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The Meaning of Elements of Reality and Quantum Counterfactuals: Reply to Kastner

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This paper is an answer to the preceding paper by Kastner, in which she continued the criticism of the counterfactual usage of the Aharonov–Bergman–Lebowitz rule in the framework of the time-symmetrized quantum theory, in particular, by analyzing the three-box "paradox." It is argued that the criticism is not sound. Paradoxical features of the three-box example are discussed. It is explained that the elements of reality in the framework of time-symmetrized quantum theory are counterfactual statements, and therefore, even conflicting elements of reality can be associated with a single system. It is shown how such "counterfactual" elements of reality can be useful in the analysis of a physical experiment (the three-box example). The validity of Kastner's application of the consistent histories approach to the time-symmetrized counterfactuals is questioned.

1. ELEMENTS OF REALITY

Quantum theory teaches us that the concepts of "reality" developed on the basis of the classical physics are not adequate for describing our world. A new language with concepts which *are* appropriate is not developed yet, and this is probably the root of numerous controversies regarding interpretation of quantum formalism. It seems to me that philosophers of science can make a real contribution for progress of quantum theory through developing of an appropriate language. A necessary condition for a success of this wisdom is that physicists and philosophers will try to understand each other. I hope that the resolution of the current controversy about the time-symmetrized quantum theory (TSQT) will contribute to such understanding.

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I took part in the development of the TSQT,[1–3] and I believe that this is an important and useful formalism. It has already helped us to find several peculiar quantum phenomena tested in laboratories in the world.[4, 5] In the framework of the TSQT, I have used terms such as "elements of reality"[6, 7] in a sense which seems to be radically different from the concept of reality considered by philosophers, and apparently, this is the main reason for the current controversy.

I *define* that there is an element of reality at time t for an observable C, "$C = c$" when it can be inferred with certainty that the result of a measurement of C, if performed, is c. Frequently, in such a situation it is said that the observable C has the value c. It is important to stress that both expressions do not assume "ontological" meaning for c, the meaning according to which the system has some (hidden) variable with the value c. I do not try to restore realistic picture of classical theory: in quantum theory observables do not possess values. The only meaning of the expressions, "The element of reality $C = c$" and "C has the value c," is the operational meaning: it is known with certainty that if C is measured at time t, then the result is c.

Clearly, my concept of elements of reality has its roots in "elements of reality" from the Einstein, Podolsky, and Rosen (EPR) paper.[8] There are numerous works analyzing the EPR elements of reality. My impression that EPR were looking for an ontological concept and their "criteria for elements of reality" is just a property of this concept. I had no intention to define such ontological concept. I apologize for taking this name and using it in a very different sense, thus, apparently, misleading many readers. I hope to clarify my intentions here and I welcome suggestions for alternative name for my concept which will avoid the confusion.

I consider elements of reality as counterfactual statements. Even if at time t the system undergoes an interaction with a measuring device which measures C, the truth of "$C = c$" is ensured not by the final reading of the pointer of this measurement, but by a counterfactual statement that if another measurement, with as short duration as we want, is performed at time t, it invariably reads $C = c$.

2. THE THREE-BOX EXAMPLE

The *actual* story is as follows.

(i) A macroscopic number N of particles (gas) were all prepared at t_1 in a superposition of being in three separated boxes,

$$|\psi_1\rangle = \frac{1}{\sqrt{3}}(|A\rangle + |B\rangle + |C\rangle) \tag{1}$$

with obvious notation: $|A\rangle$ is the state of a particle in box A, etc.

(ii) At a later time, t_2, all the particles were found in another superposition (this is an extremely rare event):

$$|\psi_2\rangle = \frac{1}{\sqrt{3}}(|A\rangle + |B\rangle - |C\rangle) \tag{2}$$

(iii) In between, at time t, *weak measurements* of a number of particles in each box, which are, essentially, usual measurements of pressure in each box, have been performed. The readings of the measuring devices for the pressure in boxes A, B, and C were

$$\begin{aligned} p_A &= p \\ p_B &= p \\ p_C &= -p \end{aligned} \tag{3}$$

where p is the pressure which is expected to be in a box with N particles.

