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Systems Competition, Vertical Merger and Foreclosure

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Abstract

In this paper we address the possibility of foreclosure in markets where the final good consists of a system composed of a hardware good and complementary software and the value of the system depends on the availability of software. Foreclosure occurs when a hardware firm merges with a software firm and the integrated firm makes its software incompatible with a rival technology or system. We find that foreclosure can be an equilibrium outcome where both the merger and compatibility decisions are part of a multistage game which permits the foreclosed hardware firm to play a number of counter-strategies. Further, foreclosure can be an effective strategy to monopolize the hardware market.

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1 Introduction

In this paper we address the incentives for, and the efficiency effects of, vertical mergers and foreclosure in markets characterized by competing systems. In particular we are interested in industries where a system consists of a durable (hardware) good and complementary products. We focus on "hardware-software" systems where the benefit of a system is increasing in the variety of complementary products or software available. In such a market, foreclosure could arise, when after a merger between a hardware firm and a software firm, the integrated firm made its software incompatible with rival hardware technologies or systems. By integrating with a software firm and foreclosing, it may be possible for a hardware firm to extend its market power in the market for *hardware* since the reduction in the number of software varieties compatible with competing hardware technologies decreases their value to consumers.

This possibility raises two questions which we address in this paper. The first concerns whether or not it is in fact true that vertical merger and foreclosure by a hardware firm is an equilibrium to a well specified game. Is it a profit maximizing strategy for a hardware firm to profitably merge with software suppliers and restrict the supply of software to competing systems? Secondly, if such an outcome is in fact possible, what are the welfare implications?

Concern precisely over the potential for this type of foreclosure has been one of the issues in a number of recent antitrust actions in the United States. In 1994 the Department of Justice (DOJ) filed a civil antitrust suit challenging the acquisition of Liberty Media Corporation by Tele-Communications, Inc. (TCI). It was the first challenge of a vertical merger under §7 of the Clayton Act by the DOJ in more than a decade.[1] TCI and Liberty are large multiple cable system operators (MSOs) with numerous programming interests.[2] One of the provisions of the consent decree enjoins TCI and Liberty

[1] "Cable TV Operator, Programmer Resolve Division's Concerns over Vertical Merger," *Antitrust and Trade Regulation Reporter* 66: 512-513 (May 5, 1994).

[2] In fact TCI is the largest MSO in the U.S.

to supply their video programming on a non-discriminatory basis to other competing multichannel television providers.[3]

On November 14, 1995, the U.S. Federal Trade Commission (FTC) approved a consent decree with Silicon Graphics, Inc. (SGI) which allowed it to acquire two of the three leading graphic entertainment software companies.[4] One of the reasons behind the challenge was that competing manufacturers of workstations would be foreclosed from two important independent providers of graphic software. One of the provisions of the consent agreement is that SGI make the two major entertainment graphics software programs it acquired from Alias Research compatible with the hardware workstations of a competitor.

On February 7, 1997 the FTC approved a restructuring of the Time Warner and Turner Broadcasting System (TBS) Inc. merger. Time Warner is the second largest cable system operator in the U.S. Time Warner is also a major supplier of cable programming networks. It owns HBO as well as other programming interests. TBS is a major supplier of cable television networks including CNN and TNT. One of the concerns in the action is that this merger would make it difficult for Time Warner's downstream cable system rivals to obtain programming. Among other provisions, the agreement prohibits Time Warner from discriminating in price or refusing to supply TBS programming to competing multichannel television providers.[5] We expect that conflicts over access to content will arise with the development of the information highway and competition between alternative technologies and vendors.[6]

[3] *United States v. Tele-Communications,* 1996-2 CCH Trade Cases ¶71, 496.

[4] See FTC News Release, "FTC Settlement Would Preserve Competition on Price and Innovation for Entertainment Graphics Software and Hardware," June 9, 1995 and Silicon Graphics, Inc.: Proposed Consent Agreement, 60 Federal Register 35032 (July 5, 1995). The final consent decree is summarized at http://www.ftc.gov/opa/1995/9511/sil2g.htm.

[5] See FTC News Releases, "FTC Requires Restructuring of Time/Warner Deal," September 12, 1996 and "FTC Gives Final Approval to Time/Warner Deal," February 7, 1997 available at http://www.ftc.gov/opa/1996/9609/timewarn.htm and http://www.ftc.gov/opa/1997/9702/twfinal.htm. The consent decree is Time Warner Inc., et al.: Proposed Consent Agreement, 61 Federal Register 50301 (July 5, 1995).

[6] Willcox (1995) emphasizes that the alleged anticompetitive effects of mergers like TCI/Liberty are not described in the prevailing vertical merger guidelines and proposes that a new set of guidelines, which articulates the theoretical foundation and enforcement

We address the question of foreclosure in system markets by modelling both the merger and foreclosure decisions as part of a multistage game. In the model we develop, a system consists of a hardware component and complementary software. The willingness of a consumer to pay for a system depends on her preferences about hardware, software and the number of compatible software varieties available. There are two hardware firms who produce differentiated products. Software is provided by two software firms. Each software firm, in the absence of foreclosure, can (and will) supply software in formats compatible with both hardware technologies.

In our setting, a hardware firm has the opportunity to merge with one of the two independent software firms. If a merger occurs, then the integrated or merged firm must determine the formats in which it will offer software. The integrated firm can elect either to make the software available only in a format compatible with its own hardware or it can make its software available in formats compatible with both hardware technologies. **Foreclosure occurs when an integrated firm makes its software incompatible.** In such a case, the consumers of the "foreclosed" hardware firm are excluded from the integrated firm's software variety.

