Interference and transmission of quantum fluxons through a Josephson ring

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(Received 22 February 1995; revised manuscript received 16 May 1995)

The influence of quantum interference on the transmission of a fluxon through an ideal long circular Josephson junction (a "Josephson ring") is studied. In the low-temperature regime the transmission is a periodic function of a gauge charge applied along the ring with a period $2e$. Around points of full period of both the gauge charge and the optical path, the transmission shows resonances as a function of the gauge charge and "antiresonances" as a function of the optical path. These resonances and antiresonances are associated with energy levels of the circular junction and with short dwelling time of the fluxon in the ring. In the high-temperature regime the interaction with plasmons dephases the fluxon wave function completely. The transmission probability in this regime is calculated in a stationary picture and in a dynamical picture and two different results are obtained. The discrepancy between the two pictures is explained via the ratio of the dwelling time to the time the thermal bath needs to change the plasmons' microscopical state. A general method that retrieves the two results is presented.

PACS number(s): 03.65.Bz

I. INTRODUCTION

The possibility of observing quantum behavior of large, composite excitations is very intriguing. In the past decade this idea was studied both theoretically [1–8] and experimentally [9] for two-dimensional superconducting vortices. Most of the studies dealt with vortices in Josephson junction arrays, while Ref. [8] considered the Abrikosov vortex. Recently it was argued that the effectively one-dimensional, long Josephson junction fluxons can exhibit measurable quantum effects [10,11]. In Ref. [10] a closed circular Josephson junction (a closed Josephson ring) was considered and it was shown that a bias charge applied along the junction acts as a gauge charge and induces a persistent motion of the fluxon, manifested in a persistent voltage. In Ref. [11] the distance over which the fluxon maintains its quantum coherence (the "dephasing length") was evaluated.

In the present work we study the transmission of fluxons through an ideal Josephson junction ring connected to two Josephson junctions leads, i.e., an open Josephson ring (see Fig. 1). We will examine two limits. When the temperature is very low, the fluxon maintains its coherence and we expect to find oscillations of the transmission as a function of the gauge charge. These oscillations are analogous to the hc/e oscillations in the transmission of an electron in a metal ring [12]. Furthermore, the transmission of an electron is known to have resonances [13,14]. We will look for resonances in the transmission of the Josephson ring. In the opposite limit, when the temperature is high enough to completely destroy the quantum coherence, the oscillations vanish. It is generally believed that a destruction of a quantum coherence by a dephasing process is equivalent to a measurement process carried out on the system. When measurement devices that detect the fluxon but otherwise leave it unchanged are coupled to the arms of the ring, the fluxon's wave function collapses. The collapse destroys the interference completely and hence acts like a dephasing process. We will show that these two pictures produce different values of the transmission probability and explain this result by the dependence of the value of the transmission probability on the different time scales in the system.

II. TRANSMISSION THROUGH A JOSEPHSON RING: A STATIONARY APPROACH

A. Description of the model

The spectrum of excitations of an ideal long Josephson junction consists of fluxons, which are topological solitons, and of plasmons, which are small amplitude plasma oscillations. These excitations are decoupled in the sense that they do not change any of their properties after interaction and the only result is a phase shift [15,16]. We will refer to this coupling as a "phase interaction." We consider the limit in which the Josephson ring is strongly connected to the two leads. This means that an incoming fluxon is certainly transmitted to one of the arms of the ring. If we also assume a symmetry between the two arms, then the S matrix describing the identical connections of the leads to the ring is

$$S=\begin{pmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/2 & -1/2 \\ 1/\sqrt{2} & -1/2 & 1/2 \end{pmatrix} . \tag{1}$$

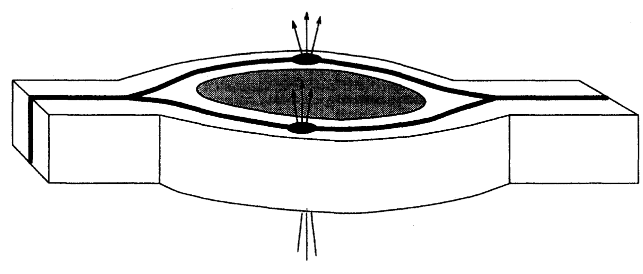

FIG. 1. Open Josephson ring. Two possible paths of a fluxon are shown.

