ON A THEORY OF THE COLLAPSE OF THE WAVE FUNCTION

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ABSTRACT. The quantum-mechanical measurement problem is reviewed, and a recent attempt (due to Ghirardi, Rimini, and Weber) to solve that problem by means of a theory of the collapse of the wave function is described. The theory is applied to the case of a Stern-Gerlach type spin-measurement, and is shown to run into some interesting difficulties there.

1. Introduction

This paper is going to be about how a certain recent attempt to solve the quantum-mechanical measurement problem, by means of a collapse of the wave function, falls somewhat short of its goal.

Let us begin by reminding you of precisely what the problem is. It arises like this: Suppose that every isolated physical system in the world invariably evolves in accordance with linear quantum-mechanical equations of motion; and suppose that M is a good measuring instrument for a certain observable A of a certain physical system S. What it means for M to be a 'good' measuring device for A is just that for all eigenvalues a_i of A:

$$|ready\rangle \; |A{=}a_i\rangle \;\rightarrow\; |M \textit{ indicates that } A{=}a_i\rangle \; |A{=}a_i\rangle \;, \qquad (1)$$

where $|ready\rangle$ is that state of measuring instrument in which M is prepared to carry out a measurement of A, ' $\rightarrow$ ' denotes the evolution of the state of M+S during the measurement-interaction between those two systems, and $|M \textit{ indicates that } A{=}a_i\rangle$ is that state of M in which, say, M's pointer is pointing to the a_i-position on its dial. That is: what it means for M to be a 'good' measuring device for A is just that M invariably indicates the correct value for A in all those states of S in which A *has* any definite value.

M. Kafatos (ed.), Bell's Theorem, Quantum Theory and Conceptions of the Universe, 1–6.

The problem is that (1), together with the linearity of the equations of motion, entails that:

$$|ready\rangle \sum_i \alpha_i \; |A{=}A_i\rangle \;\rightarrow\; \sum_i \alpha_i \; |M \text{ indicates that } A{=}a_i\rangle \; |A{=}a_i\rangle \; . \tag{2}$$

and that appears not to be what actually happens in the world. The right hand side of Eq.(2) is a *superposition* of various different outcomes of the A-measurement (and not any particular one of them); but what actually happens when we measure A on a system S in a state like the one on the right hand side of (2) is that one or the other of those outcomes does emerge.

And so, there has been a tradition of thinking that there must, in fact, be physical processes (processes associated with measurements) which do not proceed in accordance with the linear equations of motion; there has been a tradition of thinking that there must be such things as nonlinear *collapses* of the wave function.

There is a conventional wisdom about what a workable theory of the collapse of the wave function ought to be able to do, which runs roughly like this:

(i) It ought to guarantee that *measurements always have outcomes*[1] (that is: it ought to guarantee that there can never be any such thing in the world as a superposition of 'measuring that A is true' and 'measuring that B is true').

(ii) It ought to preserve the familiar statistical connections between the outcomes of those measurements and the wave functions of the measured systems just prior to those measurements (that is: it ought to guarantee that a measurement of a nondegenerate observable O on a system in the state $|\Psi\rangle$ yields the result o with probability $|\langle\Psi|\Phi\rangle|^2$, where $0\;|\Phi\rangle = o\;|\Phi\rangle$).

(iii) It ought to be consistent with everything which is experimentally known to be true about the dynamics of physical systems (for example: it ought to be consistent with the fact that isolated microscopic physical systems have never yet been observed *not* to behave in accordance with linear quantum-mechanical equations of motion; that such systems, in other words, have never yet been observed to undergo *collapses*).

Bell[2] has recently suggested that an interesting theory of the collapse of the wave function due to Ghirardi, Rimini, and Weber[3] looks as if it may be able to do all that; but the present note will show how, on closer examination, it begins to look much less so.