Having observed both the integration and compatibility decisions of the first hardware firm, the remaining hardware firm then has the opportunity to merge with the other independent software firm and if it does so, it also makes a compatibility decision. Hardware prices are then determined and consumers make their adoption decision. Our modelling highlights the possibility that a foreclosed firm can retaliate.

The profitability of foreclosure depends on the trade off between lost software profits (from not supplying the competing system) and increased hardware profits (from the increase in demand and potentially the increase

standards consistent with recent enforcement action, be created. (The U.S. DOJ 1984 Guidelines are the current ones for *vertical* mergers.) In a February 24, 1995 speech entitled, "A New Age of Antitrust Enforcement: Antitrust in 1995," given at Marina Del Rey, California, FTC Commissioner Starek also emphasized the need for enforcement to be "analytically coherent and theoretically well-founded" and that "As long as we are in the vertical enforcement business, we should provide clear guidance regarding our analytical framework and enforcement intentions."

in hardware price). Foreclosure has both a direct and an indirect effect on hardware profits. The direct effect is the increase in demand from the differential created in software availability for the two hardware systems. The indirect effect is the associated change in hardware pricing. The increase in demand can provide the foreclosing firm with incentives to charge *higher prices* for its hardware.

We find that a *unilateral foreclosure outcome*—where only one hardware firm forecloses—occurs in two (very different) cases:[7]

- Interior Foreclosure Equilibrium: In an interior foreclosure equilibrium, foreclosure does not result in monopolization of the hardware market; both hardware technologies have positive market shares. This case occurs when the hardware products are *very* differentiated and the marginal value of software variety is small. The equilibrium price of the foreclosing firm's hardware is higher than in the unintegrated structure and its market share is larger due to the direct effect of foreclosure.

- Standardization Foreclosure Equilibrium: In this case the foreclosing firm successfully monopolizes the market. This case occurs when the extent of hardware product differentiation is very small. Standardization means that the foreclosing firm does not forgo any software profits (since no consumers purchase the competing system), its hardware market share doubles and (in most instances) its price increases relative to the unintegrated industry structure.

We show that both foreclosure equilibria are inefficient because of the reduction in software variety and the increase in market coverage of the foreclosing hardware.

Ordover, Saloner, and Salop (OSS 1990) also investigate the welfare effects of foreclosure and vertical mergers in a model in which the vertical

[7]In all other cases the equilibrium industry structure is unintegrated. For *unilateral* foreclosure to be profitable it must be the case that retaliation is not profitable. A foreclosed firm can retaliate by either integrating and foreclosing or just integrating but continuing to supply software to the other system. Because retaliation does not restore the ex ante situation, it need not be profitable. See section (5).

structure of the industry is determined endogenously. The OSS result, that foreclosure can emerge in equilibrium, has been criticized because it depends on the ability of a merged firm to commit to a price.[8] Commitment is not an issue for our foreclosure equilibrium as long as the compatibility decision of the foreclosing firm is not easily reversible. In other words, foreclosure will be a credible threat if the fixed costs of eliminating compatibility, i.e. of foreclosure, are relatively small and the costs of re-establishing compatibility are relatively large (which is typically the case).[9] Alternatively, the credibility of foreclosure can arise from the incentives "multi-generation" hardware firms have to develop and preserve reputations for not supplying software for rival systems.

Our results differ from those of OSS in two significant ways, both of which suggest that the potential for foreclosure in system markets is more likely than in markets for inputs. First, our concern with systems competition means that changes in ownership affect pricing incentives. As a result while both technologies have equal market shares in the bilateral foreclosure subgame and the unintegrated industry structure, the profitability of the systems are not the same, that is, the ex ante situation is not achieved. Because of the internalization of pricing externalities, hardware prices and hence profits are significantly lower in the bilateral foreclosure subgame. This makes retaliation less likely. Second, foreclosure in systems markets can be an effective monopolization strategy. The limit on the input price rise before retaliation means that this cannot happen in OSS.

Similar concerns regarding the foreclosure effects of tying have been raised by Whinston (1990). In his analysis, tying of hardware to software results in lower hardware prices for the tying firm and leads to exclusion of the

[8]See Hart and Tirole (1990) and Reiffen (1992). In OSS the merged firm must be able to commit not to compete aggressively with the remaining supplier to supply the other downstream firm for foreclosure to be an equilibrium. This commitment allows the remaining independent input supplier to exercise its monopoly power and thus the price of the input rises.

[9]The profitability of foreclosure will then justify incurring the costs of eliminating compatibility but the large costs of re-establishing compatibility will not be justified on the basis of incremental software sales to adopters of the rival system.

rival hardware firm. The effects at work in our paper are quite different. Foreclosure is profitable when it leads the foreclosing firm to raise the price of its hardware.[10] Moreover, we show that this strategy can be profitable even if the rival is not excluded.

2 The Model

In this section we develop the model. We begin by describing technology.

2.1 Technology

We consider a situation where there are four firms: two manufacturers of hardware and two software firms. We denote the two hardware firms by A and B. The hardware products are differentiated along the unit interval. The locations of the hardware firms are fixed: technology A is at the left-end point, technology B at the right-end point.

The software firms each have a single software product. The development costs of software are sunk and we assume that the number cannot be augmented.[11] The software of both firms is initially available in formats compatible with both of the hardware technologies. Foreclosure thus means withdrawing compatible software from the software market for a rival hardware technology. Finally, we assume that the marginal cost of producing a unit of hardware or a unit of software is constant and, without loss of generality, equal to zero.

2.2 Preferences of Consumers

We now specify the preferences of consumers over the components of a system (hardware and software) and an outside good. In modelling consumer

[10] It is the direct effect, the reduction in the size of the rival's market, that is exclusionary.