It is the same S matrix that was used in Refs. [13] and [17] and was generalized in Ref. [14]. Whether such a connection is feasible for a Josephson junction is yet to be studied. We will comment about the ring connections later. The phase interaction of a fluxon propagating along an arm is represented by a barrier having zero reflection coefficients, r_i from the left and r_i' from the right ($i=u,l$, where u and l denote the upper and lower arms, respectively), and pure phase transmission coefficients t_i and t_i'. Thus a generic transmission coefficient is

$$t=e^{i\phi} . \tag{2}$$

In a stationary picture one can use the relations between the amplitudes of the different partial waves in the system ensuing from Eqs. (1) and (2) to express the total transmission amplitude of the ring D as a function of the barriers' transmission coefficients

$$D=2\,\frac{t_u t_l(t_u'+t_l')-(t_u+t_l)}{(t_u+t_l)(t_u'+t_l')-4} . \tag{3}$$

This is a special case (for $r_i=r_i'=0$) of Eq. (3) of Ref. [13].

B. Low-temperature limit

We will first examine the low-temperature limit of our system, i.e., the limit where there are no plasmons (about 0.1 K and below, as shown in Ref. [11]) and a fluxon can be considered free. When a fluxon propagates along the junction it can accumulate two phase shifts: an optical phase shift θ, which is always present, and a gauge phase shift $Q\equiv 2\pi(q/2e)$, which exists when a bias charge q is induced along the inner and outer edges of the junction [10]. Note that while the optical phase accumulates regardless of the direction of the fluxon, i.e., it is time reversible, the gauge phase cancels when the fluxon path is reversed, i.e., it is anti-time-reversible. These phase shifts can be considered as modulated transmission coefficients $t_u=e^{i(\theta_u+Q_u)}$, $t_u'=e^{i(\theta_u-Q_u)}$, $t_l=e^{i(\theta_l-Q_l)}$, and $t_l'=e^{i(\theta_l+Q_l)}$. If we assume for the moment that the two arms are equal ($\theta_u=\theta_l=\theta/2$ and $Q_u=Q_l=Q/2$), then Eq. (3) becomes

$$D=\frac{\cos(Q/2)(e^{3i\theta/2}-e^{i\theta/2})}{\cos^2(Q/2)e^{i\theta}-1} . \tag{4}$$

Thus we find that the transmission probability

$$T\equiv|D|^2=\frac{4\cos^2(Q/2)\sin^2(\theta/2)}{\cos^4(Q/2)-2\cos^2(Q/2)\cos(\theta)+1} \tag{5}$$

shows oscillations as functions of the gauge charge with a period $2e$, in analogy to the Φ_0 oscillations of the transmission of an electron in a metal ring [13,14]. There are, of course, oscillations in the transmission as a function of the optical phase.

A very interesting behavior of the transmission occurs around points of a full period of both the optical and the charge phases. When $\theta=2n\pi$ (where n is an integer) the transmission is zero, while when $Q=2m\pi$ (where m is an integer) the transmission is one. Therefore, at the points where both conditions are met there are ambiguities in the transmission. These ambiguities occur because the right-hand side of Eq. (5) is discontinuous at the points $(2n\pi,2m\pi)$. The limit values of Eq. (5) at these points depend on the way the limit is taken. If the optical phase is being kept at a full-period value and the charge phase is varied, the measured transmission will be zero everywhere, even at the points of full periods of the charge phase. Alternatively, if one holds the charge phase at a full-period value, the measured transmission will be one everywhere, even at the points of full periods of the optical phase. This behavior is a source for two kinds of resonancelike structures in the transmission: when the optical phase is near a full-period value, then there are resonances in the transmission as a function of Q around the points $Q=2m\pi$. On the other hand, when the charge phase is near a full-period value, there are "antiresonances" in the transmission as a function of θ around the points $\theta=2n\pi$.

The existence of resonances in the transmission through a ring is a well-known phenomena, which was studied extensively in Ref. [14]. We emphasize that while the antiresonances appear implicitly in Ref. [14], the resonances we have just described are not the resonances that were studied in that work. The latter are resonances of the transmission as a function of the optical phase, which exist when the coupling to the leads is weak or when the coupling is strong and the elastic scattering in the ring is also strong. The former are resonances of the transmission as a function of the gauge phase, which exist when the coupling to the leads is strong and there is no elastic scattering in the ring.