2. The Proposal of Ghirardi, Rimini, and Weber

Ghirardi, Rimini, and Weber's idea (which is formulated for nonrelativistic quantum mechanics) goes like this: The wave function of an N particle system

$$\Psi(\vec{r}_1 \ldots \vec{r}_N \; , \; t) \tag{3}$$

usually evolves in accordance with the Schrodinger equation; but every now and then (once in something like $10^{15}/N$ sec.), at random, but with fixed probability per unit time, the wave function is suddenly multiplied by a normalized Gaussian (and the product of those two *separately* normalized functions is multiplied, at that same instant, by an overall renormalizing constant). The form of the multiplying Gaussian is:

$$K \exp\left[-\frac{(\vec{r}-\vec{r}_k)^2}{2\Delta^2}\right] \tag{4}$$

where $\vec{r}_k$ is chosen at random from the arguments $\vec{r}_n$, and the width of the Gaussian, Δ, is of the order of 10^{-5} cm.. The *probability* of this Gaussian being centered at any particular point $\vec{r}$ is stipulated to be proportional to the absolute square of the inner product of (3) (evaluated at the instant just prior to this 'jump') with (4). Then, until the next such 'multiplication' or 'jump' or 'collapse' (as these sudden events have variously been called), everything proceeds, as before, in accordance with the Schrodinger equation. The probability of such jumps per particle per second (which is taken to be something like 10^{-15}, as we mentioned above), and the width of the multiplying Gaussians (which is taken to be something like 10^{-5} cm.) are new constants of nature.

That is the whole theory. No attempt is made[4] to explain the occurrence of these 'jumps'; that such jumps occur, and occur in precisely the way stipulated above, can be thought of as a new fundamental law; a beautifully straightforward and absolutely physicalist *law of collapse*, wherein (at last!) there is no talk at a fundamental level of 'measurements' or 'amplifications' or 'recordings' or 'observers' or 'minds'.

Given what is experimentally known to be true at present, this theory can very probably do (iii). Here is why: for isolated microscopic systems (i.e. systems consisting of small numbers of particles) 'jumps' will be so rare as to be completely unobservable in practice; and Δ has been chosen large enough so that the violations of conservation of energy which those jumps must necessarily produce will be very very small (over reasonable time-intervals), even in macroscopic systems.

Ghirardi, Rimini, and Weber and Bell think that this theory can very probably do (i) and (ii) too. Here is what they seem to have in mind: they suppose (if we read them correctly) that every measuring instrument must necessarily include some sort of *pointer*, which indicates the outcome of the measurement, and that the pointer (if this instrument really deserves to be called a *measuring* instrument) must necessarily be a macroscopic physical object, *and* (this is what will turn out to be problematic) that the pointer must necessarily assume macroscopically different *spatial positions* in order to indicate different such outcomes; and it turns out that if all of that is the case, then the GRW theory can do (i) and (ii).

It works like this: suppose that the GRW theory is true. Then, for measuring instruments (M) such as were just described, superpositions like

$$\alpha\ |A\rangle\ |M \textit{ indicates that 'A'}\ \rangle + \beta\ |B\rangle\ |M \textit{ indicates that 'B'}\ \rangle \tag{5}$$

(which will invariably be superpositions of macroscopically different localized states of some macroscopic physical object) are just the sorts of superpositions that do not last long. In a very short time, in only as long as it takes for the pointer's wave function to get multiplied by one of the GRW Gaussians (which will be something of the order of $10^{15}/N$ seconds, where N is the number of elementary particles in the pointer) one of the terms in (5) will disappear, and only the other will propagate, and the measurement will have an outcome. Moreover, in accordance with (ii), the probability that one term rather than another survives is proportional to the fraction of the norm which it carries. The details are spelled out quite nicely in Ref. 1.

The question, of course, is whether all measuring instruments (or, rather, whether all reasonably *imaginable* measuring instruments) really *do* work like the ones described above. That is the subject of this note.

3. Stern-Gerlach Experiments

Here is a standard sort of Stern-Gerlach arrangement for measuring the z-spin of a spin-1/2 particle: the measured particle, to begin with, is passed through a magnetic field which is non-uniform in the z direction. That field splits the wave function of the particle into spatially separated $\sigma_z = +1/2$ and $\sigma_z = -1/2$ components. Those two components move (freely, perhaps, or perhaps under the influence of additional fields) towards two different points (call one A and the other B) on a fluorescent screen. The screen works like this: a particle striking the screen at, say, point B, knocks atomic electrons in the screen in the vicinity of B into excited orbits. A short time later, those electrons return to their ground states, and (in the process) emit photons, and thus the vicinity of B becomes a luminous dot which can be observed directly by an experimenter.