[11] This is certainly reasonable in the short run where "stocks" are more important than "flows." Clearly Sony acquired Columbia Pictures Entertainment and CBS Records for their installed base of movie and music titles. Church and Gandal (1992) consider the incentives and effects for integrated hardware firms who compete against each other with closed systems to augment software variety. Our focus here is on situations where hardware firms foreclose software access by changing open systems into closed systems.

preferences for a system, we explicitly recognize the following:

- The value of a system depends on the availability of software.
- The greater the variety of software, the greater the benefit or value of a system. However, the marginal value of additional software is decreasing.
- Consumers buy at most one unit of hardware and one unit of each variety of software.[12]

We denote the benefit consumers receive from consuming N varieties of software by $W(N)$, where $W(2) > W(1)$ and $W(1) > W(2) - W(1)$.[13] The last comparison follows from decreasing marginal utility of software and it also implies that $2W(1) - W(2) > 0$.

The preferences of consumers for hardware are represented using an address model. The tastes of consumers are distributed uniformly along a line of unit length, the population is normalized to one, and all consumers have income y. The consumption of a hardware technology different from the most preferred type imposes a utility cost on the consumer that is proportional to the distance separating the two types. The utility function of a consumer located distance t_i from hardware i is therefore[14]

$$U_i = W(N_i) + x + \alpha - kt_i, \tag{1}$$

where α is the stand-alone benefit of hardware, N_i is the number of software varieties consumed, x is consumption of a competitively supplied outside good, and k measures the extent of hardware differentiation. The budget constraint for a consumer is:

[12]Negative hardware prices are possible. In such a case, we still assume that consumers buy a single unit of hardware.

[13]Of course, $W(0) = 0$.

[14]The benefit from software consumption $W(N_i)$, is only obtained if hardware i is purchased.

$$\sum_{j=1}^{N} \rho_{ji} + x = y - p_i, \tag{2}$$

where ρ_{ji} is the price of a unit of software variety j available for hardware technology i, y is the income of the consumer, p_i is the price of hardware technology i, and N is the number of software varieties purchased.

2.3 Timing of the game

We model a four-stage game. There are three actions available to hardware firm A in stage one: remain unintegrated (denoted U); integrate with one of the independent software firms, but continue to provide software compatible with both hardware technologies (denoted I); and integrate and foreclose (denoted F). In stage two, hardware firm B has the same three actions as firm A: U, F, and I.

We assume that consumers purchase hardware before purchasing software. Hence, software prices are determined after hardware purchases.[15] Thus, in the third stage, hardware prices are determined and consumers make their hardware adoption decisions. In the final stage, software firms set prices and consumers purchase software. The subgame perfect equilibrium is found by backwards induction beginning with the final stage.[16]

3 Software Pricing and Consumer Purchase of Software.

In this stage, (1) is maximized subject to (2) for each hardware technology i. Here, the consumer selects which varieties of software and the total number

[15] Causal empiricism suggests that consumers purchase hardware before software in some markets. Consider for instance compact disc players and compact discs, computers and software, and video-game players and video games.

[16] Our results are robust to changes in the timing of the compatibility decision. In particular, we would get the same results if in the third stage an integrated firm simultaneously chose its hardware price and made its compatibility determination. An integrated hardware firm that has foreclosed will not make its software available for the other hardware technology *after* the other hardware firm elects not to integrate. Even if it were technologically feasible to do so, it would not, as to do so would lower its profits.

of varieties to consume. Substituting this into (1) gives indirect utility for technology i.

Consumers may be able to purchase either two software varieties or a single variety. When a single variety is available, consumers will purchase it provided its price is equal to or less than its incremental benefit, $W(1)$. When two software varieties are available, consumers will purchase both if they are each priced equal to or below the incremental benefit of a second variety: $W(2) - W(1)$. If the price of a software variety exceeds its incremental benefit, consumers will not purchase it. Hence if the common price of software was greater than $W(2) - W(1)$ (but less than $W(1)$), consumers would only purchase a single variety.

A price less than its incremental benefit reduces profits: sales are unchanged as consumers have unit demands for a variety of software and the market size is determined by hardware sales. These two considerations also mean that software pricing is unaffected by who owns which products. The following lemma regarding the price of software follows immediately.

Lemma 1 *If a monopolist supplies a single variety of software, for technology i, then the profit maximizing software price is*

$$\rho_i = W(1). \tag{3}$$

If each of two software firms supply a single software variety for technology i, then the symmetric Nash equilibrium software price is

$$\rho_i = W(2) - W(1). \tag{4}$$

A symmetric software price equal to $W(2)-W(1)$ implies that in the case of two competing software products, the budget constraint can be written as $2W(2)-2W(1)+x = y-p_i$. Solving for x and substituting into (1) gives the indirect utility function of a consumer located distance t_i from technology i:

$$V_i = 2W(1) - W(2) + y + \alpha - p_i - kt_i. \tag{5}$$

In the case of a single software supplier, the indirect utility function of a consumer located distance t_i from technology i is:

$$V_i = y + \alpha - p_i - kt_i. \tag{6}$$

4 Hardware Pricing and Purchase

In the third stage, the consumer selects the hardware variety for which indirect utility is greatest.[17] A consumer purchases hardware A if the benefit from adopting system A (V_A) exceeds the benefit from adopting system B (V_B). If we measure t from the left-end point,[18] then the marginal consumer is defined implicitly by $V_A = V_B$. Using equation (5) and rearranging terms, the value for the equilibrium market share for technology A is

$$t = \frac{\delta_a[2W(1) - W(2)] - \delta_b[2W(1) - W(2)] - (p_A - p_B) + k}{2k}, \tag{7}$$

where δ_i is a dummy variable that takes on the value one if there are two software suppliers for technology i and zero if there is a single supplier of software. The equilibrium market share for technology B is simply $(1 - t)$. The market share for either hardware firm cannot exceed 1. If $t = 1$ then technology A is exclusively adopted. From (7), the market share of a technology depends on relative prices of hardware, the extent of hardware differentiation (k), and relative software supply.

