38

# MEASUREMENT OF NONLOCAL VARIABLES WITHOUT BREAKING CAUSALITY

LEV VAIDMAN
*School of Physics and Astronomy*
*Raymond and Beverly Sackler Faculty of Exact Sciences*
*Tel-Aviv University, Tel-Aviv, 69978 ISRAEL*

ABSTRACT

We report results of an investigation of relativistic causality constraints on the measurability of nonlocal variables. We show that measurability of certain nondegenerate variables with entangled eigenstates contradicts the principle of causality, but that there are other, certainly nonlocal, variables which can be measured without breaking causality. We show that any causal measurement of nonlocal variables must erase certain local information. For example, for a system of two spin-1/2 particles, even if we take the weakest possible definition of verification measurement, verification of an entangled state must erase all local information.

## 1. Measuring Momentum of a Particle

As early as 1931, Landau and Peierls[1] showed that relativistic causality imposes new restrictions on the process of quantum measurement. Although some of their arguments were not precise, it was commonly accepted that we cannot measure instantaneously nonlocal properties without breaking relativistic causality.

The first example is the measurement of momentum of a particle. Consider a particle localized in a small region. Measurement of its momentum, irrespective of the outcome, will spread the particle all over the space. There will be a non-zero probability to find the particle at a very large distance from its original place immediately after the (instantaneous) momentum measurement, so it seems that the particle moves faster than light. However, this argument is not decisive. Relativistic causality states that it is impossible to send a *signal* with superluminal velocity. It does not forbid instantaneous measurement of momentum, say at $t = 0$. The instantaneous measurement interaction will take place all over the space and it can create particles everywhere. Thus, the probability of finding the particle at a given location after the momentum measurement might be independent of what we did to the particle located far away before the measurement. Therefore, the possibility of instantaneous momentum measurement does not lead automatically to the possibility of sending signals with superluminal velocity.

Nevertheless, if we can measure the momentum of a spin-1/2 particle without affecting its spin, then we can violate causality. Indeed, let us assume that we know that at time $t = 0$ the momentum measurement will be performed. At the time $t = -\epsilon$ we decide to prepare the state of the particle "up" or "down" according to the signal we want to send. Then we can measure the spin component of the particle which is detected at time $t = +\epsilon$ far from its original location and thus send information with superluminal velocity. (The probability of finding the particle at a given place is very small, but we can use a large ensemble of identical particles and thus we can build a reliable superluminal transmitter.)

## 2. Constraints on Nonlocal Measurements of Two Spin-1/2 Particles

Although momentum measurement is a basic problem, it is still not the simplest example we may consider. Significant progress in understanding causality constraints on quantum measurement was made by considering an even simpler example: measurements of spin variables of two spin-1/2 particles separated in space. This is the system on which Bohm and Aharonov[2] and later Bell[3] analyzed the EPR argument and reached far-reaching conclusions regarding the nonlocal structure of quantum theory.

In order to show how measurability of nonlocal variables contradicts relativistic causality let us consider an operator with the following nondegenerate eigenstates:

$$
\begin{aligned}
|\psi_1\rangle &= |\uparrow\rangle_1 |\uparrow\rangle_2 \\
|\psi_2\rangle &= |\downarrow\rangle_1 |\downarrow\rangle_2 \\
|\psi_3\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 |\downarrow\rangle_2 + |\downarrow\rangle_1 |\uparrow\rangle_2) \\
|\psi_4\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2)
\end{aligned}
\tag{1}
$$

This operator corresponds to a nonlocal variable because its eigenstates are nonlocal. We call the state of the composite system nonlocal when it cannot be represented as a product of states corresponding to localized parts of the system; these states are also known as *entangled* states.

