Interplay of Aharonov-Bohm and Berry Phases for a Quantum Cloud of Charge[a]

SANDU POPESCU,[b] YAKIR AHARONOV,[b,c]
SIDNEY COLEMAN,[d] ALFRED S. GOLDHABER,[e]
SHMUEL NUSSINOV,[b] BENNI REZNIK,[b]
DANIEL ROHRLICH,[b] AND LEV VAIDMAN[b]

[b]*School of Physics and Astronomy*
Tel Aviv University
Ramat Aviv 69978, Israel

[c]*Department of Physics*
University of South Carolina
Columbia, South Carolina 29208

[d]*Lyman Laboratory*
Harvard University
Cambridge, Massachusetts 02138

[e]*Institute for Theoretical Physics*
State University of New York
Stony Brook, New York 11794-3840

The Aharonov-Bohm (AB) effect[1] is simple and topological: an electron encircling a solenoid containing a magnetic flux Φ acquires a geometrical phase equal to $ne\Phi/\hbar c$, where n is equal to the winding number of the electron around the solenoid. However, when a solenoid enters a quantum cloud of charge and there is no way to associate a well-defined path to the electron, the consequences of the AB effect might be complicated. For example, consider an electron bound in a potential well V, in an energy eigenstate. A solenoid crosses the well. How many times did the electron encircle the solenoid? There is no definite answer to this question. Of course, we can decompose the movement of the electron into a superposition of different Feynman paths, compute the phase acquired in each path, and resum, but no simple result will emerge. In general, the final state of the electron (once the solenoid has left the well) is different from the initial one (before the solenoid entered) and it depends on all the different parameters of the problem: the initial state, the potential V, the precise path of the solenoid and its velocity, and the value of the enclosed magnetic flux Φ. However, we have found a surprising topological effect for a solenoid containing exactly half a flux quantum [$\Phi = (\frac{1}{2})\Phi_0 = (\frac{1}{2})2\pi\hbar c/e$] when it adiabatically crosses the quantum "cloud" of an electron in a nondegenerate energy eigenstate[f] (see

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[f]Not counting the spin degeneracy.

reference 2). This topological effect is an interplay of the Aharonov-Bohm and Berry[3] phases.

Consider an electron in a nondegenerate energy eigenstate of an arbitrary potential well V and a semifluxon moving adiabatically on a closed path C that crosses the electron cloud. We take the solenoid to be linelike, that is, infinitely narrow and long. According to the adiabatic approximation, the solenoid does not induce transitions and the final state of the electron is identical to the initial one, up to a phase $e^{i\varphi}$. Now, φ contains a dynamical phase φ_d and a geometrical phase φ_g such that

$$\varphi = \varphi_d + \varphi_g. \tag{1}$$

We are interested in the geometrical part of the phase. For simplicity, we first consider the two-dimensional situation illustrated in FIGURE 1. Two limiting cases,

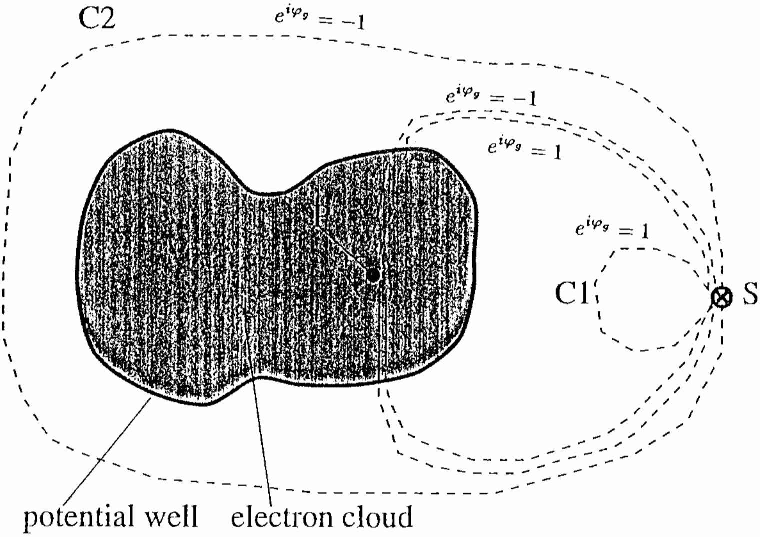

FIGURE 1. For simplicity, we consider an infinitely deep potential well so that the electron cloud vanishes outside it.