4.1 Industry Structures

There are four possible industry structures.[19]

1. Unintegrated: Neither hardware firm merges with a software firm.

[17] We assume that α is large enough so that the market is fully served, that is, all consumers purchase one of the two hardware technologies.

[18] In other words, $t_A = t$ and $t_B = 1 - t$.

[19] There might appear to be two additional industry structures: (1) both hardware firms integrate, but neither forecloses and (2) one hardware firm integrates, but does not foreclose, and the other hardware firm does not integrate. These cases are formally equivalent to the unintegrated case since they do not change the number of software varieties available for either technology or the pricing behavior of the hardware firms.

2. Bilateral Foreclosure: Both hardware firms merge with an independent software firm and foreclose.

3. Foreclosure/Integrated: Both hardware firms have merged with software firms, but only one has foreclosed.

4. Foreclosure/Unintegrated: Only one hardware firm has merged with a software firm and it has foreclosed.

Without loss of generality we assume that whenever only one of the hardware firms integrates and forecloses, it is firm A. If the industry structure is (1), then $N_A = N_B = 2$,[20] in (2), $N_A = N_B = 1$, while for (3) and (4) $N_A = 2$, but $N_B = 1$. The marginal consumer and the market shares of the two hardware firms as a function of the prices of hardware are found by substituting the relevant number of software varieties available for each hardware technology into (7). In cases (1) and (2), the number of software products supplied for each hardware platform is the same and thus,

$$t = \frac{p_B - p_A + k}{2k}. \tag{8}$$

In cases (3) and (4), the foreclosure cases,

$$t = \frac{[2W(1) - W(2)] - (p_A - p_B) + k}{2k}. \tag{9}$$

Foreclosure provides firm A with a competitive advantage. Comparing (8) and (9) illustrates that a greater supply of software leads to a market share advantage since by concavity, $2W(1) - W(2) > 0$. We refer to this term as the *demand effect* of foreclosure.[21] The industry structures differ

[20]Recall that in the absence of integration, all software products are compatible with both hardware technologies. It can easily be shown that the software firms would choose to make their products compatible with both hardware platforms (and vice versa) in the unintegrated industry structure.

[21]The magnitude of the demand effect is the increase in consumer surplus from consuming an additional software variety. It depends on the degree of software competition. When consumers place a very high value on variety, so that $W(2)$ is nearly twice $W(1)$, the demand effect will be very small because software prices will be very high (nearly $W(1)$). In this case, consumer surplus from software purchases is essentially zero with or without foreclosure. We are grateful to an anonymous referee for this point.

not only in terms of compatibility, but also in terms of ownership. Common ownership of hardware and compatible software mean that when pricing its hardware, an integrated firm will internalize the externality between software and hardware.

4.2 Industry Structure Subgames

The subgame perfect equilibrium is found by backwards induction. We now consider, in turn, the hardware pricing equilibrium in each of the four possible industry structures (subgames). We then compare the effect of the differing structures on the equilibrium prices after deriving the hardware pricing equilibria.

4.2.1 Unintegrated Industry

In this case each hardware firm only derives profits from sales of hardware. The profits of hardware firm A are, using (8),

$$\pi_A = tp_A = \frac{[(p_B - p_A) + k]}{2k} p_A. \tag{10}$$

The profit function of firm B is similar. The best-response function of firm i is simply

$$p_i \;=\; R_i^{UU}(p_j) \;\;=\;\; \frac{p_j \;+\; k}{2}, \tag{11}$$

and the following proposition summarizes the equilibrium hardware prices and profits.

Proposition 1 *The equilibrium hardware prices, market shares, and hardware profits in the unintegrated industry structure are respectively* [22]

$$p_A = p_B \equiv p^{UU} = k, t^{UU} = 1/2, \pi_A = \pi_B \equiv \pi^{UU} = k/2.$$

The profits earned by each of the independent software firms in the unintegrated industry structure are:

[22] The hardware prices, market shares, hardware profits, and software profits are identical to those in Proposition 1 if (1) both firms integrate but neither forecloses and (2) one firm integrates but does not foreclose and the other firm does not integrate.

$$\pi_s^{UU} = \rho_A t + \rho_B(1-t) = W(2) - W(1).$$

4.2.2 Bilateral Foreclosure

In this setting, each merged firm is the monopoly supplier of software for its technology. From Lemma 1, hardware firm A will charge $W(1)$ for its software product and software revenues will be $W(1)$ per consumer and $tW(1)$ in total. The profits of merged firm A from sales of hardware and software, after substituting (8) for t, are

$$\pi_A = t(p_A + W(1)) = (\frac{p_B - p_A + k}{2k})(p_A + W(1)). \tag{12}$$

The profit function for merged firm B is similar. The best-response function for firm i is

$$p_i \;=\; R_i^{FF}(p_j) \;=\; \frac{p_j \;+\; k \;-\; W(1)}{2}, \tag{13}$$

and the following proposition provides the equilibrium prices and profits.