I am pretty certain that this "actual" story never took place because the probability for successful postselection (ii) is of the order of 3^{-N}; for a macroscopic number N it is too small for any real chance to see it happens. However, given that postselection (ii) does happen, I am safe to claim that (iii) is correct, i.e., the measurements of pressure at the intermediate time with a very high probability have shown the results (3).

The description of this example in the framework of the time-symmetrized quantum formalism is as follows. Each particle at time t is described by the two-state vector

$$\langle \psi_2|\,|\psi_1\rangle = \tfrac{1}{3}(\langle A| + \langle B| - \langle C|)(|A\rangle + |B\rangle + |C\rangle) \tag{4}$$

The system of all particles (signified by index i) is described by the two-state vector

$$\langle \psi_2|\,|\psi_1\rangle = \frac{1}{3^N}\prod_{i=1}^{i=N}(\langle A|_i + \langle B|_i - \langle C|_i)\prod_{i=1}^{i=N}(|A\rangle_i + |B\rangle_i + |C\rangle_i) \tag{5}$$

The ABL formula for the probabilities of the results of the intermediate measurements yields, for each particle, probability 1 for the the following outcomes of measurements:

$$\begin{aligned} \mathbf{P}_A &= 1 \\ \mathbf{P}_B &= 1 \\ \mathbf{P}_A + \mathbf{P}_B + \mathbf{P}_C &= 1 \end{aligned} \tag{6}$$

where $\mathbf{P}_A$ is the projection operator on the state of the particle in box A, etc. Or, using my definition, for each particle there are three *elements of reality*: the particle is inside box A, the particle is inside box B, and the particle is inside boxes A, B, and C.

A theorem in the TSQT (Ref. 3, p. 2325) says that a weak measurement, in a situation in which the result of a usual (strong) measurement is known with certainty, yields the same result. Thus, from (6) it follows that

$$\begin{aligned} (\mathbf{P}_A)_w &= 1 \\ (\mathbf{P}_B)_w &= 1 \\ (\mathbf{P}_A + \mathbf{P}_B + \mathbf{P}_C)_w &= 1 \end{aligned} \tag{7}$$

Since for any variables, $(X + Y)_w = X_w + Y_w$, we can deduce that $(\mathbf{P}_C)_w = -1$.

Similarly, for the "number operators" such as $\mathcal{N}_A \equiv \sum_{i=1}^{i=N} \mathbf{P}_A^{(i)}$, where $\mathbf{P}_A^{(i)}$ is is the projection operator on the box A for a particle i, we obtain

$$\begin{aligned} (\mathcal{N}_A)_w &= N \\ (\mathcal{N}_B)_w &= N \\ (\mathcal{N}_C)_w &= -N \end{aligned} \tag{7}$$

In this rare situation the "weak measurement" need not be very weak: a usual measurement of pressure is a weak measurement of the number operator. Thus, the time-symmetrized formalism yields a surprising result (3): the result of the pressure measurement in box C is negative! It equals minus the pressure measured in boxes A and B.

The analysis of "elements of reality" in this example which are clearly counterfactual statements (in the actual world, measurements, results of which are written in Eq. (6), have not been performed) yields a tangible fruit: a shortcut for calculation of the expected outcome of an actual measurement. [2] This outcome is surprising and paradoxical. Indeed, a usual

[2] This example answers the criticism of Mermin[9] quoted by Kastner[10] in the context of my work. According to this criticism, the elements of reality I defined are "rubbish—they have nothing to do with anything."

device for measuring an observable which has only positive eigenvalues yields a negative value, the weak value in this rare pre- and postselected situation.

