right] | \chi_0 \rangle \ . \end{aligned} \tag{4.3}$$

The interpretation of this expression in terms of a scalar product of two environment states at time τ_0 is obvious. The interpretation in terms phase uncertainty emerges from the observation that Eq. (4.3) is the expectation value of a unitary operator. As all unitary operators, this operator can be expressed as the exponent of an Hermitian operator ϕ, i.e.,

$$\langle \chi_0 | \hat{T} \exp\left[-i \int_0^{\tau_0} dt V_I(x_r(t), t \right] | \chi_0 \rangle = \langle \chi_0 | e^{i\phi} | \chi_0 \rangle \ . \tag{4.4}$$

Hence the effect of the interaction with the environment is to multiply the interference term by $\langle e^{i\phi} \rangle$, where the averaging is done with respect to the phase probability distribution, as determined by the environmental state χ_0.

The phase operator ϕ was introduced here by means of the mathematical properties of unitary transformations, so that it still deserves a physical explanation. To obtain such an explanation, we first discuss the case where the potentials exerted by the environment at different points along the particle's path are commutative, i.e.,

$$[V_I(x_r(t), t), V_I(x_r(t'), t')] = 0 \ . \tag{4.5}$$

Then,

$$\begin{aligned} & \langle \chi_0 | \hat{T} \exp\left[-i \int_0^{\tau_0} dt V_I(x_r(t), t) \right] | \chi_0 \rangle \\ & \quad = \langle \chi_0 | \hat{T} \exp\left[-i \int_0^{\tau_0} dt V_I(x_r(t), t \right] | \chi_0 \rangle \ , \end{aligned} \tag{4.6}$$

and $\phi = -\int_0^{\tau_0} dt\ V_I(x_r(t), t)$. In this case $\dot{\phi}$, the rate of accumulation of the phase, is just the local potential acting on the interferring particle, independent of earlier interactions of the particle with the environment. Similar to the nondynamical case, one should distinguish here between two limits: for $\langle \delta\phi^2 \rangle \ll 1$ Eq. (4.4) yields

$$\langle e^{i\phi} \rangle \approx e^{i\langle \phi \rangle} (1 - \tfrac{1}{2} \langle \delta\phi^2 \rangle) \ , \tag{4.7}$$

and the environment's potential can be approximated by a single-particle (possibly time-dependent) potential

$$\langle V_I(x_r(t), t) \rangle = \langle \chi_0 | V_I(x_r(t), t) | \chi_0 \rangle \ . \tag{4.8}$$

For $\langle \delta\phi^2 \rangle \gg 1$, on the other hand, the interference term tends to zero. The crossover between the two regimes is then at

$$\begin{aligned} \langle \delta\phi^2 \rangle = {} & \int_0^{\tau_0} dt \int_0^{\tau_0} dt' [\langle V_I(x_r(t), t) V_I(x_r(t'), t') \rangle - \langle V_I(x_r(t), t) \rangle \langle V_I(x_r(t'), t') \rangle] \\ = {} & \int_0^{\tau_0} dt \int_0^{\tau_0} dt' \left[\int d\eta \int d\eta' \chi^*(\eta', t') V(x_r(t'), \eta') \langle \eta' | e^{-iH_{\text{env}}(t'-t)} | \eta \rangle V(x_r(t), \eta) \chi(\eta, t) \right. \\ & \left. - \int d\eta\, \chi^*(\eta, t) V(x_r(t), \eta) \chi(\eta, t) \int d\eta' \chi^*(\eta', t') V(x_r(t'), \eta') \chi(\eta', t') \right] \sim 1 \ , \end{aligned} \tag{4.9}$$

where $\chi_0(\eta,t)\equiv e^{-iH_{\text{env}}t}\chi_0(\eta)$ is the environment state as it evolves in time under H_{env}.

When is the condition in Eq. (4.5) valid, and what happens when it is not? A typical case where the potentials at different points along the path are commutative is the case of an interfering electron interacting with a free electromagnetic field. In that case the interaction is

$$V_I(x_r(t),t)=-\frac{e}{c}\dot{\mathbf{x}}_r(t)\cdot\mathbf{A}(x_r(t),t)\ , \tag{4.10}$$

where $\mathbf{A}(\mathbf{x},t)$, the electromagnetic free field, is

$$\mathbf{A}(\mathbf{x},t)=\sum_{k,\lambda}\epsilon_{k,\lambda}\left[\frac{2\pi c^2}{\omega_k}\right]^{1/2}(ae^{i\mathbf{k}\cdot\mathbf{x}-i\omega t}+a^{\dagger}e^{-i\mathbf{k}\cdot\mathbf{x}+i\omega t})\ , \tag{4.11}$$

and $[V_I(\mathbf{x},t),V_I(\mathbf{x}',t')]=0$ unless $|\mathbf{x}-\mathbf{x}'|=c|t-t'|$. Since $\dot{x}_r(t)<c$, condition (4.5) is valid. Generally, this condition is valid when there is no amplitude for an environment excitation created at $(x_r(t),t)$ to be annihilated at $(x_r(t'),t')$, i.e., when a change induced in the environment's state at $(x_r(t),t)$ does not influence the potential the environment exerts on the interfering particle at $(x_r(t'),t')$. In the above example, a photon emitted by the electron at $(x_r(t),t)$ will not be at $(x_r(t'),t')$ when the electron gets there.

If instead of discussing the interaction with a photons field we turn our attention to the interaction with phonons, the speed of light in Eqs. (4.10) and (4.11) is replaced by the sound velocity. Then, a phonon emitted by the electron at $(x_r(t),t)$ might be encountered again by the electron at $(x_r(t'),t')$. Hence lattice excitations excited by the electron along its path affect the potential it feels at a later stage of the path. The potential the electron feels at a given point of its path is now not a local function of that point, but depends on the path since it includes a "back reaction" of the environment to the potential exerted by the electron. Therefore this potential will be different from $V_I(x_r(t),t)$, and consequently, the rate of phase accumulation will also differ from $V_I(x_r(t),t)$. However, in many-body environments the potential exerted by the environment on the interfering particle is usually independent of the particle's history since the environment's memory time is very short. Therefore Eq. (4.5) can be assumed to hold.