Let us show that the measurability of this variable contradicts relativistic causality. To this end we perform the following set of measurements:

i) We prepare state $|\uparrow\rangle_2$ of particle number 2 a long time before the time $t = 0$.

ii) At time $t = -\epsilon$ we prepare state $|\uparrow\rangle_1$ or $|\downarrow\rangle_1$ of particle number 1 according to the message we want to send from particle 1 to particle 2.

iii) At time $t = 0$ we measure the variable defined by the nondegenerate eigenstates of Eq. (1).

iv) At the time $t = \epsilon$ we measure the spin component of particle 2.

The two events, choosing the spin of particle 1 and measurement of the spin of particle 2, are space-like separated, and therefore must be causally disconnected. But if we choose spin "up" for particle 1, then the state of the composite system before the time $t = 0$ is $|\uparrow\rangle_1 |\uparrow\rangle_2$, the measurement at the time $t = 0$ does not change it (since it is an eigenstate), and thus the spin measurement of particle two will yield "up" with probability one. If, instead, at the time $t = -\epsilon$, we put, the particle 1 in the state "down," then the state of the composite system before the measurement (iii) is $|\downarrow\rangle_1 |\uparrow\rangle_2$. This state is not one of the eigenstates of the nonlocal operator, and therefore the measurement at time $t = 0$ will change it. Since the scalar product between $|\downarrow\rangle_1 |\uparrow\rangle_2$ and the eigenstates is not vanishing only for the eigenstates $|\psi_3\rangle$ and $|\psi_4\rangle$, the state after $t = 0$ will be one of those. But for both $|\psi_3\rangle$ and $|\psi_4\rangle$ the probability to find the spin "up" for particle 2 is just 1/2. We have shown that the possibility of measuring nonlocal variable described by eigenstates (1) allows us to change the probability of the result of a spin measurement performed on particle 2 by acting on particle 1 a time only $2\epsilon$ before the measurement on particle 2; and since the distance between the particles might be larger than $2\epsilon c$, this procedure represents a superluminal signal transmitter.

## 3. Measurable Nonlocal Variables

The examples above may lead us to believe that measurement of any nonlocal variable breaks relativistic causality. This, in fact, was generally believed until Aharonov and Albert[4] found a method involving solely local interactions (hence consistent with the causality principle) which does allow us to measure certain nonlocal variables. In particular, we can measure the variable $\sigma_{1z} + \sigma_{2z}$. The method applies the standard von Neumann measuring procedure to a measuring device consisting of two parts which were prepared in an *entangled* state before the measurement. Each part of the measuring device interacts with one of the particles for a short time, and is observed immediately after by a local observer. The combined observations of the two observers (one at each particle) determines whether the state is $|\psi_1\rangle$, $|\psi_2\rangle$ or belongs to the subspace spanned by $|\psi_3\rangle$ and $|\psi_4\rangle$. The feature of this method is that while it measures $\sigma_{1z} + \sigma_{2z} = 0$, it does *not* measure the spin of each particle separately. The details of the method of nonlocal measurements can be found in Ref. (5).

It might seem that the measurability of the operator $\sigma_{1z} + \sigma_{2z}$ has something to do with its having a complete set of eigenstates which are not entangled. But this is not the explanation. The next example shows an operator with nondegenerate eigenstates that are all entangled but which is, nevertheless, measurable by local interactions. The eigenstates of the nondegenerate operator are

$$\begin{aligned}
|\psi_1\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 |\uparrow\rangle_2 + |\downarrow\rangle_1 |\downarrow\rangle_2) \\
|\psi_4\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 |\uparrow\rangle_2 - |\downarrow\rangle_1 |\downarrow\rangle_2) \\
|\psi_3\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 |\downarrow\rangle_2 + |\downarrow\rangle_1 |\uparrow\rangle_2) \\
|\psi_4\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2)
\end{aligned} \tag{2}$$

