MEASUREMENTS OF NONLOCAL VARIABLES

LEV VAIDMAN

School of Physics and Astronomy
Raymond and Beverly Sackler Faculty of Exact Sciences
Tel-Aviv University, Tel-Aviv 69978, Israel

Nonlocal variables are briefly reviewed and it is shown that all nonlocal variables related to two or more separate sites can be measured instantaneously, provided we restrict ourselves to verification measurements. The method is based on quantum teleportation.

Seventy years ago Landau and Peierls [1] claimed that the measurability of nonlocal variables contradicts relativistic causality. Twenty years ago, Aharonov and Albert [2] showed that some nonlocal variables can be measured and that this does not contradict causality. The question: "What are the *observables* of relativistic quantum theory?" remains topical even today [3]. In the thriving field of quantum communication this question is relevant for quantum cryptography and quantum computation performed with distributed systems. Here, using the techniques of the process of teleportation [4], I will show that *all* nonlocal variables related to two or more separate sites are measurable.

Although there are many papers on nonlocal measurements, there is no clear and unique definition of the concept of *nonlocal variable.* I will start with the discussion of a *nonlocal variable* of a compound quantum system consisting of several separated parts. One possible definition is:

Definition 1
Variable O of a compound system is nonlocal if it cannot be measured (in a nondemolition way) using measurements of local variables of all separate parts of the system.

According to this definition, for a system of two separated spin-$\frac{1}{2}$ particles, variable $\sigma_{Az} + \sigma_{Bz}$ is nonlocal, while $\sigma_{Az} + 2\sigma_{Bz}$ is local. Indeed, measurements of σ_{Az} and σ_{Bz} separately yield values of both variables of the composite system, but an eigenstate of $\sigma_{Az} + \sigma_{Bz}$, $\frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle)$ is disturbed by local measurements while all four eigenstates of $\sigma_{Az} + 2\sigma_{Bz}$ are not.

The requirement that the measurement is *nondemolition* might not be relevant for some considerations. Then we can modify the definition:

Definition 2
Variable O of a compound system is nonlocal if it cannot be verified (maybe in a demolition way) using measurements of local variables of all separate parts of the system.

According to this definition, the above variables are both local, but there are other variables of two spin-$\frac{1}{2}$ particles which are nonlocal. Probably the most popular example is the Bell operator which is defined by its four nondegenerate eigenstates:

$$\begin{aligned}
|\Psi_-\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B),\\
|\Psi_+\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\downarrow\rangle_B + |\downarrow\rangle_A|\uparrow\rangle_B),\\
|\Phi_-\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\uparrow\rangle_B - |\downarrow\rangle_A|\downarrow\rangle_B),\\
|\Phi_+\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\uparrow\rangle_B + |\downarrow\rangle_A|\downarrow\rangle_B).
\end{aligned} \tag{1}$$

It is interesting that entanglement of the eigenstates is not a necessary condition for the nonlocality. The variable with the following set of eigenstates which are all product states is also nonlocal [6]:

$$\begin{aligned}
|\Psi_1\rangle &= |\uparrow_z\rangle_A|\uparrow_z\rangle_B,\\
|\Psi_2\rangle &= |\uparrow_z\rangle_A|\downarrow_z\rangle_B,\\
|\Psi_3\rangle &= |\downarrow_z\rangle_A|\uparrow_x\rangle_B,\\
|\Psi_4\rangle &= |\downarrow_z\rangle_A|\downarrow_x\rangle_B.
\end{aligned} \tag{2}$$

The question I want to discuss here is the measurability of nonlocal variables. I consider instantaneous von Neumann measurements relaxing the requirement of repeatability, i.e., as in Definition 2, the requirement that the measurement is nondemolition. The existence of a measurement which yields the eigenvalue of a variable with certainty, if prior to the measurement the quantum system was in the corresponding eigenstate, gives the physical meaning for such a variable. (The need to relax the requirement of repeatability was clear before [7], when it has been shown that measurements of some nonlocal variables erase local information and, therefore, cannot be nondemolition.)