where we know how much charge the solenoid encircles, are easily computed.[g] When the solenoid moves along the path $C1$, it does not encircle any charge at all. The AB phase is 0. On the other hand, when the solenoid moves along $C2$, the electron is encircled with certainty and the AB phase is $e\Phi/\hbar c$, which in the case of a semifluxon (half a quantum of flux) yields π. However, how are we to interpolate the phase for intermediate paths that cross the electron cloud? Apparently, as the solenoid moves

[g]The AB effect is usually illustrated by an electron that encircles a solenoid. Here, a solenoid encircles an electron. However, whether we consider the movement in the reference frame of the solenoid or of the electron, the phase is the same.

on paths encircling larger and larger portions of the cloud, the effective charge it encircles gradually increases from 0 to e. Thus, the AB phase should gradually change from 0 to π. Closer inspection, though, leads to a different conclusion. The reason is that the physics manifests time-reversal symmetry. The initial wave function of the electron is nondegenerate and therefore unchanged under time reversal. (Assume that, initially, the semifluxon is infinitely far from the electron and no other vector fields act on it.) Under time reversal, the magnetic field inside the solenoid changes sign and thus also the magnetic flux ($\Phi \rightarrow -\Phi$); however, in the particular case of half a flux quantum, this change is not observable as long as the electron cannot penetrate into the solenoid because the difference between $(\frac{1}{2})\Phi_0$ and $-(\frac{1}{2})\Phi_0$ is exactly a flux quantum. Consequently, for any path C, the geometric phase $e^{i\varphi_g}$ must be the same in whatever direction the solenoid moves. On the other hand, φ_g must change sign when the solenoid changes direction (because it can be written as a line integral along the path C). Thus, we obtain

$$e^{i\varphi_g} = e^{-i\varphi_g}, \tag{2}$$

implying $e^{i\varphi_g} = \pm 1$. This result, corresponding to φ_g equal to an integer multiple of π, contradicts our naive expectation that the AB phase gradually changes from 0 to π.

What happened? Let us try to interpolate between the paths $C1$ and $C2$. We can gradually distort the path $C1$ into $C2$ by many steps that enlarge the loop by an infinitesimal region. In a certain region, the phase factor jumps from 1 to -1. We probe this particular infinitesimal region until we encounter a point P with the property that the geometrical phase jumps by π when P is encircled. However, what is the phase when the solenoid moves on a path crossing P? Our best guess is that the phase is not well defined. Our assumption that the solenoid moves adiabatically breaks down on this path. In other words, although initially the wave function of the electron was a nondegenerate energy eigenstate, it is no longer nondegenerate when the solenoid goes through the point P: at P, the solenoid induces a degeneracy.

Thus, we find a clue to the puzzle of abrupt phase changes. Indeed, we can say that our argument for a gradual change in the Aharonov-Bohm phase was correct. However, we neglected a second contribution to the geometric phase. As the solenoid crosses, it distorts the wave function of the electron and generates a Berry phase that adds to the AB phase:

$$\varphi_g = \varphi_{\mathrm{AB}} + \varphi_{\mathrm{Berry}}. \tag{3}$$

The Berry phase is responsible for maintaining the total geometric phase factor (1 or -1) despite gradual changes in the AB phase. The Berry phase is also responsible for the jump in the total geometric phase around the point P: as Berry[3] showed, isolated energy degeneracies can add π to the geometric phase.

Returning to our original problem, we can add a few details. First, the existence of points of degeneracy, for any arbitrary potential well V, can be proved by using time-reversal symmetry in the context of a Born-Oppenheimer approximation. Nevertheless, finding such points appears to be a difficult problem; the only explicit examples we know are for rotationally symmetric potential wells and for wells with even discrete rotational symmetries $[V(r, \theta) = V(r, \theta + 2\pi/2n)]$, where the center of the well is such a point. Second, it is clear that there might be more than one such point. Any odd number of points is consistent with the phases of the extremal paths $C1$ and $C2$. Third, the adiabatic approximation might break down not only at some isolated point P, but in a whole region if the initial nondegenerate state becomes degenerate with states in the continuum. Last, but not least, a similar effect of phase jumps and energy level crossings arises even if the solenoid is not straight and also when several solenoids, each carrying half a flux quantum, enter the electron cloud.