We thus see that the loss of interference due to an interaction with a dynamical environment can be understood in the same two ways used in the former examples. The interference is destroyed either when the state of the environment coupled to the right wave is orthogonal to that coupled to left wave, or, alternatively, when the width of the phase distribution function exceeds a magnitude of order unity. The interaction with the dynamical environment turns the phase into a statistical variable, and this, together with the phase being defined only over a range of 2π, determines the conditions for the phase to become completely uncertain. If the potential exerted by the environment on the interfering particle at a given point along its path is assumed to be independent of the path, the phase uncertainty is given by

$$\begin{aligned}\langle\delta\phi^2\rangle=\int_0^{\tau_0}dt\int_0^{\tau_0}dt'[&\langle V_I(x_r(t),t)V_I(x_r(t'),t')\rangle\\ &-\langle V_I(x_r(t),t)\rangle\\ &\times\langle V_I(x_r(t'),t')\rangle]\ .\end{aligned} \tag{4.12}$$

The exact behavior of the interference term for $\langle\delta\phi^2\rangle\gg 1$, i.e., the value of $\langle e^{i\phi}\rangle$ for broad distribution functions, depends on the phase distribution $P(\phi)$. However, the description of the phase as a statistical variable enables us, under appropriate conditions, to apply the central limit theorem, and conclude that $P(\phi)$ is a normal distribution. The central limit theorem is applicable, for example, when the phase is accumulated in a series of uncorrelated events (e.g., by a series of scattering events off different, noninteracting, scatterers), or, more generally, whenever the potential-potential correlation function decays to zero with a characteristic decay time much shorter than the duration of the experiment. In particular, the central limit theorem is usually applicable for coupling to a heat bath. For a normal distribution,

$$\langle e^{i\phi}\rangle=e^{i\langle\phi\rangle-(1/2)\langle\delta\phi^2\rangle}\ , \tag{4.13}$$

and the phase is accumulated in a diffusionlike process. This expression is exact for the model of an environment composed of harmonic oscillators with a linear coupling to the interfering waves. The evaluation of $\langle e^{i\phi}\rangle$ by Eq. (4.13) reproduces the result obtained by Feynman and Vernon[1] for that model. Feynman and Vernon's result was obtained by integration of the environment's paths. This model was proven in recent years to be very useful in the investigation of the effect of the environment on quantum phenomena.[2,5] Equation (4.13) is therefore a convenient way to calculate the influence functional for many-body environments, where the central limit theorem is usually applicable.

This is an appropriate point to discuss the time dependence of the phase uncertainty $\langle\delta\phi^2\rangle$. Due to unitarity, this uncertainty is a positive quantity; but it is not necessarily a monotonic function of time. This was demonstrated in the spin thought experiment (Sec. II), where $\langle\delta\phi^2\rangle$ oscillates when τ is varied, due to the cyclic definition of the phase. The following thought experiment supplies another example for a nonmonotonic $\langle\delta\phi^2\rangle$.[6] Consider an environment made of an Einstein model oscillator with frequency ω. Calculating the phase uncertainty for this case [Eq. (4.9)] we find that the propagator $\langle\eta'|e^{-iH_{\text{env}}(t'-t)}|\eta\rangle$ is a periodic function of $\omega(t'-t)$, whose average over a period is zero. The integrals over t and t' have contributions only during a period τ_{int} when a significant part of the interfering particle's wave packet is in the region of nonzero V. So, there exist two different limits for the phase uncertainty.

(a) When $\omega\tau_{\text{int}}\ll 1$ the propagator can be approximated by 1. Then, the environment is almost stationary during the interaction and the phase uncertainty is just the potential uncertainty (which is also constant during the interaction) multiplied by τ_{int}, or

$$\langle\delta\phi^2\rangle_a\sim V^2\tau_{\text{int}}^2\ . \tag{4.14}$$

(b) When $\omega\tau_{\text{int}}\gg 1$, the integrals over t and t' cover many periods of the environment propagator, accumulating a significant contribution only during a period of the order of $1/\omega$. In that case the phase uncertainty will be of order of

$$\langle \delta\phi^2 \rangle_b \sim \frac{V^2 \tau_{\text{int}}}{\omega} \sim \frac{\langle \delta\phi^2 \rangle_a}{\omega \tau_{\text{int}}} \ , \tag{4.15}$$

i.e., very small.

Now, if the wave packet's width Δx is larger than the region where V is nonzero, then $\tau_{\text{int}} = \Delta x / v$, where v is the wave packet's velocity. If Δx is of the order of $1/p$, the inverse momentum of the wave packet then $1/\tau_{\text{int}} = pv$ is of the order of the energy of the wave packet. Therefore the condition that distinguishes case (a) from (b) is the frequency of the oscillator being smaller [case (a)] or larger [case (b)] than the energy of the wave packet.

It is then easy to understand the crossover at $\omega\tau \sim 1$ from the point of view of the change in the state of the oscillator. Case (a) is the case where the partial wave has enough energy to excite the oscillator, while case (b) is the case where the wave does not have this energy, i.e., it cannot change the state of its environment. Similar considerations are known in the context of the Mossbauer effect.[7]

As seen from Eq. (4.9) the phase uncertainty remains constant when the interfering wave does not interact with the environment. Thus, if a trace is left by a partial wave on its environment, this trace cannot be wiped out after the interaction is over. Neither internal interactions of the environment, nor a deliberate application of a classical force on it, can reduce back the phase uncertainty after the interaction with the environment is over. This statement can be proved also from the point of view of the change the interfering wave induces in its environment.[3] This proof follows simply from unitarity. The scalar product of two states that evolve in time under the same Hamiltonian does not change in time. Therefore, if the state of the system (electron plus environment) after the electron environment interaction took place is

$$|r(t)\rangle \otimes |\chi_{\text{env}}^{(1)}\rangle + |l(t)\rangle \otimes |\chi_{\text{env}}^{(2)}\rangle \ , \tag{4.16}$$

then the scalar product $\langle \chi_{\text{env}}^{(1)}(t) | \chi_{\text{env}}^{(2)}(t) \rangle$ does not change in time. The only way to change it is by another interaction of the electron with the same environment. Such an interaction keeps the product $\langle \chi_{\text{env}}^{(1)}(t) | \chi_{\text{env}}^{(2)}(t) \rangle \otimes \langle r(t) | l(t) \rangle$ constant, but changes $\langle \chi_{\text{env}}^{(1)}(t) | \chi_{\text{env}}^{(2)}(t) \rangle$. The interference will be retrieved only if the orthogonality is transferred from the environment wave function to the electronic wave functions, which are not traced on in the experiment.