This operator can be measured[6] using a set of nonlocal operators with degenerate eigenstates (such as $\sigma_{1z} + \sigma_{2z}$), where the particles 1 and 2 are far from one another. Recently[7] the measurability of operators for two spin-1/2 particles has been analyzed, and it was shown that the only measurable nondegenerate operators are those with eigenstates of two possible types:

$$
\begin{aligned}
|\psi_1\rangle &= |\uparrow_z\rangle_1 |\uparrow_{z'}\rangle_2 \\
|\psi_2\rangle &= |\uparrow_z\rangle_1 |\downarrow_{z'}\rangle_2 \\
|\psi_3\rangle &= |\downarrow_z\rangle_1 |\uparrow_{z'}\rangle_2 \\
|\psi_4\rangle &= |\downarrow_z\rangle_1 |\downarrow_{z'}\rangle_2
\end{aligned}
\tag{3a}
$$

or

$$
\begin{aligned}
|\psi_1\rangle &= \frac{1}{\sqrt{2}}(|\uparrow_z\rangle_1 |\uparrow_{z'}\rangle_2 + |\downarrow_z\rangle_1 |\downarrow_{z'}\rangle_2) \\
|\psi_2\rangle &= \frac{1}{\sqrt{2}}(|\uparrow_z\rangle_1 |\uparrow_{z'}\rangle_2 - |\downarrow_z\rangle_1 |\downarrow_{z'}\rangle_2) \\
|\psi_3\rangle &= \frac{1}{\sqrt{2}}(|\uparrow_z\rangle_1 |\downarrow_{z'}\rangle_2 + |\downarrow_z\rangle_1 |\uparrow_{z'}\rangle_2) \\
|\psi_4\rangle &= \frac{1}{\sqrt{2}}(|\uparrow_z\rangle_1 |\downarrow_{z'}\rangle_2 - |\downarrow_z\rangle_1 |\uparrow_{z'}\rangle_2)
\end{aligned}
\tag{3b}
$$

with spin polarized "up" or "down" along directions $z$ and $z'$.

Operators of type (3a), although they refer to two separated spins, are effectively local. They can be measured simply by measuring the $z$ component of spin of the first particle and the $z'$ component of spin of the second particle. Operators with the eigenstates (3b) are truly nonlocal. They can be measured[7] in the same way as an operator with eigenstates given in Eq. (2) (a particular case of Eq. (3b)).

On the other hand[7] measurability of any nondegenerate operator with eigenstates not equivalent to the forms (3a) or (3b) implies the possibility of superluminal communication, i.e., violation of relativistic causality.

## 4. State Verification Measurements

A measurement of a nondegenerate operator is also a state verification measurement for all its eigenstates. The weakest possible definition of a state verification measurement which requires only *reliability* of the measurement is: the verification measurements of the state $|\psi_0\rangle$ must always yield the answer "yes" if the measured system has the initial state $|\psi_0\rangle$, and must always yield "no" if the system is initially in an orthogonal state. One may suspect that the verification of a state with canonical form (Schmidt decomposition) different from

$$\frac{1}{\sqrt{2}}(|\uparrow_z\rangle_1 |\uparrow_{z'}\rangle_2 + |\downarrow_z\rangle_1 |\downarrow_{z'}\rangle_2) \tag{4}$$

(the form of the eigenstates in (3b)) contradicts relativistic causality; i.e., that verification of a state

$$|\psi_1\rangle = \alpha|\uparrow_z\rangle_1 |\uparrow_{z'}\rangle_2 + \beta|\downarrow_z\rangle_1 |\downarrow_{z'}\rangle_2, \qquad |\alpha| \neq |\beta| \neq 0 \tag{5}$$

allows superluminal communication. Indeed, it has been shown[6] that the type of measurements of entangled states described above, i.e. nondemolition operator measurements with solely local interactions, cannot measure the state given by the form (5).