Proposition 2 *The equilibrium hardware prices, market shares, and profits in the bilateral foreclosure case are:*

$$p_A = p_B \equiv p^{FF} = k - W(1), t^{FF} = 1/2, \pi_A = \pi_B = \pi^{FF} = k/2.$$

4.2.3 Foreclosure/Integrated

The two merged firms compete as duopolists in the software market for technology A and from Lemma 1, the price of software will be $\rho_A = W(2) - W(1)$, while in the software market for technology B, firm B is a monopolist and the price of software will be $\rho_B = W(1)$. The profits of merged firm A from its sales of hardware and software for its system are

$$\pi_A = (p_A + W(2) - W(1))t. \tag{14}$$

The best-response function for firm A is found by substituting (9) for t into (14) and maximizing with respect to p_A:

$$p_A \;=\; R_A^{FI}(p_B) \;=\; \frac{p_B \;+\; k \;+\; 3W(1) \;-\; 2W(2)}{2}. \tag{15}$$

Similarly, the profits of merged firm B are

$$\pi_B = (p_B + W(1))(1-t) + [W(2) - W(1)]t. \tag{16}$$

Substituting in (9) for t into (16) and maximizing with respect to p_B gives the reaction function for firm B:

$$p_B \;=\; R_B^{FI}(p_A) \;=\; \frac{p_A \;+\; k \;+\; 2W(2) \;-\; 4W(1)}{2}. \tag{17}$$

In the following proposition, we summarize the equilibrium prices, market shares and profits for the integrated case. In advance of the proposition, define

$$k^{FI} = \frac{W(2) - W(1)}{3}. \tag{18}$$

Proposition 3 *When $k > (\leq) k^{FI}$, an interior (standardization) equilibrium obtains. When both hardware firms integrate, but only firm A forecloses (the foreclosure/integrated case), the hardware prices, market shares, and profits in an "interior" equilibrium (an equilibrium in which both hardware technologies have positive market shares) are:*

$$p_A^{FI} = \frac{3k + 2W(1) - 2W(2)}{3}, p_B^{FI} = \frac{3k + 2W(2) - 5W(1)}{3}, t^{FI} = \frac{W(2) - W(1) + 3k}{6k},$$

$$\pi_A^{FI} = \frac{[W(2) - W(1) + 3k]^2}{18k}, \pi_B^{FI} = \frac{[W(1) - W(2) + 3k]^2}{18k} + W(2) - W(1).$$

In a standardization equilibrium (an equilibrium in which all consumers purchase the same hardware technology) for the foreclosure/integrated case, hardware prices, market shares, and profits are:[23]

[23] In a standardization equilibrium, an integrated firm B continues to earn profits from

$$p_A^{FI} = -k, p_B^{FI} = [W(2) - 2W(1)], t^{FI} = 1,$$

$$\pi_A^{FI} = W(2) - W(1) - k, \pi_B^{FI} = W(2) - W(1).$$

4.2.4 Foreclosure/Unintegrated

The best-response function for merged firm A is once again given by (15). The profits of the unintegrated hardware firm B equal

$$\pi_B \ = \ (1 \ - \ t)p_B \tag{19}$$

where t is given by (9). As a result B's best-response function is

$$p_B \ = \ R_B^{FU}(p_A) \ = \ \frac{p_A \ + \ k \ \ + \ W(2) \ - \ 2W(1)}{2}. \tag{20}$$

In the following proposition we summarize the equilibrium prices, market shares and profits for this industry structure. In advance of the proposition, define

$$k^{FU} = \frac{W(1)}{3}. \tag{21}$$

Proposition 4 *When $k > (\leq) k^{FU}$, an interior (standardization) equilibrium obtains. The equilibrium hardware prices, market shares, and profits in an interior equilibrium when only firm A integrates and forecloses (the foreclosure/unintegrated case) are:*

$$p_A^{FU} = \frac{4W(1) - 3W(2) + 3k}{3}, p_B^{FU} = \frac{-W(1) + 3k}{3}, t^{FU} = \frac{W(1) + 3k}{6k},$$

its "technology A" compatible software. For parameter values where $k^{FI} \geq k$, when firm A prices according to (15) $t^{FI} \geq 1$. The best-response of firm A is to raise its price until $t^{FI} = 1$ when B is pricing according to (17). The equilibrium price in the standardization equilibria for firm A follows from substituting B's best response function, (17) into the expression for market share, (9), setting the resultant expression equal to one, and solving for p_A. Substituting the standardization equilibrium price of firm A into (17) yields the standardization equilibrium price for firm B. The standardization profits follow immediately.

$$\pi_A^{FU} = \frac{[W(1)+3k]^2}{18k}, \pi_B^{FU} = \frac{[-W(1)+3k]^2}{18k}.$$

The independent software firm earns

$$\pi_s^{FU} = [W(2)-W(1)](\frac{W(1)+3k}{6k}) + W(1)(\frac{-W(1)+3k}{6k}).$$

For the standardization equilibrium in the foreclosure/unintegrated case, the equilibrium prices, market shares and profits are:[24]

$$p_A^{FU} = 2W(1) - W(2) - k, p_B^{FU} = 0, t^{FU} = 1,$$

$$\pi_A^{FU} = W(1) - k, \pi_B^{FU} = 0.$$

The profits of the independent software firm are

$$\pi_s^{FU} = W(2) - W(1).$$

4.3 Implications of the Alternative Industry Structures

In this section we summarize the results of the impact of the differing industry structures on the equilibrium hardware prices and market shares.

Corollary 1 *The hardware market shares for firm A under the different industry structures are ordered as follows:*

$$t^{FU} \geq t^{FI} \; > \; t^{FF} \; = \; t^{UU} \; = \; \frac{1}{2}. \tag{22}$$

As expected the effect of foreclosure is an increase in market share. Indeed, if the variety advantage is large enough relative to the degree of hardware differentiation, then a standardization equilibrium results and all consumers purchase technology A. If firm B responds by foreclosing, then the

[24] A standardization equilibrium will result if $k^{FU} \geq k$. Note that $k^{FU} > k^{FI}$. The standardization price for technology A is found by substituting B's reaction function, (20), for p_B into the expression for market share, setting the result equal to one, and solving for p_A. Substituting into B's reaction function yields $p_B = 0$. The standardization profits follow immediately.

demand effect is negated and the two firms have equal market shares. Provided hardware is not minimally differentiated, integration by firm B partially offsets the demand effect.[25] This occurs as a result of the effect that integration has on the incentives of B to price hardware aggressively.