V. INTERACTION OF THE ENVIRONMENT WITH TWO PARTIAL WAVES

The next step in our generalization scheme is concerned with an environment that interacts with both partial waves. Such a typical case was discussed in our previous work.[3] There we considered the spin thought experiment discussed in Sec. II, with one path being time reversed with respect to the other. A typical example of such interference is found in *A*-*B* rings, when the interference is examined after each of the partial waves encircles the whole ring once (see Fig. 3). This is the weak localization, $h/2e$ flux periodic, contribution. It was shown there that in this case *each* of the partial waves flips the spin so that the final state of the spin is the same for both partial waves, and the interference is not affected. This process was also described in terms of phase uncertainty—even though the flip of the spin by each of the partial waves corresponds to a complete uncertainty of each of the two partial waves' phases, the *relative* phase remains well defined, and therefore, the interference, which depends on the relative phase of the two partial waves, is not affected. We then have an environment which dephases the interference of two half circle paths (the h/e flux periodic interference), but does not dephase the interference of two full circle paths ($h/2e$ flux periodic). Experimentally, this is an interesting phenomenon, since it yields different phase breaking times τ_ϕ's for h/e and $h/2e$ conductance oscillations. For this phenomenon to survive in a more realistic environment, this environment has to satisfy the following two conditions: first, the interaction with this environment has to be one that does not distinguish between the two time reversed, full circle, paths [it cannot, e.g., be proportional to $\dot{x}(t)$]; and second, the internal dynamics of the environment has to be one that does not distinguish between the two paths. In the spin case, for example, the interaction of the spin with the two waves is the same interaction, but it happens at different times. If the spin is coupled to some other degree(s) of freedom, its state might change during the time between the interactions with the two partial waves. Then the final spin

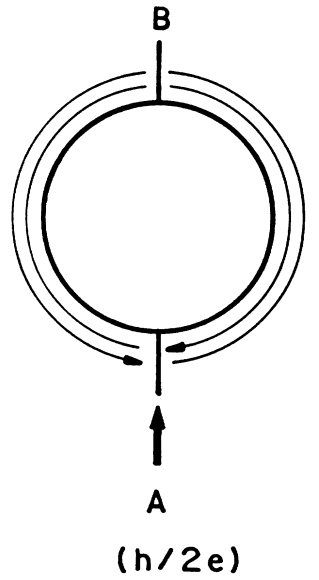

FIG. 3. Full circle interference experiment in an *A*-*B* ring. Both partial waves interact with the same environment degrees of freedom, possibly at different times. This kind of interference gives rise to $h/2e$ oscillations of the conductance.

states coupled to the two waves are not necessarily identical, and the interference might be affected.

Assuming that the interaction of electrons with magnetic impurities in a mesoscopic ring is a spin-spin interaction,

$$V=V(x)\tau\cdot\sigma\ , \tag{5.1}$$

where τ and σ are the electron and impurity spins, respectively, and x is the electron-impurity separation, the conditions for the full circle interference to survive the dephasing by magnetic impurities are then long spin-lattice relaxation times (long compared to the time it takes an electron to traverse the ring) and weak spin-spin scattering (or low magnetic impurities concentration—we consider here coupling to effectively one spin only). The latter is needed since, if each partial wave interacts with more than one spin, the change in the state of the electron's spin is different for each of the two waves, and the interference is affected. When these two conditions are fulfilled, the dephasing time due to magnetic impurities is expected to be significantly longer for full circle interference compared to half circle one.

We will now generalize this description of an environment interacting with the two waves for the dynamical environment of Sec. IV. In a derivation similar to that of Sec. IV

$$\begin{aligned}\psi(\tau_0)=l(\tau_0)\otimes e^{-iH_{\text{env}}\tau_0}\hat{T}&\\ \times\exp\left[-i\int_0^{\tau_0}dtV_I(x_l(t),t)\right]&\chi_0(\eta)\\ +r(\tau_0)\otimes e^{-iH_{\text{env}}\tau_0}\hat{T}&\\ \times\exp\left[-i\int_0^{\tau_0}dtV_I(x_r(t),t)\right]&\chi_0(\eta)\ ,\end{aligned} \tag{5.2}$$

and the interference term is then multiplied by

$$\begin{aligned}\langle\chi_0|\left[\hat{T}\exp\left[i\int_0^{\tau_0}dtV_I(x_l(t),t)\right]\right]&\\ \times\left[\hat{T}\exp\left[-i\int_0^{\tau_0}dtV_I(x_r(t),t)\right]\right]|\chi_0\rangle&\\ =\langle\chi_0|e^{-i\phi_l}e^{i\phi_r}|\chi_0\rangle\ .&\end{aligned} \tag{5.3}$$

If the two phase operators commute, then Eq. (5.3) is

$$\langle\chi_0|e^{i(\phi_r-\phi_l)}|\chi_0\rangle\ . \tag{5.4}$$

Hence the phase ϕ of Eq. (4.4) is replaced, in the case of an environment interacting with both partial waves, by the *relative* phase $\phi_r-\phi_l$, and it is the uncertainty in *that* phase which determines the loss of quantum interference. This uncertainty is always smaller than, or equal to, the sum of uncertainties in the two partial waves' phases. The case of noncommuting phases will be discussed elsewhere.[8]

A typical example where the same environment interacts with the two interfering waves is the interaction of an interfering electron with the electromagnetic fluctuations in vacuum.[3] In this case, if the two waves follow parallel paths with equal velocities, their dipole radiation, despite the energy it transfers to the field, does not dephase the interference. This radiation makes each of the partial waves' phases uncertain, but does not alter the relative phase. This example demonstrates that an exchange of energy is also not a sufficient condition for dephasing.