We want to inquire whether or not the GRW theory entails that a measurement such as this has an outcome. That will depend on whether or not there ever necessarily comes a time in the course of such a measurement, when the position of a macroscopic object, or the positions of some gigantic collection of microscopic objects, is *correlated* to the measured z-spin. With all this in mind, let us rehearse the stages of the measuring process again:

First, the wave function of the particle is magnetically separated into $\sigma_z = +1/2$ and $\sigma_z = -1/2$ components. No outcome of the z-spin measurement (no collapse, that is) will be precipitant by that, since, as yet, nothing in the world save the position of that particle[5] (nothing, that is, save a single microscopic degree of freedom) is correlated to the z-spin. Let's keep looking.

Next, the particle hits the screen, and at that stage the fluorescent electrons get involved. Consider however, whether those fluorescent electrons get involved in such a way as to precipitate (via GRW) an outcome of the z-spin measurement. Here is the crucial point: the GRW 'collapses' are invariably collapses onto eigenstates of position (or, more precisely, onto narrow Gaussians in position-space); but it is the *energies* of those fluorescent electrons, and *not* their positions, that get correlated, here, to the z-spin to be measured! The GRW collapses are not the right *sorts* of collapses to precipitate an outcome of the measurement here.

Let us make this point somewhat more precise. Suppose that the initial state of the measured particle is an eigenstate of x-spin. Then, just after the impact of the particle on the screen, the state of the particle and of the various fluorescent electrons in the vicinities of A and B will look (approximately; ideally) like this:

$$\frac{1}{\sqrt{2}} \left|\sigma_z=+\frac{1}{2}, \vec{r}=A \right\rangle_{MP} \cdot |\uparrow\rangle_{e_1} \cdots |\uparrow\rangle_{e_N} \cdot |\downarrow\rangle_{e_{N+1}} \cdots |\downarrow\rangle_{e_{2N}} +$$

$$\frac{1}{\sqrt{2}} \left|\sigma_z=-\frac{1}{2}, \vec{r}=B \right\rangle_{MP} \cdot |\downarrow\rangle_{e_1} \cdots |\downarrow\rangle_{e_N} \cdot |\uparrow\rangle_{e_{N+1}} \cdots |\uparrow\rangle_{e_{2N}} \quad (6)$$

where 'MP' is the measured particle, $e_1 ... e_N$ are fluorescent electrons in the vicinity of A, $e_{N+1} ... e_{2N}$ are fluorescent electrons in the vicinity of B, $|\uparrow\rangle$ represents an excited electronic state, and $|\downarrow\rangle$ is a ground state. Suppose, now, that a GRW 'collapse' (i.e. a multiplication of (6) by a Gaussian of the form (4), where $\vec{r}_n$ is the position-coordinate of one of the fluorescent electrons) occurs. Consider whether this sort of collapse will make one of the terms in (6) go away, and allow only the other to propagate. The problem, once again, is that these are not the right sorts of collapses for that job;

because $|\uparrow\rangle$ can not be distinguished from $|\downarrow\rangle$ in terms of the *position* of anything. Here is a somewhat more precise way to put it: the position *differences* between $|\uparrow\rangle$ and $|\downarrow\rangle$, which do, in fact, exist, are far smaller than the 10^{-5} cm widths of the multiplying Gaussians. Indeed, such a collapse will leave (6) almost entirely unchanged (except, perhaps, in the wave function of some single one of the many many fluorescent electrons).

We have left aside the whole question of the *probability* of such a collapse here, but it ought to be noted in passing that that probability might well be extremely *low*. It is well known, after all, that the unaided human eye is capable of detecting very small numbers of photons; so perhaps only very small numbers of fluorescent electrons need, in principle, be involved here! It would be interesting to calculate those numbers; but however that calculation comes out, it appears (for the reasons described in the previous paragraph) that the GRW theory would not entail that an outcome of the z-spin measurements emerges at this stage, either.