Corollary 2 *In an interior equilibrium, the ranking of hardware prices for the foreclosed firm (firm B) is:*

$$p_B^{UU} \;>\; p_B^{FU} \;>\; p_B^{FI} \;>\; p_B^{FF}. \tag{23}$$

The ranking of hardware prices for the foreclosing firm (firm A) is:

$$p_A^{UU}, p_A^{FU} \;>\; p_A^{FI} \;>\; p_A^{FF}. \tag{24}$$

$$p_A^{FU} > p_A^{UU} \;\; \textit{iff} \;\; W(2) - W(1) < W(1)/3. \tag{25}$$

The intuition for (25) in Corollary (2) is that there are two off-setting indirect strategic effects associated with foreclosure. First, the demand effect makes the foreclosing firm "softer" (in the terminology of Fudenberg and Tirole (1984)); for any p_B firm A will find it optimal to charge a higher price relative to the unintegrated industry structure.[26] However, integration and foreclosure also provides firm A with an incentive to price its hardware more aggressively since it will internalize the pricing externality between the hardware and software components which comprise its system. Lowering hardware price to increase hardware market share will increase software sales. The price effect associated with integration and foreclosure makes the foreclosing firm "tougher."

Hence, the effect of foreclosure on the pricing behaviour of firm A is ambiguous. From (11) and (15), for a given p_B, firm A's finds it optimal to charge lower prices (higher prices) if $W(2) - W(1) > (<) \frac{W(1)}{2}$. Since

[25] Note that $k^{FU} > k^{FI}$.

[26] It also makes the foreclosed firm "tougher": for any p_A firm B finds it optimal to charge a lower price.

foreclosure always makes firm B's pricing tougher, the *equilibrium* price of the foreclosing firm is higher than under the unintegrated industry structure if $W(2) - W(1) < \frac{W(1)}{3}$. We show in Proposition (5) that firm A finds it profitable to foreclose when the above condition holds.

5 Equilibrium

In the preceding section, we determined the equilibrium prices and profits for each of the four possible industry structures. In this section we use that analysis to determine the equilibrium to the full (four-stage) game.

We introduce the following notation to simplify the analysis. Let $\Pi_i^{s_A,s_B}$ denote the sum of the payoffs of hardware firm i and a software firm, where $i = A, B$ and s_A is the action (U, I, or F) played by firm A and s_B is the action played by firm B. For the cases in which a hardware firm chooses either to integrate (I) or integrate and foreclose (F), these payoffs are simply the profits of the merged firm. For the case in which a firm chooses to remain unintegrated (U), these payoffs are the *stand-alone* payoffs, that is the sum of the profits of the hardware firm and an independent software firm.

For foreclosure to be an equilibrium we must establish the following:

1. Firm B does not find it profitable to retaliate by integrating, i.e., the sum of the unintegrated profits of hardware firm B and the unintegrated software firm are greater than the joint profits of a merged firm B. The required condition is $\Pi_B^{FU} > \max(\Pi_B^{FI}, \Pi_B^{FF})$.

2. It must be profitable to foreclose. This means that the merged firm A's profits from software and hardware exceed the sum of the profits of the unitegrated hardware firm A and the unintegrated software firm. The required condition is $\Pi_A^{FU} > \Pi_A^{UU}$.[27]

[27]Recall that if firm A chooses not to merge with the software firm, firm B has a chance to merge with the software firm. Due to the symmetry in the model, however, if a merger is unprofitable for firm A, it will also be unprofitable for firm B. That is, firm B (endogenously) chooses not to merge in equilibrium. Thus the correct comparison for firm A is between the profits under foreclosure (Π_A^{FU}) vs. the profits under the unintegrated industry structure (Π_A^{UU}).

3. The acquired software firm must find it profitable to merge. The potential for a hold-out problem exists. In a foreclosure equilibrium, the independent software firm will be a monopoly provider of software for the foreclosed hardware technology as well as a duopolist in the software market for the integrated firm's hardware. The constraint on participation by a software firm requires that it be profitable for the hardware firm to pay an amount for the software firm such that it prefers to merge rather than be independent. This constraint is $\Pi_A^{FU} - \pi_B^{FU} > \pi_s^{FU}$ or $\Pi_A^{FU} > \Pi_B^{FU}$. The aggregate profits of the foreclosing system must exceed the aggregate profits of the foreclosed system, ensuring that it is preferable to be part of the merging coalition.[28]

5.1 Interior (Non-Standardization) Equilibrium

In this section, we consider parameter values under the assumption that the resulting equilibrium in any of the subgames does not entail standardization, that is we consider parameter values for which $k > k^{FU}$. We first show that if firm B finds it profitable to respond to foreclosure by "retaliating," i.e., by integrating or by integrating and foreclosing, firm A will not integrate and foreclose.[29]

Lemma 2 *Firm B 's optimal retaliation strategy is to integrate.*[30] *If firm B finds it profitable to retaliate by integrating, firm A prefers not to integrate and foreclose.*

If firm B retaliates, it will do so by integrating since this is an effective way for B to commit to aggressively price its hardware, *without* forgoing software sales for system A. In such a case, A no longer finds foreclosure profitable since integration by B reduces both A's hardware market share and hardware price.

[28] We do not model the acquisition stage and are therefore agnostic about the distribution of the total surplus from foreclosure. What we do care about is that the benefits to be distributed from foreclosing are greater than from not.

[29] To reduce clutter, proofs for Lemma 2 and Propositions 5-7 are in the appendix.

[30] Of course it may not be profitable for firm B to retaliate.

Using the above Lemma, we can state the following proposition. For ease of presentation of our results, we employ the notation $\gamma = W(2) - W(1)$, the marginal benefit of the second software variety, where $0 < \gamma < W(1)$.