The meaning of "instantaneous measurement" is that in a particular Lorentz frame, at time t, we perform local actions for a duration of time which can be as short as we wish. At the end of the procedure (arbitrary small period of time after t) there are local records which together yield the outcome of the measurement of the nonlocal variable. The question I ask: "Is it possible to measure nonlocal variables (defined by Definition 2) in instantaneous measurements of this type?" We assume that it is allowed to perform, beyond local measurements, arbitrary local interactions and to use *prior entanglement* between the sites of different parts of the system.

In fact, it was known before [8] that the Bell operator variable can be measured, and even in a nondemolition way. The variable (2) cannot be measured in a nondemolition way: its measurability would allow superluminal communication. The question of measurability (in a demolition way) of nonlocal variable of all types has been answered only recently [9]. Here I will report this result, showing that *all* nonlocal variables related to two or more separate sites are measurable.

I will start explaining the method by describing the measurement of a nonlocal variable with nondegenerate eigenstates (2). The first step of the measurement is the teleportation of the state of the spin from B (Bob's site) to A (Alice's site). Bob and Alice do not perform the full teleportation (which invariably requires a finite period of time), but only the Bell measurement at Bob's site which might last, in principle, as short a time as we wish. (I will continue to use the term "teleportation" just for this first step of the original proposal [4].) This action teleports the state of particle B except for a possible rotation by π (known to Bob) around one of the axes of teleportation: $\hat{x}_1$, $\hat{x}_2$, or $\hat{x}_3$.

The second step is taken by Alice. She can perform it at time t without waiting for Bob. She measures the spin of her particle in the z direction. If the result is "up", she measures the spin of the particle teleported from Bob in the z direction and if the result is "down", she measures the spin in the z direction. This completes the measurement. Indeed, the eigenstates of the spin in the z direction are teleported without leaving the z line and the eigenstates of the spin in the x direction are teleported without leaving the x line. Thus, Bob's knowledge about possible flip together with Alice's results distinguish unambiguously between the states Ψ_i.

Next, consider the measurement of a nonlocal variable of two spin-$\frac{1}{2}$ particles located in separate locations A and B, whose eigenstates are the following generalization of (2):

$$
\begin{aligned}
|\Psi_1\rangle &= |\uparrow_z\rangle_A\,|\uparrow_z\rangle_B,\\
|\Psi_2\rangle &= |\uparrow_z\rangle_A\,|\downarrow_z\rangle_B,\\
|\Psi_3\rangle &= |\downarrow_z\rangle_A\,|\uparrow_\theta\rangle_B,\\
|\Psi_4\rangle &= |\downarrow_z\rangle_A\,|\downarrow_\theta\rangle_B,
\end{aligned}
\tag{3}
$$

where $|\uparrow_\theta\rangle$ is an eigenstate of a spin pointing in a direction $\hat{\theta}$ making angle θ with the z axis. The method of measurement of this variable was found recently using a different approach [10] and this result inspired the current work.

The first step is, again, the Bell measurement at Bob's site which teleports the state of the spin from B to A except for a possible rotation by π (known to Bob) around one of the axes of teleportation: $\hat{x}_1$, $\hat{x}_2$, or $\hat{x}_3$. This time Bob modifies the axes of teleportation (which define the eigenstates of the Bell measurement) in the following way: $\hat{x}_3 = \hat{z}$ and $\hat{x}_1$ is such that $\hat{\theta}$ lies in the plane of $\hat{x}_3$ and $\hat{x}_1$, see Fig. 1.

The second step is taken by Alice at time t. As in the previous case, she measures the spin of her particle in the z direction.

If the result is "up", she measures the spin of the particle teleported from Bob in the z direction and this completes the measurement since the eigenstates of the spin in the z direction are teleported without leaving the z line and, therefore, Bob's knowledge about possible flip together with Alice's results distinguish unambiguously between $\Psi_1$and Ψ_2.

If the result is "down", Alice cannot perform a measurement on Bob's teleported particle because it has spin either along the line of $\hat{\theta}$ (corresponding to teleportation without rotation or rotation around $\hat{x}_2$) or along the line of $\hat{\theta}'$ obtained from the line of $\hat{\theta}$ by π rotation around $\hat{x}_1$ (or $\hat{x}_3$). In this case, Alice teleports Bob's teleported state back to Bob using a new teleportation axes defined by $\hat{x}'_3 = \hat{\theta}$ and $\hat{x}'_2 = \hat{x}_2$.