We find a simple rule for the geometric phase of an atom with a heavy nucleus, initially in a spherically symmetric eigenstate, moving around semifluxons. (In this case, geometrical phases arise for both the electron and the nucleus, but the wave function of the nucleus is much more concentrated and semifluxons rarely penetrate it. Thus, its geometrical phase is simply the usual AB phase, which we neglect in the following.) The rule allows that we can replace the electron cloud with a point charge at the center of the atom and the semifluxons with "shadow" fluxons. A shadow fluxon is a point at which two electronic energy levels cross, if the center of the atom sits there. The winding number of the path of the point charge around the shadow fluxons gives the geometric phase accumulated by the atom.

To derive this rule, consider two straight and parallel solenoids situated a distance L apart. Two extreme cases are easily solved. When the distance between the solenoids is much larger than the size of the atom, we can move the atom in the vicinity of one of the solenoids without the electron cloud touching the other solenoid. In this case, the atom collects a phase of π each time its center encircles the solenoid, exactly as if the other solenoid were not present. There are thus two shadow fluxes, coinciding with the original solenoids. On the other hand, for $L = 0$, the two solenoids are at the same point, with their magnetic fluxes adding to an integer flux quantum. However, an integer flux quantum has no effect on an electron. There are therefore no energy level crossings and thus no shadow fluxons. When the solenoids are slightly separated, they do affect energy levels, but, by continuity, this effect is small and does not induce energy level crossing; rather, a minimal distance L^* is required. Thus, we conclude that, in adiabatic motion, the geometric phase accumulated by the atom due to two parallel semifluxons is zero once their separation is less than some critical distance L^*. We can now interpolate between these two extreme cases (large and small L) (FIGURE 2). When the distance between the solenoids is very large, the shadows coincide with the original solenoids. When the distance is still large, but comparable to the size of the atom, the shadow fluxes no longer coincide with the original solenoids. Instead, the shadow fluxon associated to each solenoid is shifted towards the other solenoid. When the two solenoids are at a critical distance L^*, their shadows overlap and therefore have no effect whatsoever on the atom. For separations smaller than L^*, the shadow fluxons disappear.

It is amusing to consider more general patterns of semifluxons carrying half a quantum of flux and the resulting shadows. Even in the case of a single solenoid, the shadow need not coincide with the original, if the solenoid is not straight. For example, a solenoid in the form of a ring should have a circular shadow, but of smaller radius. Just as in the case of two parallel solenoids, there is a critical radius for the ring (depending on the electron cloud) below which there will be no shadow at all. As a consequence, there will be no topological scattering of the atoms from small toroidal solenoids. For two intersecting straight solenoids, we expect hyperbolic shadows situated in the plane of the solenoids, in the acute angles. When the

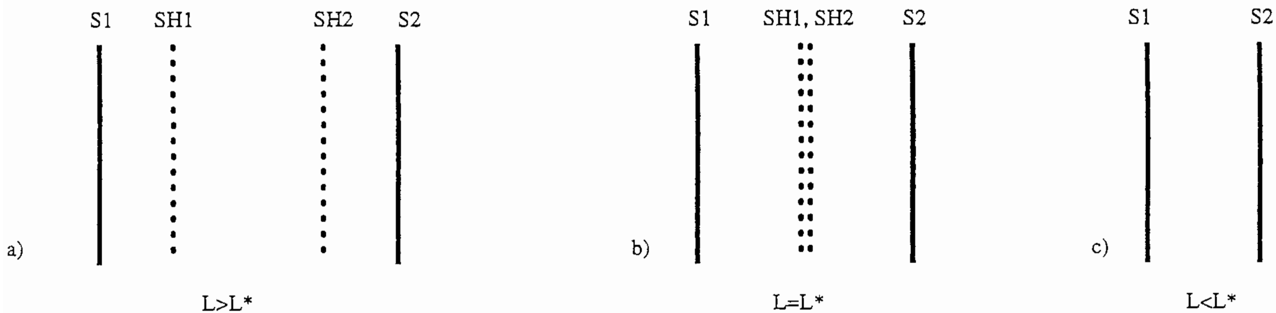

FIGURE 2. S1 and S2 represent the original solenoids and SH1 and SH2 represent their shadows (three cases—a, b, c—for varying lengths of L).

solenoids are perpendicular to each other, the shadows will coincide with the solenoids.

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