VI. THERMAL STATE VERSUS COHERENT STATE—AN APPLICATION

We now turn to a discussion of several applications of the general principle. In the first application the environment is a photon mode. We examine the effect of this environment on the interference pattern. First, we derive general expressions for the first two moments of the phase probability distribution. Then we analyze these expressions for the cases of the photon mode being in a coherent state and in a thermal state. Then we interpret the results in terms of changes the interfering wave induces in the state of the photon mode.

The interaction of the interfering particle with this photon mode, assuming (for brevity) that the dipole approximation can be made and that the mode's polarization is parallel to the particle's momentum, is of the form

$$V_I(x_r(t),t)=\frac{ep(t)}{m}\left[\frac{2\pi}{\omega}\right]^{1/2}(a^\dagger e^{i\omega t}+ae^{-i\omega t})\ , \tag{6.1}$$

where $p(t)=m\dot{x}(t)$ is the interfering particle's momentum, and a and $a^\dagger$ are photon annihilation-creation operators.

The environment's state, in its most general form, is

$$\chi_0=\sum_n c_n e^{i\gamma_n}|n\rangle\ , \tag{6.2}$$

where c_n and γ_n are real and positive, and $\sum_n|c_n|^2=1$.

The average phase shift is then

$$\begin{aligned}\langle\phi\rangle&=\int_0^{\tau_0}dt\langle\chi_0|V_I(x_r(t),t)|\chi_0\rangle\\ &=\int_0^{\tau_0}dt\frac{ep(t)}{m}\left[\frac{2\pi}{\omega}\right]^{1/2}\sum_n(c_{n+1}^*c_n\sqrt{n+1}e^{i(\gamma_{n+1}-\gamma_n+\omega t)}+c_{n-1}^*c_n\sqrt{n}\,e^{i(\gamma_{n-1}-\gamma_n-\omega t)})\ ,\end{aligned} \tag{6.3}$$

and the average of ϕ^2 is

$$\begin{aligned}\langle\phi^2\rangle=\int_0^{\tau_0}dt\int_0^{\tau_0}dt'\frac{2\pi e^2p(t)p(t')}{m^2\omega}\sum_n\{&c_{n+2}^*c_ne^{i(\gamma_n-\gamma_{n+2})}\sqrt{(n+1)(n+2)}e^{i\omega(t+t')}\\ &+c_{n-2}^*c_ne^{i(\gamma_n-\gamma_{n-2})}\sqrt{n(n-1)}e^{-i\omega(t+t')}\\ &+|c_n|^2[ne^{i\omega(t-t')}+(n+1)e^{-i\omega(t-t')}]\}\ .\end{aligned} \tag{6.4}$$

We now focus on two specific sets of c_n and γ_n. The first set describes a coherent Glauber state[9] $|Ce^{i\gamma}\rangle$,

$$c_n=\frac{C^n}{(n!)^{1/2}}e^{-c^2/2},\quad \gamma_n=n\gamma\ . \tag{6.5}$$

Substituting these values into the expressions (6.3) and (6.4), we obtain

$$\langle\phi\rangle=\frac{2e}{m}\left[\frac{2\pi}{\omega}\right]^{1/2}C\int_0^{\tau_0}dt\,p(t)\cos(\omega t+\gamma) \tag{6.6}$$

and

$$\langle\delta\phi^2\rangle\equiv\langle\phi^2\rangle-\langle\phi\rangle^2=\frac{2\pi e^2}{m^2\omega}\left|\int_0^{\tau_0}dt\,p(t)e^{-i\omega t}\right|^2 . \tag{6.7}$$

One interesting aspect of Eq. (6.7) is that it is independent of C, the coherent-state amplitude. The phase uncertainty results from the uncertainty in the "position" of the oscillator (i.e., the uncertainty in $a+a^\dagger$), and this uncertainty is the same for all coherent states. This result can also be explained in terms of an exchange of photons, i.e., a change in the state of the photon mode. In order to be shifted to a state orthogonal to its initial one, the number of photons in the mode has to change by n, where n has to be larger than the uncertainty in the number of photons in the original state. For a state $|Ce^{i\gamma}\rangle$, this condition is

$$n>C\ , \tag{6.8}$$

so that as C gets larger, the number of photons needed gets higher. Since the emission and absorption of photons is a random process, it takes n^2 emissions and absorptions until the number of photons changes by n. On the other hand, as C gets larger, the time needed for an emission or absorption of one photon gets shorter by a factor of C^{-2}. Hence the time needed for a shift of the mode to an orthogonal state does not depend on C.

The situation is different for a photon mode in a thermal state. For the state in Eq. (6.2) to be a thermal state, the c_n's have to be distributed thermally, i.e.,

$$c_n=e^{-\beta\omega n/2}(1-e^{-\beta\omega})^{1/2}\ . \tag{6.9}$$

The γ_n's have to be distributed in such a way that makes the photon mode exert a zero average potential, with finite fluctuations. Looking at Eq. (6.3), we find that this condition is satisfied when the average value of $e^{i(\gamma_n-\gamma_{n+1})}$ is zero. It happens at thermal equilibrium when the γ_n's are completely uncertain. This can be understood in terms of the coupling of the mode to a still larger heat bath. The phases γ_n are uncertain when the bath's states coupled to different energy states of the photon mode are orthogonal. Hence we describe the photon mode as a subsystem in a mixed state, where the whole system (the photon mode and the whole heat bath to which it is coupled) is in a pure state. Then, the first moments of the phase distribution are