Proposition 5 *Suppose that $k > k^{FU}$. A foreclosure equilibrium exists if and only if*

- *The marginal benefit of the second software variety is relatively small so that $\gamma < W(1)/3$ and*
- *The degree of hardware differentiation is relatively large so that $k > \frac{(2W(1)-\gamma)}{3} \equiv k_{min}$.*

The region of the parameter space for which the interior foreclosure equilibrium exists is region B_1 in Figure 1. The first condition in the proposition provides the restriction on the parameter space to ensure that firm A finds it profitable to foreclose. This condition is identical to that required for A's equilibrium hardware price to be higher in the foreclosure/unintegrated industry structure than in the unintegrated industry structure. (See Corollary 2).[31] This means that when A forecloses its hardware profits must rise since both its hardware price and its hardware market share increase when it forecloses and B does not respond. Furthermore, if the condition is satisfied, duopoly software prices are relatively low. Consequently, the costs of foreclosing, lost software profits, are small. The second condition of the Proposition states that if a merged firm forecloses, the unintegrated hardware firm will find it profitable to integrate if the degree of product differentiation is small, that is, $k < k_{min}$. Here firm B can effectively restore its market share by committing to lower the price of its hardware.

An interesting feature of this equilibrium is that the market share for the foreclosing firm is relatively small. By substituting the expression for k_{min} and the maximum value of the marginal benefit of software ($\gamma = W(1)/3$)

[31] Hence, in contrast to Whinston, in this instance foreclosure is profitable when it leads the foreclosing firm to raise the price of its hardware.

into t^{FU} from Proposition 4, the upper bound on the market share of the foreclosing firm is 80 percent. As the degree of product differentiation k increases, the market share advantage of the foreclosing firm is reduced.

5.2 Standardization Equilibrium

In this section, we consider parameter values under the assumption that the resulting equilibrium in the foreclosure/unintegrated subgame entails standardization, that is, $k < k^{FU}$. We find that there exist parameter values for which foreclosure is an equilibrium. There are two subcases: (i) $k^{FI} < k < k^{FU}$ and (ii) $k \leq k^{FI}$.

Case (i): $k^{FI} < k < k^{FU}$.

In the absence of a response by B there will be de facto standardization on technology A. Firm B can restore some market share and increase its profits by integrating. Integration by B gives it an incentive to price its hardware more aggressively to create a market for B compatible software, in which it is a monopolist, and by not foreclosing it does not forgo software sales for system A. Lower hardware prices and a smaller market share reduce the profitability of foreclosure, making it non-optimal for A:

Proposition 6 *Whenever $k^{FI} < k < k^{FU}$ the equilibrium industry structure is unintegrated.*

Case (ii): $k \leq k^{FI}$

In this case the degree of product differentiation between the hardware products is very small, and foreclosure will result in de facto standardization unless firm B responds by foreclosing as well. In advance of the following proposition, let

$$k_{max} = \frac{2(W(1) - \gamma)}{3}.$$

Proposition 7 *Suppose that $k < k^{FU}$. Further suppose that the degree of hardware differentiation is small so that $k < \min[k_{max}, k^{FI}]$. Then the unique equilibrium actions are $s_A = F$, $s_B = U$ and technology A is exclusively adopted. If $k > \min[k_{max}, k^{FI}]$ then the unique equilibrium actions are $s_A = U$, $s_B = U$.*

The region of the parameter space for which the standardization foreclosure equilibrium exists is region A_1 in Figure 1. In the foreclosure equilibrium, firm A makes the same profits from software as it would if it did not foreclose. Foreclosure is thus profitable when hardware profits increase. This depends on the effect on profits from doubling market share (from half the market to the entire market) and the change in hardware prices. It can be shown that whenever $k < 3k_{max}/4$, the price of the foreclosing hardware firm increases. When $3k_{max}/4 < k < k_{max}$, the hardware price under foreclosure is lower than the unintegrated industry structure, but the increase in market share associated with foreclosure more than offsets the price decline. When $k > k_{max}$, the reverse is true and unilateral foreclosure is not profitable. Foreclosure by B in retaliation is not profitable due to the pricing externality and the loss in software profits. There is always a sufficiently small extent of product differentiation which insures that foreclosure will be a standardization equilibrium.

5.3 Summary

Figure 1 provides a convenient summary of our results. It shows that foreclosure emerges under two very different sets of circumstances. Regions A_1, A_2, A_3, and A_4 are for the case when foreclosure would lead to standardization ($k < k^{FU}$). Regions B_1, B_2, B_3, and B_4 are for the case when foreclosure would not lead to standardization ($k > k^{FU}$).

- *Standardization Foreclosure Equilibrium:* Provided hardware products are sufficiently similar, then regardless of the size of the marginal benefit of software, all consumers purchase the foreclosing technology (Proposition 7). This is region A_1 in Figure 1.

- *Interior Foreclosure Equilibrium:* When the hardware technologies are highly differentiated and when the marginal value of the second software product is relatively small, an interior foreclosure equilibrium emerges (Proposition 5). This is region B_1 in Figure 1.

- *Unintegrated Equilibrium:* In all other regions in Figure 1, the equilibrium industry structure is unintegrated. In regions A_2 and B_2 in Figure 1, firm B would respond to foreclosure by integrating; this makes foreclosure unprofitable. In regions A_3 and B_3, firm B would not respond by integrating; in these regions, however, unilateral foreclosure is unprofitable. In regions A_4 and B_4, unilateral integration is also unprofitable; in any case, in these regions, firm B would respond by integrating.

6 Social Welfare

In this section we evaluate the social desirability of the foreclosure equilibrium. We do this by comparing the surplus associated with the unintegrated equilibrium to the surplus associated with the foreclosure equilibrium.