There are other paradoxical aspects discussed in relation to this example. The first paradoxical issue which was discussed[11] concerns contextuality. Consider an observable X which tells us the location of the particle: Is it in box A, B, or C? The eigenstate of this observable corresponding to finding the particle in box A is identical to the eigenstate of the projection operator on A: $|X = A\rangle = |\mathbf{P}_A = 1\rangle$. However, in this example there is no element of reality $X = A$ (if we measure X by opening all boxes at time t, we have only the probability $\frac{1}{3}$ to find the particle in box A), despite the fact that $\mathbf{P}_A = 1$ is an element of reality. Kastner discussed another paradoxical aspect of the three-box example. It is discussed in the next section.

3. KASTNER'S ANALYSIS OF THE THREE-BOX EXAMPLE

In the three-box example there are two elements of reality for the *same* particle: "The particle is in box A," and "The particle is in box B." Kastner[12] considers this situation as a paradox which she resolves by rejecting the legitimacy of my concept of elements of reality. She does not mention my resolution of the "paradox." Elements of reality are counterfactual statements. To be more explicit, "The particle is in box A" means that if the particle is searched for in box A (and if it is not searched for in box B!), then it is certain that the particle will be found in box A. Obviously, the two elements of reality cannot be considered together. Each element of reality assumes that the antecedent of the other element of reality is false. Thus, both elements of reality exist separately, but we should not conclude from this that there is an element of reality consisting of the union of these elements of reality: the antecedent, "The particle is searched for in A and it is not searched for in B and the particle is searched for in B and it is not searched for in A," is logically inconsistent. The fact that we cannot consider the union of elements of reality does not make the whole exercise empty. We still can consider consequences of all true elements of reality together. In particular, in the three-box example the consequences of elements of reality (6) are the statements about weak values (7) and weak measurements which yield these weak values *can* be performed together.

Kastner finds the elements of reality "The particle is in box A" and "The particle is in box B" to be "highly peculiar and counterintuitive." This is indeed so, especially because there is no element of reality, "The particle is in box A and in box B," as explained above. This peculiar situation is an example of the failure of the "product rule" for pre- and postselected elements of reality.[6] From $A = a$ and $B = b$ it does not follow that $AB = ab$. The element of reality, "The particle is in box A and in box B," corresponds to the definite value of the product of projection operators: $\mathbf{P}_A \mathbf{P}_B = 1$. But in the three-box examples, $\mathbf{P}_A \mathbf{P}_B = 0$, despite the fact that $\mathbf{P}_A = 1$ and $\mathbf{P}_B = 1$.

Kastner's main objection is that the elements of reality, "The particle is in box A" and "The particle is in box B," cannot be interpreted as applying to an individual particle because "in any given run of the experiment in which a given particle X is postselected, we can measure only one of the two observables A and B." She does not take into account that "elements of reality" are just counterfactual statements. She does not pay attention to the word "instead" in my writings, which she herself quotes in her paper: "If in the intermediate time it was searched for in box A, it has to be found there with probability one, and if, instead, it was searched for in box B, it has to be found there too with probability one... ."

For demonstration that Kastner's criticism is unfounded, let me repeat here an example[13] in which attributing properties which cannot be observed together to an individual system is not controversial.

Consider a system of two spin-$\frac{1}{2}$ particles prepared, at t_1, in a singlet state,

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2) \tag{9}$$

We can predict with certainty that the results of measurements of spin components of the two particles fulfill the following two relations:

$$\{\sigma_{1x}\} + \{\sigma_{2x}\} = 0 \tag{10}$$

$$\{\sigma_{1y}\} + \{\sigma_{2y}\} = 0 \tag{11}$$

where $\{\sigma_{1x}\}$ signifies the result of measurement of the spin x component of the first particle, etc. Relations (10) and (11) cannot be tested together: the measurement of σ_{1x} disturbs the measurement of σ_{1y} and the measurement of σ_{2x} disturbs the measurement of σ_{2y} (not necessarily in the same way). According to the standard approach to quantum theory, we accept that there are two matters of fact—"The outcomes of the spin x components for the two particles have opposite values" and "The outcomes of the spin y components for the two particles have opposite values"—despite the fact that the two statements cannot be tested together. If the spin x components have been measured at time t, we know that y components of spin were not measured at time t. Note that if they were measured at a later time, after the spin x component measurement, then the outcomes might not fulfill Eq. (11). According to Kastner's line of argument, the application of statements (10) and (11), which I named "generalized elements of reality" (because they are not just about the values of observables, but about *relations* between these values), to a single quantum system should also be rejected. However, physicists do not reject such statements. There are innumerable works analyzing counterfactuals related to incompatible measurements on a single system of correlated spin-$\frac{1}{2}$ particles. Similarly, Kastner's argument is not valid for the three-box example.

Kastner (Ref. 12) explains her claim in the following sentence:

> Since we cannot say for sure that particle X [which was observed at the intermediate time in A] would have been postselected [in the particular state $|\psi_2\rangle$] via an intervening measurement of B and since attributing the ABL probability of unity to particle X crucially *depends* on such an outcome for particle X, it is clearly incorrect to say of any such particle X that it also, with certainty, had a "counterfactual" probability 1 of being found in box B [at the intermediate time].

But in the ABL setup "such particle" means exactly that it *was* postselected in the state $|\psi_2\rangle$. In the time-symmetric approach preselection and postselection have the same status. Kastner's approach, in which only preselection is fixed, is explicitly time-asymmetric. Note also that fixing the postselection by fiat is the only known option in the framework of quantum theory: we "cannot say for sure that particle X would have been postselected in *any* counterfactual world, even in the world in which the particle was searched in box A, but, say, by another observer." See more discussion of this point in Section 9 of Ref. 13.

4. QUANTUM COUNTERFACTUALS

I try here to clarify my statements which are criticized in Section 4 of Kastner paper.[12]

First, the meaning of the quotation from my work, "Indeterminism is crucial for allowing nontrivial time-symmetric counterfactuals," is just the following. Time-symmetric counterfactuals are related to time-symmetric background conditions; i.e., the state of the system is fixed both before and after the time about which the counterfactual statement is given. In a deterministic theory everything is fixed by conditions at a single time, and therefore, no novel (nontrivial) features can appear in the time-symmetric approach.

To clarify the meaning of my continuation, "Lewis's and other general philosophical analyses are irrelevant for the issue of counterfactuals in quantum theory," let me quote Lewis' "system of weights or priorities" for similarity relation of counterfactual worlds (Ref. 14, p. 47):

(1) It is of the first importance to avoid big, widespread, diverse violations of [physical] law

(2) It is of the second importance to maximize the spatio-temporal region throughout which perfect match of particular facts prevails.

(3) It is of the third importance to avoid even small, localized, simple violations of law.

(4) It is of little or no importance to secure approximate similarity of particular fact, even in matters that concern us greatly.

This priorities might be helpful in the analysis of the truth value of a widely discussed counterfactual: "If Nixon had pressed the nuclear war button, the world would be very different." The purpose of the priorities is to "resolve the vagueness of counterfactuals." In physics context, however, the counterfactuals are not vague. (At least, I hope that the counterfactuals I have defined are not vague.) The truth value of quantum counterfactuals can be calculated from the equations of quantum theory. The above priorities cannot help in deciding the truth value of the counterfactual, "The outcomes of the spin y components measurement at time t for the two particles have opposite values," in the example discussed above. Priorities (1) and (3) are not relevant because violations of physical laws are not considered. The counterfactual worlds are different from the actual world not because of "miracles," i.e., violations of physical laws, but because different measurements on the system are considered. And the question about how it was decided which measurement to perform, is not under discussion. Priorities (2) and (4) are not relevant because quantum theory fixes everything.