However, the observation made in Ref. [14] that resonances in the transmission probability are associated with the energy levels of the closed ring applies also to our case. The resonance conditions $\theta=2n\pi$ and $Q=2m\pi$ are just the Bohr-Sommerfeld condition for the existence of an energy level in the ring. If we relax our assumption that the two arms are equal, then taking the excess phase of one of the arms to be Δ, we have instead of Eqs. (4) and (5), respectively,

$$D=\frac{\cos[(Q+\Delta)/2](e^{3i(\theta+\Delta)/2}-e^{i(\theta+\Delta)/2})}{\cos^2[(Q+\Delta)/2]e^{i(\theta+\Delta)}-1} \tag{6}$$

and

$$T=\frac{4\cos^2[(Q+\Delta)/2]\sin^2[(\theta+\Delta)/2]}{\cos^4[(Q+\Delta)/2]-2\cos^2[(Q+\Delta)/2]\cos(\theta+\Delta)+1} . \tag{7}$$

The resonance conditions are now met when $\theta+\Delta=2n\pi$ and $Q+\Delta=2m\pi$. Thus the Bohr-Sommerfeld condition is the resonance condition for a general ring, i.e., resonances occur at energy levels of the closed ring. One question is left open though. In Ref. [14] the resonances are explained by a long dwelling time of the electron in the ring, which results in a coherent build up of its wave function. The long dwelling time was due either to a weak coupling to the leads or to a strong elastic scattering in the ring. In the case described here, the coupling to the leads is strong *and* there is no elastic scattering. Thus there is no apparent mechanism that guarantees a long dwelling time.

Any practical probing of these resonances will undoubtedly involve fluctuations of the optical phase and/or the gauge charge. When trying to measure the transmission at the points of double resonances, the two kinds of fluctuations will compete. The optical phase fluctuations strongly increase the transmission, while the gauge phase fluctuations strongly decrease it. Thus the measured value will drastically depend on the fluctuations' distribution. General phase fluctuations can be decomposed into time-reversible and anti-time-reversible fluctuations. The time-reversible part will act like optical phase fluctuations and the anti-time-reversible part will act like gauge phase fluctuations. Since the phase shift acquired by a fluxon due to an interaction with plasmons is neither time reversible nor anti-time-reversible [16], the measured value of the transmission in the presence of plasmons will be between one and zero.

C. High-temperature limit

We turn now to the high-temperature limit. In Ref. [11] we have shown that at high temperature dephasing occurs even for a particle having only a phase interaction, e.g., a fluxon. The thermal distribution of plasmons causes uncertainty in the phase accumulated by the fluxon during the interaction. When the uncertainty reaches 2π, quantum coherence is completely lost. The dephasing temperature of the long Josephson junction fluxon is a function of the ratio between the length of the junction L and the Josephson penetration depth Λ_J. For a typical junction with $L=10\Lambda_J$, the dephasing temperature is about 5 K. This high-temperature regime can be represented in our model by a random distribution of the phases of the transmission coefficients of the barriers. The total transmission amplitude is given now by the average value of Eq. (3). As discussed in Ref. [11], one can distinguish between two possible plasmons ensembles. In the first case there are two different ensembles of plasmons in the two arms. This scenario can be achieved if the leads and the connections are kept at zero temperature. Hence all the four transmission coefficients are independent. On the other hand, there can be one ensemble of plasmons pertaining to the whole ring, i.e., the plasmons are in eigenstates of the ring. Since now a fluxon encircling the ring encounters the same plasmons in the two arms, one can see immediately that $t_u=t_l'$ and $t_l=t_u'$. Note that we may not take $t_i=t_i'$, as done in Refs. [13] and [14], since time-reversal symmetry is not guaranteed, even in the absence of an external gauge field. A lack of time-reversal symmetry results whenever the transmission properties are an outcome of an interaction between a certain particle and the other degrees of freedom of the system. In such a case, reversing the particle time alone does not render the system invariant. This is indeed the situation for the fluxon in a long Josephson junction.

We calculate the transmission probability in the two above-mentioned scenarios by a numerical average of Eq. (3), using equally spaced phases of the transmission coefficients. The result converges rapidly and we find that the total transmission probability in the independent arms scenario is

$$T\approx 0.7267 \ , \tag{8}$$

while its value in the whole ring scenario is

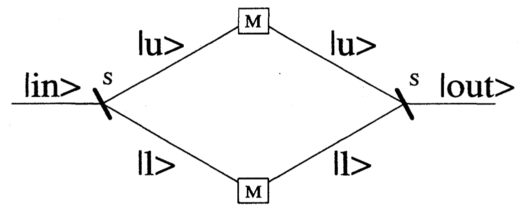

FIG. 2. Schematic quantum mechanical system analogous to the Josephson junction ring in the high-temperature regime. S denotes the scattering matrix at the connections to the leads and M denotes a measuring device that detects the particle, but does not alter its motion.

