PROTECTIVE MEASUREMENTS

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The content of this lecture has already appeared in several proceedings so here we will present only a short abstract together with the list of references.

At present, the commonly accepted interpretation of the Schrödinger wave is that which leads to the probability density. This interpretation stems from the belief that the Schrödinger wave can only be tested for an ensemble of particles. We have proposed new type of measurements: "protective measurements" which allow direct measurement of the Schrödinger wave density on a single particle. We have shown that one can simultaneously measure the density and the current of the Schrödinger wave in seveal positions. The results of these measurements then allow to reconstruct the Schrödinger wave.

As an example of a simple protective measurement, let us consider a particle in a discrete nondegenerate energy eigenstate $\Psi(x)$. The standard von Neumann procedure for measuring the value of an observable A in this state involves an interaction Hamiltonian, $H = g(t)PA$, coupling the system to a measuring device, or pointer, with coordinate and momentum denoted, respectively, by Q and P. The time-dependent coupling $g(t)$ is normalized to $\int g(t)dt = 1$, and the initial state of the pointer is taken to be a Gaussian centered around zero. In standard impulsive measurements, where $g(t) \neq 0$ for only a very short time interval, the interaction term dominates the rest of the Hamiltonian, and the time evolution $e^{-\frac{i}{\hbar}PA}$ leads to a correlated state: eigenstates of A with eigenvalues a_n are correlated to measuring device states in which the pointer is shifted by these values a_n. By contrast, the protective measurements of interest here utilize the opposite limit of extremely slow measurement. We take $g(t) = 1/T$ for most of the time T and assume that $g(t)$ goes to zero gradually before and after the period T. We choose the initial state of the measuring device such that the canonical conjugate P (of the pointer variable Q) is bounded. We also assume that P is a constant of motion not only of the interaction Hamiltonian, but of the whole Hamiltonian. For $g(t)$ smooth enough we obtain an adiabatic process

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in which the particle cannot make a transition from one energy eigenstate to another, and, in the limit $T \to \infty$, the interaction Hamiltonian does not change the energy eigenstate. For any given value of P, the energy of the eigenstate shifts by an infinitesimal amount given by first order perturbation theory: $\delta E = \langle H_{int} \rangle = \langle A \rangle P/T$. The corresponding time evolution $e^{-iP\langle A \rangle/\hbar}$ shifts the pointer by the average value $\langle A \rangle$. This result contrasts with the usual (strong) measurement in which the pointer shifts by one of the eigenvalues of A. By measuring the averages of a sufficiently large number of variables A_n, the full Schrödinger wave $\Psi(x)$ can be reconstructed to any desired precision.

The main idea is presented in Ref. 4 and elaborated in Ref. 5. In Ref. 1 an apparent contradiction with causality is resolved. In Refs. 6-9 there are (partially critical) discussions of the proposal, and our reply appears in Ref. 10. More discussions and theoretical analyses of possible realistic experiments are presented in Refs. 11-12. Refs. 13-14 present experimental work which is close to what we propose. Experiments with single trapped atoms, which conceptually are exact realization of protective measurements of the Schrödinger wave, are given in Ref. 15. However, the resolution in these experiments is still one order of magnitude too large, so the observed "cloud" is not the wave of the atom. Our proposal to a a further development of the idea, the protective measurements of a two-state vector, will be presented in Ref. 16.

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