We do not have the freedom of interpretation in the framework of quantum counterfactuals after *defining* the similarity criteria. For the case of preselected counterfactuals it is simply the identity of quantum description of the system before the measurement and this is not controversial. For time-symmetrized counterfactuals there is no consensus. I have my definition. Its advantage is that it yields the standard definition as a particular case for the preselected-only situation and it allows us to analyze and derive useful results for pre- and postselected quantum systems. I am aware of other proposals.[15, 16] Each proposal should be judged according to its consistency and usefulness for the purpose for which it has been defined. The success or failure of various definitions of similarity criteria of counterfactuals in exact sciences is not measured by maximizing priorities (1)–(4), but by its effectiveness in the framework of a particular theory. Priorities (1)–(4) are relevant outside the framework of exact sciences, where we have no laws which determine unambiguously the truth values of counterfactual statements.

Contrary to Kastner's writing, I never claimed that Lewis' theory is not applicable in an indeterministic universe. On the contrary, I have used Lewis' framework of possible worlds for defining counterfactuals in quantum theory. I only claimed that most of Lewis' analysis is irrelevant because counterfactuals in the context of quantum theory are of a very specific form and the majority of aspects discussed in the general philosophical literature on counterfactuals is not present in the quantum case. To make things even more clear, I add another quotation from Lewis' writings (Ref. 14, p. 33) with an example of argumentation for which I cannot find any counterpart in the analysis or quantum counterfactuals:

> Jim and Jack quarreled yesterday, and Jack is still hopping mad. We conclude that if Jim asked Jack for help today, Jack would not help him. But wait: Jim is a prideful fellow. He never would ask for help after such a quarrel; if Jim were to ask Jack for help today, there would have to have been no quarrel yesterday. In that case Jack would be his usual generous self. So if Jim asked Jack for help today, Jack would help him after all....

Kastner continues by criticizing my definition of time-symmetrized counterfactual regarding results of a measurement performed on pre- and postselected quantum system:

> If it were that a measurement of an observable A has been performed at time t, $t_1 < t < t_2$, then the probability for $A = a_i$ would be equal to p_i, provided that the results of measurements performed on the system at times t_1 and t_2 are fixed.

Her criticism[17] regarding "problematicity" of the fixing requirement is answered in another paper.[13] The latter was also criticized by Kastner.[10] She claims that fixing the results of measurements at t_1 and t_2 is "*ad hoc* gerrymanddering" which relies on accidental similarity of individual facts." But these facts are the physical assumptions in the pre- and postselected situations for analysis in which the above concept of time-symmetrized counterfactuals has been introduced. Disregarding these facts is similar to deciding that there has been no quarrel between Jim and Jack even though the counterfactual statement starts with "Jim and Jack quarreled yesterday...." The definitions in physics have no ambiguity which might allow such free reading of the text.

In her present paper[12] Kastner criticizes the syntax of the definition, in particular, that it reflects "a confusion between the noncounterfactual and the counterfactual usage of the ABL rule." In fact, I feel very unsure about the grammatical correctness of tenses in my definition. Also, I was not able to find an exact philosophical definition according to which one can decide if a certain statement is "counterfactual." However, it seems to me that the meaning of my definition is unambiguous and the name counterfactual is appropriate in the context of situations in which this definition was applied. For example, in the three-box example described above, the definition is applied when it is known that in the actual world the observable A (e.g., $\mathbf{P}_A$) has not been measured.

Kastner suggests two possible "usages" of my definition. The difference, apart form using various tenses (the difference between which is beyond my linguistic understanding) is that only the second one includes the word "instead." This word is essential. According to my understanding it is implicit in every counterfactual statement, but maybe it is helpful to state it explicitly, modifying the definition as follows.

> If it were that a measurement of an observable A was performed at time t, $t_1 < t < t_2$, instead of whatever took place at time t in the actual world, then the probability for $A = a_i$ would be equal to p_i, provided that the results of measurements performed on the system at times t_1 and t_2 are fixed.