$$\langle\phi\rangle=0\ , \tag{6.10}$$

$$\langle\phi^2\rangle=\int_0^{\tau_0}dt\int_0^{\tau_0}dt'\frac{2\pi e^2}{m^2\omega}p(t)p(t')\times(\langle n\rangle_{\text{th}}e^{i\omega(t-t')}+\langle n+1\rangle_{\text{th}}e^{-i\omega(t-t')})\ , \tag{6.11}$$

where $\langle\ \rangle_{\text{th}}$ means a thermal averaging. The phase uncertainty $\langle\delta\phi^2\rangle$ is then just the sum of probabilities of an emission and an absorption of a photon. Therefore, when the photon mode is in a thermal state, any emission or absorption of a photon turns the phase to be completely uncertain, and destroys the interference. Clearly, coupling of a partial wave to a thermal state destroys the interference faster than a coupling to a coherent state. One photon is enough in order to shift any thermal state into an orthogonal state, while it is usually not enough for such a shift in a coherent state. The two expressions for the phase uncertainty coincide only when the temperature of the thermal state is zero. For any finite temperature, the thermal state dephases more effectively than the coherent state. Therefore, if the interfering particle is driven by, e.g., a "classical" ac electric field exerted by photon modes in coherent states, the dephasing due to these modes will be weaker than that of the thermal radiation present in the experiment. This result is valid for an arbitrary strength of the driving electric field.

Looking at the dephasing due to a coupling to a thermal state from the point of view of the changes in the photon mode's state one might be concerned with the following dilemma: As the temperature gets higher, the uncertainty in the number of photons in the state gets higher, but still, the time it takes the state to be shifted into an orthogonal state gets shorter. This dilemma is settled by a closer inspection of the thermal state. Since the phases γ_n's are randomly distributed, an emission or absorption of one photon, even though much less than the uncertainty in the number of photons, is enough for the state to be shifted into an orthogonal one. This argument is true for all strongly nonminimal wave packets. A

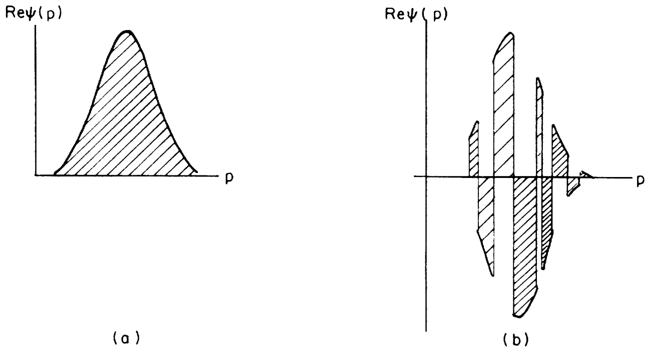

FIG. 4. Minimal vs nonminimal wave packet. (a) The real part of a minimal wave packet's wave function. In order to shift this function to an orthogonal state, it has to be shifted by more than its width. (b) The real part of a strongly nonminimal wave packet. Due to the rapidly changing signs, a small shift of the wave packet creates a state orthogonal to the original one.

nonminimal state has the property $\Delta p \Delta x \gg 1$ and consequently, its wave function has rapidly varying phases. The shift in phase space needed in order to shift such a wave function to an orthogonal state is much smaller than its width [see Figs. 4(a) and 4(b)].

The results of this section can be easily generalized to dephasing by phonon modes by replacing the electron-photon potential [Eq. (6.1)] with the appropriate electron-phonon potential. The difference between dephasing by a thermal state and dephasing by a coherent state is then found to be valid for phonon modes too.

VII. DEPHASING BY ELECTROMAGNETIC FLUCTUATIONS IN METALS

As the second application of our general principle we examine dephasing of interference phenomena in metals. This problem was previously discussed in the pioneering work of Alt'shuler, Aronov, and Khmlenitskii[4] (AAK), and later by Chakravarty and Schmid.[10] In Ref. 4 the problem was treated by deriving and solving an equation of motion for the cooperon. Some of the predictions of that work were convincingly confirmed in experiments by Wind *et al.* and later by Pooke *et al.*[11]

In this section of our work, we rederive the results of AAK by our phase uncertainty approach. Our derivation further clarifies the physical interpretation of these results. We also use our derivation to analyze the dephasing of conductance oscillations in an A-B ring, and conclude that the decay of these oscillations in a one-dimensional ring is not a simple exponential function of the ring's circumference.

We start by considering the interference of two electron paths $x_1(t)$ and $x_2(t)$ in a metal. These two paths start together at $t=0$ at $x=x_i$, and end together at $t=\tau_0$ at $x=x_f$. Out of all the environment degrees of freedom in the metal, we focus here on the interaction of the interfering electron with photons and other electrons. We investigate the dephasing of the interference due to these degrees of freedom by investigation the phase uncertainty it yields. Choosing an electromagnetic gauge where the scalar potential is identically zero, the first-order interaction of the interfering electron with the electromagnetic potentials is of the form

$$\frac{e}{c}\dot{\mathbf{x}}(t)\cdot \mathbf{A}(x(t),t) \ . \tag{7.1}$$

Thus the relative phase accumulated by the two interfering waves is

$$\phi=\frac{e}{c}\int_0^{\tau_0} dt[\dot{\mathbf{x}}_1(t)\cdot \mathbf{A}(x_1(t),t)-\dot{\mathbf{x}}_2(t)\cdot \mathbf{A}(x_2(t),t)] \ . \tag{7.2}$$

We have calculated the first two moments of ϕ for an electron in vacuum in our previous work.[3] Those moments involve two averages of the electromagnetic potential, i.e., $\langle A(x,t)\rangle$, and $\langle A(x,t)A(x',t')\rangle$. For a vacuum, as well as for an insulator, these averages can be easily calculated since they involve only photon modes. For a metal the calculation is more complicated. Following previous authors,[12] we find the average electromagnetic potential to be zero, so that $\langle\phi\rangle=0$, and we calculate $\langle A(x,t)A(x',t')\rangle$ by an application of the fluctuation-dissipation theorem. The details of the calculation are given in the Appendix. It is found there that

$$\langle A_\alpha A_\beta\rangle_{\mathbf{k},\omega}=\frac{\coth(\omega/2k_BT)c^2}{\sigma\omega}\times\left[\frac{k_\alpha k_\beta}{k^2}+\left[\delta_{\alpha,\beta}-\frac{k_\alpha k_\beta}{k^2}\right]\times\frac{1}{1+[k\delta(\omega)]^4}\right] , \tag{7.3}$$

where $\delta(\omega)\equiv c/\sqrt{4\pi\sigma\omega}$ is the skin depth.