In the third step, Bob performs an action similar to that of Alice in step 2. He knows whether the spin state in θ direction was teleported to Alice along the $\hat{\theta}$ line

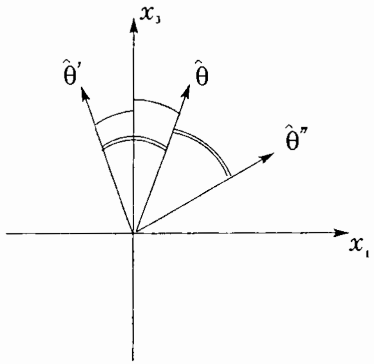

Figure 1. The axes of teleportations. Bob starts with teleportation choosing $\hat{x}_3$ and $\hat{x}_1$ such that $\hat{\theta}$ lies in the plane defined by the axes. Alice teleports back with $\hat{x}'_3 = \hat{\theta}$. Bob continues with $\hat{x}''_3 = \hat{\theta}'$, etc.

or along the $\hat{\theta}'$ line. In the former case, the state teleported to him is still along the $\hat{\theta}$ line, so he completes the procedure by spin measurement in this direction. In the latter case, he receives the spin either along the $\hat{\theta}'$ line or along the $\hat{\theta}''$ line obtained by π rotation around the $\hat{\theta}$ axis. In this case, he teleports the particle back with the teleportation axes $\hat{x}''_3 = \hat{\theta}'$ and $\hat{x}''_2 = \hat{x}_2$.

Alice and Bob continue this procedure. If $\theta = \frac{k}{2^n}\pi$, the line $\hat{\theta}^{(n)}$ coincides with the line $\hat{\theta}^{(n-1)}$ because the angle between the lines is: $\theta^{(n)} - \theta^{(n-1)} = (2^n\theta)\text{mod}\pi$. In this particular case the process is guaranteed to stop after n teleportation steps. If the lines do not coincide, the probability that after n teleportations the result of the measurement is not known is 2^{-n}, so the probability of success can be made as large as we wish. There is no minimal time for performing all the steps of this procedure. Bob and Alice need not wait for each other: they only have to specify before the measurement the teleportation channels they will use. Note, that usually Alice and Bob will use only a small number of teleportation channels: they stop when both Alice and Bob make teleportations which do not change the line of the spin. Thus, this method requires less resources than the alternative approach [10].

Groisman and Reznik [10] showed also how to measure other nonlocal variables of two spin-$\frac{1}{2}$ particles. The method I presented above can be modified for these variables too. However, I will turn now to another, universal, method which is applicable to any nonlocal variable $O(q_A, q_B, ...)$, where q_A belongs to region A, etc. I will not try to optimize the method or consider any realistic proposal: my task is to show that, given unlimited resources of entanglement and arbitrary local interactions, any nonlocal variable is measurable.

I will start with the case of a general variable of a composite system with two parts. First, (for simplicity), Alice and Bob perform unitary operations which swap the states of their systems with the states of sets of spin-$\frac{1}{2}$ particles. In this way Alice and Bob will need the teleportation procedure for spin-$\frac{1}{2}$ particles only.

Teleporting the states of all individual spins teleports the state of the set, be it entangled or not.

The general protocol is illustrated in Fig. 2. Bob teleports his system to Alice. Again, he does not send to Alice the results of his Bell measurements, but keeps them for his further actions; we signify the possible outcomes of these measurements by $i = 1, \ldots N$. The outcome $i = 1$ corresponds to finding singlets in all Bell measurements and in this case the state of Bob's system is teleported without distortion.

Alice performs a unitary operation on the composite system of her spins and the teleported spins which, under the assumption of non-distorted teleportation, transforms the eigenstates of the nonlocal variable (which now actually are fully located in Alice's site) to product states in which each spin is either "up" or "down" along the z direction. Then she teleports the *complete* composite system consisting of her spins and Bob's teleported spins to Bob. From now on this is the system which will be teleported back and forth between Bob and Alice. In all these teleportations the usual z, x, y basis is used. Hence, if the state is in the one of the product states in spin z basis, then it will remain in this basis.