$$T\approx 0.5857 \ . \tag{9}$$

III. DYNAMICAL APPROACH

The transmission probability in the high-temperature regime can be calculated in a different way. A phase interaction that results in a total loss of coherence can be represented by a measuring device coupled to an arm, which detects with certainty any fluxon passing through that arm, but does not alter its motion. In other words, the wave function collapses in that arm. Thus the Josephson ring, whose connections to the leads are described by Eq. (1), seems to be equivalent in the high-temperature regime to the schematic construction shown in Fig. 2. When a particle enters the system from the left, it splits into two partial waves, each passing along one of the arms with equal probability 1/2. One of the waves is detected and the wave function collapses in that arm. Using Eq. (1) again, we find that when the collapsed wave reaches the right-hand side of the system, it is transmitted to the right lead with probability 1/2, transmitted to the other arm with probability 1/4, and reflected with probability 1/4. The same process is carried out back and forth and one can see that the transmission probability to the right lead is

$$T=\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n=\frac{2}{3} \ . \tag{10}$$

The apparent contradiction between Eq. (8) or (9) and Eq. (10) is resolved when one notices that in the stationary method one implicitly assumes that there are autocorrelations between the transmission coefficients, e.g., each time a fluxon is moving from left to right in the upper arm it is affected by the same transmission coefficient. On the other hand, in the picture involving collapse, there are no correlations at all. This discrepancy is clearly seen when one uses the following general dynamical method for calculating the total transition amplitude, which includes the two different results as special cases.

In this method we prepare the fluxon as a narrow wave packet (i.e., the relative uncertainty in the position $\Delta x/L$ is small). We like the wave packet to remain narrow through the whole transmission process, so we assume that the relative uncertainty in the momentum $\Delta p/p$ is also small. We denote the wave packet in the left lead as $|\,\text{in}\rangle$, in the upper arm as $|u\rangle$, in the lower arm as $|l\rangle$, and in the right lead as $|\text{out}\rangle$. A fluxon is sent from the left lead (state $|\text{in}\rangle$). According to the S matrix Eq. (1) the fluxon penetrates the ring in the state $(1/\sqrt{2})(|u\rangle+|l\rangle)$, i.e., it splits symmetrically into two wave packets. When the wave packets pass through the arms, they accumulate different phases, which are determined by the states of the plasmons at the instant of passing. Thus the fluxon reaches the right connection in the state $(1/\sqrt{2})(t_u^{(1)}|u\rangle+t_l^{(1)}|l\rangle)$. The upper index (1) means that it is the first time the fluxon traverses the ring in the right direction. After scattering on the connection, there is an outgoing state in the right lead with an amplitude

$$A^{(1)}=\frac{1}{2}(t_u^{(1)}+t_l^{(1)}) \tag{11}$$

and two wave packets are scattered back into the ring. After the next scattering on the left connection, there is an outgoing state in the left lead with an amplitude

$$A'^{(1)}=\frac{1}{4}(t_u^{(1)}-t_l^{(1)})(t_u'^{(1)}-t_l'^{(1)}) \ . \tag{12}$$

In a similar way we obtain the amplitude of the second outgoing state in the right lead

$$A^{(2)}=\frac{1}{8}(t_u^{(1)}-t_l^{(1)})(t_u'^{(1)}+t_l'^{(1)})(t_u^{(2)}-t_l^{(2)}) \ . \tag{13}$$

Continuing along this line of argumentation, we can obtain two infinite sequences of amplitudes A_n and A'_n, corresponding to right and left outgoing states, respectively. In order to calculate the total transmission T and the total reflection R, one should note that all the outgoing states are created at different moments of time. Since the wave packets are narrow, the outgoing states are spatially separated. Thus there are no interferences between these states and T and R are given by

$$T=\sum_{n=1}^{\infty}|A^{(n)}|^2 \ , \tag{14}$$

$$R=\sum_{n=1}^{\infty}|A'^{(n)}|^2 \ . \tag{15}$$

If the temperature is high enough, the phases of the transmission coefficients are homogeneously distributed over the interval $[0,2\pi]$. Therefore, in order to calculate the transmission and the reflection one has to average the right-hand side of (14) and (15) over all possible phases.