This expression will now be the basic ingredient for the calculation of $\langle\phi^2\rangle$. As seen in Eq. (7.2), $\langle\phi^2\rangle$ includes four terms. For brevity, we calculate one of them, and then sum all the four. The first term is

$$\frac{e^2}{c^2}\int_0^{\tau_0}dt\int_0^{\tau_0}dt'\dot{x}_1^{\alpha}(t)\dot{x}_1^{\beta}(t')\langle A_\alpha(\mathbf{x}_1(t),t)A_\beta(\mathbf{x}_1(t'),t')\rangle$$
$$=\frac{e^2}{c^2}\int_0^{\tau_0}dt\int_0^{\tau_0}dt'\int d\mathbf{k}\int d\omega\, \dot{x}_1^{\alpha}(t)\dot{x}_1^{\beta}(t')e^{i\mathbf{k}\cdot[\mathbf{x}_1(t)-\mathbf{x}_1(t')]-i\omega(t-t')}\langle A_\alpha A_\beta\rangle_{\mathbf{k},\omega} \ . \tag{7.4}$$

Following AAK, we discuss the case in which most of the contribution to the $\mathbf{k}$ and ω integrals comes from values for which $k\delta(\omega)\gg 1$. For this case, most of the contribution to the phase uncertainty comes from longitudinal fluctuations of the electromagnetic potential. Then, Eq. (7.4) becomes

$$e^2\int_0^{\tau_0}dt\int_0^{\tau_0}dt'\int d\mathbf{k}\int d\omega\, \dot{x}_1^{\alpha}(t)\dot{x}_1^{\beta}(t')e^{i\mathbf{k}\cdot[\mathbf{x}_1(t)-\mathbf{x}_1(t')]-i\omega(t-t')}\frac{\coth(\omega/2k_BT)}{\sigma\omega}\frac{k_\alpha k_\beta}{k^2} \ . \tag{7.5}$$

Most of the contribution to the integral over ω comes from $\omega \ll k_BT$. Approximating $\omega\coth(\omega/2k_BT)$ by $2k_BT$, with a cutoff of the integral at $\omega\sim k_BT$, and evaluating the t and t' integrals by parts, we get

$$e^2\int_0^{\tau_0}dt\int_0^{\tau_0}dt'\int d\mathbf{k}\int_{-k_BT}^{k_BT}d\omega\, e^{i\mathbf{k}\cdot[\mathbf{x}_1(t)-\mathbf{x}_1(t')]-i\omega(t-t')}\frac{2k_BT}{\sigma k^2} \ . \tag{7.6}$$

Assuming that the duration of the experiment is much longer than $(k_B T)^{-1}$, the ω integral can be approximated by $\delta(t-t')$. This assumption is, in fact, the assumption that $k_B T \tau_\phi \gg 1$. Summing together all the four terms of the phase uncertainty, we obtain

$$\langle \delta\phi^2 \rangle = \int_0^{\tau_0} dt \int d\mathbf{k} \frac{e^2 k_B T}{\sigma k^2} \sin^2\{\mathbf{k}\cdot[\mathbf{x}_1(t)-\mathbf{x}_2(t)]\} \ , \tag{7.7}$$

and τ_ϕ, the phase breaking time, is the value of τ_0 for which the phase uncertainty is of order unity.

There are two important points that should be emphasized regarding this expression. The first is that $\langle \delta\phi^2 \rangle$ is not necessarily a linear function of time. Since the intensity of the interference term is reduced by the factor $e^{-1/2\langle \delta\phi^2 \rangle}$, this means that the interference term reduction does not have to be a simple exponential function of time. This result is important in the analysis of the conductance of a mesoscopic ring as a function of the magnetic flux inside the ring. It was shown in previous works[4,13] that if the interference term is reduced by a factor e^{-t/τ_ϕ}, then the flux sensitivity of the conductance decays exponentially as a function of the ring's circumference, i.e., it is proportional to e^{-L/L_ϕ}. This assertion is no longer valid when $\langle \delta\phi^2 \rangle \sim (t/\tau_\phi)^\alpha$ and $\alpha \neq 1$. We will shortly see that for typical paths in one dimension, $\alpha \neq 1$.

The second point is the strong dimensionality dependence of the phase uncertainty.[4] As seen in Eq. (7.7), for $d=1,2$ (where d is the dimensionality of the sample),

$$\langle \delta\phi^2 \rangle \sim \frac{e^2 k_B T}{\sigma} \int_0^{\tau_0} dt |x_1(t)-x_2(t)|^{2-d} \ . \tag{7.8}$$

Most of the contribution to the $\mathbf{k}$ integral of Eq. (7.7) comes from $k \sim |x_1(t)-x_2(t)|^{-1}$, and large values of k do not contribute. Since for typical paths in a diffusive medium $|x_1(t)-x_2(t)| \sim \sqrt{Dt}$, we obtain for these paths

$$\langle \delta\phi^2 \rangle \sim \frac{e^2 k_B T}{\sigma} D^{(2-d)/2} t^{(4-d)/2} \ , \tag{7.9}$$

and for the phase breaking time

$$\tau_\phi \sim \left[\frac{\sigma}{e^2 k_B T D^{(2-d)/2}} \right]^{2/(4-d)} \ . \tag{7.10}$$