If, indeed, Bob happened to teleport his spins without any distortion, i.e., $i = 1$ (the probability for which is $\frac{1}{N}$), Bob gets the composite system in one of the spin z product states and his measurements in the spin z basis that he now performs, complete the measurement of the nonlocal variable. If $i \neq 1$, Alice's operation does not bring the eigenstates of the nonlocal variable to the spin z basis, so Bob cannot perform the measurement and he teleports the system back to Alice following a protocol that we explain below.

Alice and Bob have numerous teleportation channels arranged in $N-1$ clusters numbered from 2 to N. Each cluster consists of two teleportation channels capable to teleport the complete system and $M-1$ clusters of a similar type, where M is the number of possible outcomes of the Bell measurement for teleportation of the complete system. In turn, each of the $M-1$ clusters consists of two teleportation channels and $M-1$ further nested clusters, etc.

If in his first teleportation the result of Bob's Bell measurements is i, he teleports now the composite system back to Alice in the teleportation channel of cluster i. Alice does not know in which channel she gets the system back (if she gets it back at all). So she must work on all of them. She knows that if she does get the system in channel i, the result of the Bell measurement in Bob's first teleportation was i. Thus, she knows all the transformations performed on this system except for the last teleportation. Alice performs a unitary operation that transforms eigenstates of the nonlocal variable to product states under the assumption that the last teleportation was without distortion and teleports the system back to Bob.

Let us denote the result of the Bell measurement in Bob's last teleportation by i', $i' = 1, \ldots, M$. Again, for $i' = 1$ which corresponds to finding singlets for all Bell measurements, Bob performs the spin z measurement on the system which he receives in the teleportation channel of the cluster i. This then completes the nonlocal measurement. Otherwise, he teleports the system back in the channel of the sub-cluster i'. Alice and Bob continue this procedure. The nonlocal measurement is completed when, for the first time, Bob performs a teleportation without

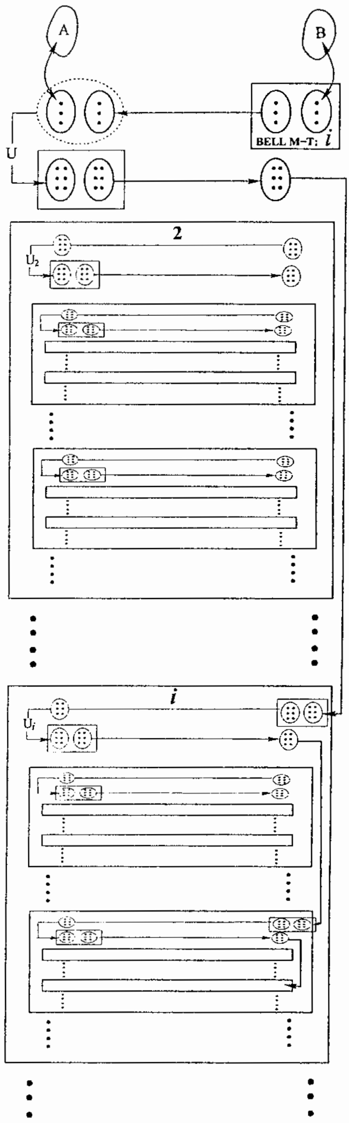

Figure 2. The scheme of the measurement of a nonlocal variable of a two-part system.

distortion. Since, conceptually, there is no limitation for the number of steps, and each step (starting from the second) has the same probability for success, $\frac{1}{M}$, the measurement of the nonlocal variable can be performed with probability arbitrarily close to 1.