However, an unmeasurable quantity should not represent physical reality. If we want to consider the quantum state as a physical (versus purely mathematical) concept, it must be measurable. We do know how to *prepare* this state (the preparation procedure is also frequently called measurement). But the state (5) can also be measured using a new type of verification measurement named an *exchange* measurement[6] The idea is to make simultaneous short local interactions with parts of the measuring device such that the states of the system and the measuring device will be exchanged. The novel point in this method is that *local* interactions exchange *nonlocal* states. The result of the measurement cannot be read by two local observers; we must bring the two parts of the measuring device to one place. In addition, this procedure has another unconventional property. The final state of the system is completely independent of its initial state: it is just the initial state of the measuring device. The state of the system *is completely erased* by this state verification measurement.

It has recently been proven[7] that *any* verification of the state

$$|\psi_1\rangle = \alpha|\uparrow_z\rangle_1|\uparrow_{z'}\rangle_2 + \beta|\downarrow_z\rangle_1|\downarrow_{z'}\rangle_2, \qquad \alpha, \beta \neq 0 \tag{6}$$

erases all local information. The probable outcome of a local spin measurement performed after the state verification measurement is independent of the state of the composite system prior to the state verification. The example considered above of a measurable nondegenerate operator (2) trivially fulfills this result: for all eigenstates we have the property that the probability for any outcome of local spin measurement is the same. There is no local information after this nonlocal measurement.

## 5. Conclusions

Let us formulate the last result for the somewhat more general case of a system of two separated particles with several orthogonal states. Consider the Schmidt decomposition of a state $|\psi_0\rangle$ of this composite system:

$$|\psi_0\rangle = \sum_i \alpha_i |i\rangle_1 |i\rangle_2. \tag{7}$$

Here $|i\rangle_1$ and $|i\rangle_2$ are local orthonormal bases of states of the two particles. Let us denote by $H^{(1)}$ and $H^{(2)}$ the Hilbert spaces of part 1 and part 2 respectively, and by $H_0^{(1)}$ and $H_0^{(2)}$ the subspaces of $H^{(1)}$ and $H^{(2)}$ which are spanned by the base vectors $|i\rangle_1$ and $|i\rangle_2$ corresponding to coefficients $\alpha_i \neq 0$. Then for all initial states which belong to the Hilbert space $H_0^{(1)} \otimes H^{(2)}$, the probabilities $p(\psi)$ for results of local measurements in part 1, performed after verification of the state $|\psi_0\rangle$, have no dependence on the initial state.

Thus, the erasing effect of the proposed "exchange" measurements is a generic property of any reliable, causal state verification measurement. The full implications of this result are not yet clear. It already has helped complete the analysis of measurability of nondegenerate operators discussed above. It also has been used to show[7] that measurability of certain *ideal measurements of the first kind* contradicts relativistic causality, thus placing a serious doubt concerning the possibility of generalizing axiomatic quantum theory to the relativistic domain.

We would like to conclude by stressing the importance of measuring nonlocal properties via local interactions (with separate parts of the measuring device prepared in an entangled state). The same method can be used for so-called "multiple-time" measurements[8] which open the way to many new quantum phenomena[9]

## Acknowledgements

The research was supported by grant 425/91-1 of the the Basic Research Foundation (administered by the Israel Academy of Sciences and Humanities).

## References

1. L. Landau and R. Peierls, *Z. Physik* **69** (1931) 56.
2. D. Bohm and Y. Aharonov, *Phys. Rev.* **108** (1957) 1070.
3. J. S. Bell, *Physics* **1** (64) 195.
4. Y. Aharonov and D. Albert, *Phys. Rev.* **D21** (1980) 3316.
5. Y. Aharonov and D. Albert, *Phys. Rev.* **D24** (1981) 359.
6. Y. Aharonov, D. Albert, and L. Vaidman, *Phys. Rev.* **D34** (1986) 1805.
7. S. Popescu and L. Vaidman, "Causality restrictions on nonlocal quantum measurements", preprint of Tel-Aviv University TAUP-2011-92 (1992).
8. Y. Aharonov and D. Albert, *Phys. Rev.* **D29** (1984) 223.
9. Y. Aharonov and L. Vaidman, *J.Phys. A: Math. Gen.* **24** (1991) 2315.