For $d=3$, the $\mathbf{k}$ integral of Eq. (7.7) diverges at the upper limit. It is cut off by the condition $Dk^2 < \omega < k_B T$, i.e,. by $|\mathbf{k}| = (k_B T/D)^{1/2}$ (see the Appendix). Then

$$\langle \delta\phi^2 \rangle \sim \frac{e^2 k_B T}{\sigma} \left[\frac{k_B T}{D} \right]^{1/2} t \ , \tag{7.11}$$

where we assume that $(k_B T/D)^{1/2} |x_1(t)-x_2(t)| \gg 1$ for most values of t. Therefore, for $d=3$,

$$\tau_\phi \sim \frac{\sigma D^{1/2}}{e^2 (k_B T)^{3/2}} \ . \tag{7.12}$$

Thus, in metals, interference is dephased mainly by longitudinal fluctuations of the electromagnetic potential. Unlike the transverse fluctuations that originate in the photon modes and exist also for $\sigma \to 0$, the longitudinal modes originate from electron-electron interactions, and are diminished when the metal becomes an insulator. The most effective fluctuations in metals of one and two dimensions are those of wavelengths comparable to the distance between the two interfering paths, i.e., those where $k \sim l_\phi^{-1}$. Longer wavelengths contribute to the uncertainty in each wave's phase, but keep the relative phase well defined. Shorter wavelengths make the relative phase uncertain, but their magnitude is relatively small.

In two and three dimensions, the phase uncertainty increases linearly with time, and therefore the flux sensitive correction to the conductance of a ring decreases as a simple exponential function of the ring's circumference. In a one-dimensional ring,

$$\langle \delta\phi^2 \rangle \sim \left[\frac{t}{\tau_\phi} \right]^{3/2} \ . \tag{7.13}$$

The decrease of the above-mentioned correction with the increase of L is then different.[8]

CONCLUSIONS

In this paper we considered the loss of quantum interference of two paths, due to an interaction with the environment. We used two approaches and established their equivalence. We believe that besides their conceptual importance, the descriptions of phase accumulation as a statistical process and the loss of interference as resulting from an uncertainty in the phase are powerful tools in the analysis of quantum phenomena in the presence of an environment. As demonstrated in Secs. VI and VII, these descriptions allow us to analyze the effect of the environment on the interference via the statistical properties of the potential the environment exerts on the interfering particle, *with no need to analyze the detailed internal structure of the environment.* The dephasing is described in terms of equilibrium properties of the environment. On the other hand, the description of dephasing in terms of the changes induced by the interfering particle in its environment is a description of dephasing in terms of nonequilibrium processes. In this sense, the equivalence of the two approaches to dephasing relates equilibrium properties to nonequilibrium phenomena. Dissipation and/or exchange of energy, are not essential for this relation. As discussed in Sec. IV, the interference can only be retrieved by further interactions with the interfering particle.

Our analysis in this paper considered the interference of two *given* paths $x_r(t)$ and $x_l(t)$, and we have not yet commented on the way these two paths are chosen. In principle, the time evolution of the interfering particle is determined by a sum over all paths, so that its probability distribution $|\psi|^2$ involves a double sum over paths, i.e., a sum over many two-wave interference terms. Each one of these two-wave terms is dephased according to the principles discussed in this paper. In practice, however, one usually sums only the classical paths. In the presence of an environment, the classical paths should be calculated with the effect of the environment taken into account. We will elaborate on this point elsewhere.[8]

Our last comment regards the experimental relevance of our work. The relation between equilibrium fluctuations and dephasing, the different features of dephasing by thermal and coherent states, the possibility of different dephasing times for h/e and $h/2e$ flux periodic interferences, and the dependence of the flux sensitive correction of a ring's conductance on the ring's circumference can all be tested experimentally.

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APPENDIX: ELECTROMAGNETIC CORRELATOR IN METALS

In this appendix we calculate the correlation function $\langle A(x,t)A(x',t')\rangle$ in metals, needed for the calculation of the dephasing time (Sec. VII). The calculation is based[4] on an application of the fluctuation-dissipation (FD) theorem.[12] Assuming that the metal is uniform, $\langle A_\alpha(x,t)A_\beta(x',t')\rangle$ (where α and β are Cartesian components) depends only on $x-x'$ and $t-t'$. Thus its Fourier transform is $\langle A_\alpha A_\beta\rangle_{\mathbf{k},\omega} \equiv \langle A_\alpha(\mathbf{k},\omega)A_\beta(-\mathbf{k},-\omega)\rangle$, and it is convenient to proceed with the calculation in $\mathbf{k},\omega$ space. We use an electromagnetic gauge where the scalar potential is identically zero.

As a first step towards the application of the FD theorem, we calculate the linear response of a metal to an *external* driving current density $(1/c)\mathbf{j}^{\text{ext}}(\mathbf{k},\omega)$, i.e., we calculate the linear response function $\Sigma_{\alpha\beta}(\mathbf{k},\omega)$ satisfying the relation

$$A_\alpha(\mathbf{k},\omega)=\frac{1}{c}\Sigma_{\alpha\beta}(\mathbf{k},\omega)j_\beta^{\text{ext}}(\mathbf{k},\omega) \ . \tag{A1}$$

In the presence of such a driving current $\mathbf{j}^{\text{ext}}$, the total current in the metal $\mathbf{J}$ is

$$\mathbf{J}=\sigma\mathbf{E}+iD\mathbf{k}\rho+\mathbf{j}^{\text{ext}} \ . \tag{A2}$$

We assume that the conductivity tensor σ is diagonal, and is given by

$$\sigma_{\alpha\beta}(\mathbf{k},\omega)=\begin{cases}\sigma\delta_{\alpha\beta} & \text{for } a^{-1}\gg k\gg L_\alpha^{-1}, \quad \omega\ll\tau_d^{-1}\\ 0 & \text{otherwise}\end{cases} \tag{A3}$$

where L_α is the sample length in the α direction, a is the interatomic distance, and τ_d is the Drude relaxation time. Due to the diffusive nature of the conductance, the above expression for $\sigma_{\alpha\beta}$ is valid as long as $Dk^2<\omega$. Equation (A2) can be written in terms of components parallel ($\|$) and perpendicular ($\perp$) to $\mathbf{k}$,

$$J_\|=\sigma E_\|+iDk\rho+j_\|^{\text{ext}} \ , \tag{A4}$$

$$J_\perp=\sigma E_\perp+j_\perp^{\text{ext}} \ . \tag{A5}$$

Maxwell's equations in $\mathbf{k},\omega$ space are

$$\frac{\omega}{c}k\epsilon A_\|=4\pi\rho \ , \tag{A6}$$

$$k^2A_\perp=\frac{4\pi}{c}J_\perp+\frac{\epsilon\omega^2}{c^2}A_\perp \ , \tag{A7}$$

$$0=\frac{4\pi}{c}J_\|+\frac{\epsilon\omega^2}{c^2}A_\| \ , \tag{A8}$$

where ϵ is the real dielectric constant (defined by $\mathbf{D}=\epsilon\mathbf{E}$), and J is the *total* current in the metal.