I hope that this clarifies my definition and makes its meaning unambiguous, even though grammatically it might not be perfect. Again, Kastner's arguments presented in her other paper,[17] that this usage of my definition is "generally incorrect," have been answered in detail elsewhere.[13] Here I want only to comment on Kastner's concluding sentence, in which she writes: "[Vaidman's] definition, as it stands, is grammatically incorrect in a way that reflects its lack of clarity and rigor with respect to the physically crucial point concerning which measurement has *actually* taken place." According to my definition of time-symmetrized counterfactuals, the measurement performed at time t is not "the physically crucial point;" on the contrary, it plays no role in calculating the truth value of the counterfactual statement. I noted this feature of my definition in the paper[18] which Kastner criticized. The countertactual statement is about the counterfactual world in which at time t some action was performed *instead* of the measurement which was performed in the actual world. Thus, the question which measurement has actually been performed is clearly irrelevant. The result of the measurement in the actual world does not add any information either, because in the framework of standard quantum theory to which the time-symmetrized formalism is applied, the results of measurements at t_1 and t_2 (which are fixed by definition) yield a complete description of the system at time t.

5. WHAT DOES IT MEAN: PROBABILITY OF A HISTORY?

I want to add a comment about a connection to the consistent histories approach[19] advocated by Kastner and presented in the Appendix to her paper. Following Cohen,[20] Kastner claims that the counterfactual usage of the ABL rule is valid only for cases corresponding to "consistent" histories. Since for my counterfactuals the ABL rule is always valid, I find this approach to be an unnecessary limitation which prevents seeing interesting results.

In addition, I have to admit that I have never been able to understand the meaning of a basic concept in the consistent history approach: probability of a history. A particular history associates a set of values of observables in a sequential set of times. If the meaning of probability is the probability of this set being the results of the measurements of these observables at the appropriate times, then this is a well-defined question in the framework of standard quantum theory. [The corresponding formula is given in the ABL paper.[1]] Apparently, the meaning is something different. Indeed, in the example considered by Kastner, she uses the following expression:

> What is the probability that the system is in state C_k at time t_1, given that it was preselected in state D and postselected in state F?

What is the meaning of "the system is in state C_k?" In this example the system (up to known unitary transformation) *is* in state D. This is a standard quantum state evolving toward the future. In the framework of the TSQT, one can also associate with the system at time t_1 the backward-evolving state F and say that the system is described by the two-state vector $\langle F| \, |D\rangle$. However, from the text of Kastner's paper it is obvious that she considers something different. She writes, "we consider a framework in which the system has some value C_k associated with an arbitrary observable." As I mentioned in Section 1, quantum observables do not possess values. Thus, I cannot understand the meaning of Kastner's sentence: "... We cannot use the ABL rule to calculate the probability of any particular value of either A or B at time t_1..." because the "probability of a value" is not defined.

6. CONCLUSION

In this paper I have clarified the meaning of the concepts from the time-symmetrized quantum formalism: quantum counterfactuals and elements of reality (which are particular quantum counterfactuals). I have answered the criticism of these concepts in the preceding paper by Kastner.[12] Kastner has claimed that the three-box example is a paradox arising from an invalid counterfactual usage of the ABL rule. I have argued here that if one adopts my definition of quantum counterfactuals, the ABL rule is valid. Peculiarities of this example do not represent a true paradox, but the unusual features of pre- and postselected elements of reality, such as the failure of the product rule.[6]

Current controversy can be added to the list of examples which led Bell to suggest abandoning the usage of the word "measurement" in quantum theory.[21] However, I do not think that abstaining from using problematic concepts is the most fruitful approach. I believe that physical and philosophical concepts which are vague and ambiguous should continue to be discussed until the concepts and the structure of the physical theory are clear. I hope that the current discussion brings us closer to constructing solid foundations for quantum theory.

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