We have to look elsewhere. The next stage of the measuring process involves the decay of the excited electronic orbits and the emission of photons. If the first term in (6) obtained, the photons would be emitted at A; if the second term obtained, the photons would be emitted at B. Those two photon states, then, *can* be distinguished, at least at the moment of emission, in terms of the *positions* of the photons. Now, so far, GRW's theory has been applied by them only to a nonrelativistic system of distinguishable particles. Photons, on the other hand, are relativistic, indistinguishable particles, and it is not completely clear how GRW might treat them. If photons can not experience GRW collapses, then of course no outcome can *possibly* emerge at this stage. But let us suppose that the photons *can* experience GRW collapses. The problem at this stage of the measurement will be that that distinguishability in terms of positions will be *extremely* short-lived. In almost no time, too little for a GRW collapse to be likely to occur (supposing that A and B are, say, a few centimeters apart, on a flat screen) the two photon wave functions described above will almost entirely overlap in position-space, the distinguishability in terms of positions will go away, and we shall be in just such a predicament as we found ourselves at the previous stage of the measurement. No outcome, it seems, will emerge here, either.

But now we are running out of stages. The measurement (according to the conventional wisdom about measurements) is already *over*! By now, after all, we have a recording; by now genuinely macroscopic changes (that is: changes which are thermodynamically irreversible, changes which are directly visible to the unaided human eye) have already taken place in the measuring apparatus. The technical details of real Stern-Gerlach experiments have of course been oversimplified or idealized or just left out of the present account, but these details are beside the point (any number of *other* experimental arrangements, which, like this one, are free of macroscopic moving parts, would have served our purpose here equally well); the *point* is simply that genuine recordings need *not* entail macroscopic changes in the *positions* of anything. Changes in the *internal* states of large numbers of microsystems (changes, say, in atomic energy levels) can be recordings too.

That is what's overlooked in the GRW proposal. What the GRW theory requires in order to produce a collapse is not merely that the recording in the measuring apparatus be macroscopic (in any or all of the senses of 'macroscopic' just described), but rather that the recording process involve macroscopic changes in the *position* of something. The problem is that *no* changes of that latter sort are involved in the kinds of measurements we have considered here.

Suppose, after all this, that we wanted to stick with the GRW theory anyway. What would that entail? Well, we would have to deny that the measurement described above is over even once a macroscopic recording exists. And we would have to go on looking for an outcome, albeit we have already looked right up to the retina of the human experimenter, and not found one. The only place left to look would be inside of that experimenter's nervous system.[6]

So it would turn out (if we wanted to stick with this theory in spite of everything) that the possibility of entertaining a certain proposal about the fundamental laws of the world would hinge on certain details of the neurophysiology of human brains, and of the brains of whatever other sentient beings there may happen to be.

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References and Footnotes

1 Of course, measurements need not have outcomes until they are *over*, until a *recording* exists in the measuring device! So, if (i) is to be a meaningful physical requirement of a satisfactory theory of the collapse, then something is going to have to be said about *what* a recording *is*. It will be best (it will make our argument as strong and as general as possible, as the reader will presently see) to be very *conservative* about that: so no change in the physical state of a measuring device will be called a recording, here, unless that change is macroscopic, thermodynamic, irreversible, and visible to the unaided eye of a human experimenter.

2 Bell, J.S. (1987) *Are There Quantum Jumps?* in 'Speakable and Unspeakable in Quantum Mechanics', Cambridge University Press, Cambridge, pp. 201-212.

3 Ghirardi, G.C., Rimini, A., and Weber, T. (1986) *Unified Dynamics for Microscopic and Macroscopic Systems,* Phys. Rev. **D34**, 470.

4 Recently, a theory which leads to continuous 'jumps' was developed: Pearle, P. (1988) *Combining Stochastic Dynamical Statevector Reduction with Spontaneous Localization*, to be published in Phys. Rev. **A**.

5 Actually, the *first* thing that gets correlated to the *z*-spin in an arrangement like this is the momentum, or something approximating the momentum, of the measured particle; but that momentum (since the initial wave-function of the particle is taken to be reasonably well localized) quickly (before the particle hits the screen) gets translated into a position, which can then be 'read off' from the screen.

6 Perhaps such a possibility ought not to be lightly dismissed. Bell, for example, has indicated (in private conversation) a willingness to take it seriously.