The generalization to a system with more than two parts is more or less straightforward. Let us sketch it for a three-part system. First, Bob and Carol teleport their parts to Alice. Alice performs a unitary transformation which, under the assumption of undisturbed teleportation of both Bob and Carol, transforms the eigenstates of the nonlocal variable to product states in the spin z basis. Then she teleports the complete system to Bob. Bob teleports it to Carol in a particular channel i_B depending on the results of the Bell measurement of his first teleportation. Carol teleports all the systems from the teleportation channels from Bob back to Alice in the channels (i_B, i_C) depending on her Bell measurement result i_C. The system corresponding to $(i_B, i_C) = (1, 1)$ is not teleported, but measured by Carol in spin z basis. Alice knows the transformation performed on the system which arrives in her channels (i_B, i_C) except for corrections due to the last teleportations of Bob and Carol. So she again assumes that there were no distortions in those, and teleports the system back to Bob after the unitary operation which transforms the eigenstates of the variable to product states in the spin z basis. Alice, Bob and Carol continue the procedure until the desired probability of successful measurement is achieved.

The required resources, such as the number of teleportation channels and required number of operations are very large, but this does not concern us here. We have shown that there are no relativistic constraints preventing instantaneous measurement of any variable of a quantum system with spatially separated parts, answering the above long standing question.

Can this result be generalized to a quantum system which itself is in a superposition of being in different places? The key to this question is the generality of the assumption of the possibility to perform any local operation. If a quantum state of a particle which is in a nonlocal superposition can be locally transformed to (an entangled) state of local quantum systems, then any variable of the particle is measurable through the measurement of the corresponding composite system. However, while for bosons it is clear that there are such local operations (transformation of photon state to entangled state of atoms has been achieved in the laboratory [11]), for fermion states the situation is different [12]. If the transformation of a superposition of a fermion state to local variables is possible, then these local separated in space variables should fulfill anti-commutation relations. This is the reason to expect super-selection rules which prevent such transformations.

It is a pleasure to thank Yakir Aharonov, Shmuel Nussinov and Benni Reznik for helpful discussions. This research was supported in part by grant 62/01 of the Israel Science Foundation and by the Israel MOD Research and Technology Unit.

References

1. L. Landau and R. Peierls, Z. Phys. **69**, 56 (1931).
2. Y. Aharonov, and D. Z. Albert, Phys. Rev. D **24**, 359 (1981).

3. D. Beckman *et al.*, Phys. Rev. A **64**, 052309 (2001).
4. C. H. Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993).
5. C. H. Bennett *et al.*, Phys. Rev. A **59**, 1070 (1999).
6. B. Groisman and L. Vaidman, J. Phys. A **34**, 6881 (2001).
7. S. Popescu and L. Vaidman, Phys. Rev. A **49**, 4331 (1994).
8. Y. Aharonov, D. Albert and L. Vaidman, Phys. Rev. D **34**, 1805 (1986).
9. L. Vaidman, quant-ph/0111124.
10. B. Groisman and B. Reznik, Phys. Rev. A, to be published [quant-ph/0111012] (2002).
11. X. Maitre, *et al.*, Phys. Rev. Lett. **79**, 769 (1997).
12. Y. Aharonov, and L. Vaidman, Phys. Rev. A **61**, 052108 (2000).

DISCUSSION

Chairman: R. Chiao

G. Hegerfeldt: I have a very simple question. As I understand you are working in non-relativistic quantum mechanics. Then you are using the finite velocity of light. How can you bring them together?

L. Vaidman: The context of the work is following. We have non-relativistic quantum mechanics and then, when people talk about relativistic quantum mechanics, they usually go to field theory, string theory and so on. The question is: Can we go and use the concept of non-relativistic quantum mechanics correctly in relativistic quantum mechanics? If we have a variable of the form $O(q_a, q_b, q_c, t)$, does it have meaning in relativistic quantum mechanics? The claim is that it does have a meaning. It is clearly physical and it is measurable in Nature. There is a procedure, which will tell us exactly what it is. It will transform accordingly from one Lorentz frame to another because it is measurable. So, this variable has a meaning in relativistic quantum mechanics.

E. Polzik: Your results imply that if there is an entanglement shared between two particles, the presence of this entanglement can be verified instantaneously.

L. Vaidman: No. When you make a quantum measurement, you measure an operator. You find an eigenvalue. You don't know that before the measurement the state was the eigenstate with this eigenvalue. You know that the state was not orthogonal to this eigenstate. So, if I found an entangled state $\frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B$ it might be that before the measurement it was just $|\uparrow\rangle_A|\downarrow\rangle_B$. I had a 50% chance to find it entangled.