Substituting Eqs. (A4) and (A5) into Maxwell's equations (A6)–(A8), we relate the electromagnetic potential $\mathbf{A}$ to the external current perturbation $\mathbf{j}^{\text{ext}}$,

$$A_\perp=\frac{4\pi c j_\perp^{\text{ext}}}{(kc)^2-\epsilon\omega^2-i4\pi\omega\sigma} \ , \tag{A9}$$

$$A_\|=\frac{4\pi c j_\|^{\text{ext}}}{i\omega(i\epsilon\omega-\epsilon Dk^2-4\pi\sigma)} \ , \tag{A10}$$

Equations (A9) and (A10) define the two response functions $\Sigma_\|$ and $\Sigma_\perp$ satisfying $A_{\|,\perp}=(1/c)\Sigma_{\|,\perp}j_{\|,\perp}^{\text{ext}}$. These two functions are related to the Cartesian response function $\Sigma_{\alpha\beta}$ by noting the following relations:

$$\begin{aligned}A_\alpha&=(\Sigma_\| j_\|^{\text{ext}})_\alpha+(\Sigma_\perp j_\perp^{\text{ext}})_\alpha\\ &=\frac{k_\alpha}{|k|^2}(\mathbf{k}\cdot\mathbf{j}^{\text{ext}})\Sigma_\|+\left[\mathbf{j}^{\text{ext}}-\frac{\mathbf{k}\cdot\mathbf{j}^{\text{ext}}}{|k|^2}\mathbf{k}\right]_\alpha\Sigma_\perp\\ &=\left[\frac{k_\alpha k_\beta}{|k|^2}\Sigma_\|+\left[\delta_{\alpha\beta}-\frac{k_\alpha k_\beta}{|k|^2}\right]\Sigma_\perp\right]j_\beta^{\text{ext}} \ ,\end{aligned} \tag{A11}$$

so that

$$\Sigma_{\alpha\beta}=\frac{k_\alpha k_\beta}{|k|^2}\Sigma_\|+\left[\delta_{\alpha\beta}-\frac{k_\alpha k_\beta}{|k|^2}\right]\Sigma_\perp \ . \tag{A12}$$

Finally, with the expression for $\Sigma_{\alpha\beta}$, we are in a position to apply the FD theorem. According to the theorem,[12]

$$\langle A_\alpha A_\beta\rangle_{\mathbf{k},\omega}=\text{Im}[\Sigma_{\alpha\beta}(\mathbf{k},\omega)]\coth\frac{\omega}{2k_BT} \ . \tag{A13}$$

Substituting Eqs. (A9), (A10), and (A12) into Eq. (A13), we obtain the electromagnetic potential correlation function (in $\mathbf{k},\omega$ space),

$$\langle A_\alpha A_\beta \rangle_{\mathbf{k},\omega}=\coth\left[\frac{\omega}{2k_BT}\right]\left[\frac{(4\pi c)^2\sigma\omega+4\pi c^2\omega\epsilon Dk^2}{\omega^2[\epsilon^2\omega^2+(\epsilon Dk^2+4\pi\sigma)^2]}\frac{k_\alpha k_\beta}{k^2}+\frac{(4\pi c)^2\sigma\omega}{[(kc)^2-\epsilon\omega^2]^2+(4\pi\omega\sigma)^2}\left[\delta_{\alpha,\beta}-\frac{k_\alpha k\beta}{k^2}\right]\right] . \tag{A14}$$

This is a general expression. Limiting ourselves to the case of metals, we can simplify it by a few approximations. First, the thermal factor $\coth(\omega/2k_BT)$ makes the correlator significant only for $\omega < k_BT$. For metals, such frequencies are much smaller than the conductivity. Moreover, $Dk^2 \ll 4\pi\sigma$ unless k is of the order of the screening wavelength. As we shall later see, the contribution of such wavelengths to the phase uncertainty is negligible. Therefore Eq. (A14) can be approximated by

$$\begin{aligned}\langle A_\alpha A_\beta \rangle_{\mathbf{k},\omega}&=\sigma\omega\coth\left[\frac{\omega}{2k_BT}\right]\left[\frac{c^2}{\omega^2\sigma^2}\frac{k_\alpha k_\beta}{k^2}+\frac{(4\pi c)^2}{[(kc)^4+(4\pi\omega\sigma)^2]}\left[\delta_{\alpha,\beta}-\frac{k_\alpha k_\beta}{k^2}\right]\right]\\&=\frac{\coth(\omega/2k_BT)c^2}{\sigma\omega}\left[\frac{k_\alpha k_\beta}{k^2}+\left[\delta_{\alpha,\beta}-\frac{k_\alpha k_\beta}{k^2}\right]\frac{1}{1+[k\delta(\omega)]^4}\right] ,\end{aligned} \tag{A15}$$

where $\delta(\omega)\equiv c/\sqrt{4}\pi\sigma\omega$ is the skin depth.

It might be instructive to summarize at this point the assumptions made while calculating this expression,

$$\omega < k_BT \ll \sigma , \tag{A16}$$

$$Dk^2 \ll 4\pi\sigma , \tag{A17}$$

$$Dk^2 < \omega \ll \tau_D^{-1} . \tag{A18}$$

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