Let us assume that all the phases are not correlated. In this case we obtain that $\overline{|A^{(n)}|^2}=\frac{1}{2}(\frac{1}{4})^{n-1}$ and $\overline{|A'^{(n)}|^2}=(\frac{1}{4})^n$. This yields $\bar{T}=\frac{2}{3}$ and $\bar{R}=\frac{1}{3}$, which is the result of the calculation involving collapse given by Eq. (10). On the other hand, we can assume a completely correlated situation, which means that each time the fluxon traverses the ring it encounters the same plasmons' state. Thus $t^{(n)}=t^{(1)}$ and $t'^{(n)}=t'^{(1)}$ for each n. Here again we have the same two scenarios of plasmons ensembles discussed above. In the independent arms scenario these correlations manifest themselves for the first time in the average value of $|A^{(2)}|^2$, which becomes now equal to 3/16 instead of 1/8 in the uncorrelated case. Summing the series Eq. (14) and averaging the sum over the phases, we obtain $\bar{T}\approx 0.7267$, thus explaining the result of the stationary picture calculation given by Eq. (8). A similar calculation in the one ensemble scenario also produces the result obtained for this case by the stationary method Eq. (9).

IV. DISCUSSION AND FUTURE STUDY

The presence or the absence of the autocorrelations described above is determined by two characteristic time scales of the system. One is the dwelling time τ_d, which is the time the fluxon stays in the ring. Since the coupling between the ring and the leads is strong, τ_d can be estimated as $10\tau_{\rm tr}$, where $\tau_{\rm tr}$ is the time the fluxon traverses an arm. The second characteristic time is the time the thermal bath needs to change the plasmons' microscopic state, which we will denote as $\tau_{\rm bath}$. When $\tau_{\rm bath}\ll\tau_d$, the state of the plasmons is different every time the fluxon traverses the arm, thus there are no autocorrelations. In the opposite limit $\tau_{\rm bath}\gg\tau_d$, the state of the plasmons practically does not change during the time the fluxon stays in the ring. Therefore, there are total autocorrelations. If the system connected to the heat bath is macroscopic, $\tau_{\rm bath}$ is very short. The Josephson ring is, however, a *mesoscopic* system, hence the existence or absence of autocorrelations in our case is not clear. This question can be answered experimentally.

In order to get a more realistic description of the system, one should look more closely at the connections of the leads to the ring. It is known that a connection between three or more long Josephson junctions serves as a pinning center for the fluxon. Hence the fluxon can radiate its kinetic energy and stop at the connection. Depending on the fluxon's energy, more complicated phenomena can occur, like backscattering or fluxon-antifluxon pair creation. The trapping of classical fluxons and pair creation at a connection between three long Josephson junctions were studied numerically in [18]. In the quantum limit these inelastic phenomena will reduce the interference. An additional fluxon residing in the pinning center can be used to "fill" the potential well, thus overcoming the trapping effect. However, as calculated in [18], a second fluxon that comes to this filled connection encounters an effective potential barrier of height 4 (in the usual dimensionless sine-Gordon units). In order to eliminate this barrier, the pinning center should be deepened by enlarging the width of the junctions in the connection area. This enlargement will also help to enhance the quantum splitting between the arms since such a splitting is possible only if the fluxon has an additional degree of freedom in the transverse direction of the junction. In this direction the fluxon behaves as a particle bound to a potential well, and the wider the well, the larger the probability of the quantum splitting.

To conclude, we would like to mention that our results are not restricted to fluxons. The resonances in the low-temperature limit will appear for other particles interacting with a gauge field in the ballistic regime. The dephasing and the importance of autocorrelations in the high-temperature limit are general features of objects having only phase interaction.

ACKNOWLEDGMENTS

We would like to thank Y. Aharonov and A. Ustinov for very fruitful discussions. One of us (Z. H.) was supported by the Ben-Gurion Fund via the Israeli Ministry of Science. This research was supported in part by the Wolfson foundation via the Israeli Academy of Sciences.

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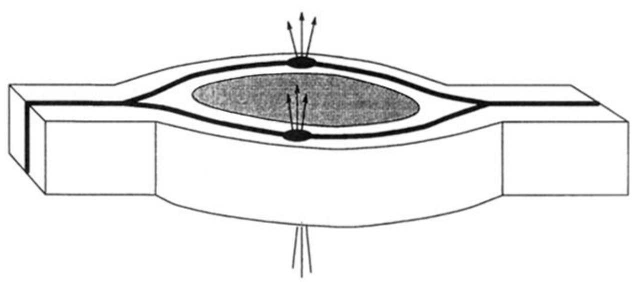

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