6.1 Total Surplus

Total surplus is the sum of hardware profits, software profits, and consumers' surplus. If foreclosure results in standardization then while the software consumption of all consumers is unchanged, the reduction in hardware variety reduces welfare relative to the unintegrated equilibrium. In the interior foreclosure equilibrium, the benefit from software consumption is lower than in the unintegrated equilibrium, since some consumers purchase a hardware technology with less software variety. A second loss arises from other consumers who switch to the foreclosing system, even though in the absence of foreclosure they prefer the foreclosed system. Since demand for a variety of software and hardware is perfectly inelastic, there is no inefficiency associated with hardware and software pricing. In light of the above discussion, we can state the following proposition.

Proposition 8 *The foreclosure equilibrium is inefficient.*

6.2 Consumer Surplus

The following proposition (which is proved in the appendix) shows how foreclosure affects consumer surplus.

Proposition 9 *(i) Consumer surplus is always lower under an interior foreclosure equilibrium than under the unintegrated industry structure. (ii) Consumer surplus is higher when foreclosure leads to standardization (region A_1) than in the unintegrated equilibrium if and only if* $4[2W(1) - W(2)]/7 < k < 2[2W(1) - W(2)]/3 \equiv k_{max}$.[32]

The intuition for (ii) is that when k is relatively large, the equilibrium hardware price under foreclosure is relatively low; this benefits consumers. However, the same result does not hold for an interior foreclosure equilibrium: consumers are always worse off. This result is not surprising since the equilibrium hardware price for A is higher in the interior "foreclosure" equilibrium than in the unintegrated industry structure.

7 Conclusion

In this paper we developed a model to address whether or not foreclose in the markets for systems, when the value of a system depends on the availability of software or complementary services, is an equilibrium outcome and, if it is, what the welfare implications are. We find that for certain parameter values, in equilibrium, one hardware firm will merge with a software firm and discontinue software support for a rival system of hardware technology. Moreover, the remaining hardware firm will not respond in kind. Our model provides an analytical basis for concern over vertical mergers in systems markets due to the potential for foreclosure or discriminatory access to software.

Foreclosure equilibria arise in two sets of circumstances: (i) when hardware is relatively differentiated, but software is not (interior) and (ii) software is relatively differentiated, but hardware is not (standardization). Foreclosure is not an equilibrium when both are differentiated, either because it is not profitable or it would invite retaliation. In these circumstances, foreclosure due to strategic or market power reasons is unlikely. When hardware and

[32]Recall that region A_1 is defined by $k < min[k_{max}, k^{FI}]$. If $k_{max} < k^{FI}$, by definition, part of region A_1 will have higher consumer surplus under the standardization equilibrium and part of region A_1 will have lower consumer surplus under the standardization equilibrium. Similarly, both regions exist if $k_{max} > k^{FI}$, since $k^{FI} > 4[2W(1) - W(2)]/7$.

software are relatively differentiated, integration and foreclosure is instead likely a response to coordination and contracting problems.

In the cases discussed in the introduction, there appears to be little product differentiation among the hardware products. This suggests that the consent decrees that require integrated "hardware/software" firms to make software available on a non-discriminatory basis for other hardware technologies might prevent foreclosure that would lead to socially inefficient standardization on one of the platforms.

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Appendix: Proofs

Proof of Lemma 2:

For the first part of the Lemma we need to show that $\Pi_B^{FI} > \Pi_B^{FF}$. From Propositions 3 and 4 this is true if

$$\frac{(W(1) - W(2) + 3k)^2}{18k} + W(2) - W(1) > \frac{k}{2}.$$

Simple algebra shows that the above inequality holds for all parameter values. For the second part of the Lemma, firm A will not find it profitable to foreclose if B retaliates (by integrating) if

$$\Pi_A^{UU} = k/2 + W(2) - W(1) > \Pi_A^{FI} = \frac{[W(2) - W(1) + 3k]^2}{18k}.$$

Rearranging terms, the above inequality holds for all $k > \frac{W(2)-W(1)}{12}$. Since $\frac{W(2)-W(1)}{12} < k^{FI} < k^{FU}$, the result holds for all $k > k^{FU}$. *Q.E.D.*

Proof of Proposition 5:

(i) Firm A will find it profitable to foreclose if B does not retaliate if $\Pi_A^{FU} \geq \Pi_A^{UU}$, which is true for all $k > k^{FU}$ if $W(2) - W(1) < W(1)/3$. If $W(2) - W(1) > W(1)/3$, $\Pi_A^{FU} \geq \Pi_A^{UU}$ if and only if $k < \frac{W(1)^2}{[6(3W(2)-4W(1))]} \equiv k_c$.

(ii) There is no hold-out problem if $\Pi_A^{FU} \geq \Pi_B^{FU}$. This is true for all $k > k^{FU}$ if $W(2) - W(1) < W(1)/3$. For $W(2) - W(1) > W(1)/3$ there is no hold-out problem if $k > \frac{W(1)[W(2)-2W(1)]}{[4W(1)-3W(2)]} \equiv k_{ho}$.

(iii) Firm B will not respond to foreclosure by integrating if $\Pi_B^{FU} \geq \Pi_B^{FI}$, which is true if $k > \frac{2W(1)-[W(2)-W(1)]}{3} \equiv k_{min}$.

From Lemma 2 firm B's optimal strategy if it chooses to retaliate is to integrate. The necessary and sufficient condition for no response by firm B is $k > k_{min}$. Moreover, from Lemma 2 if B does integrate A will not foreclose.

From (i) and (ii) $W(2) < 4W(1)/3$ is a sufficient condition for no commitment and no hold-out problems. We now show that it is also necessary. Hence suppose that $W(2)-W(1) > W(1)/3$. For there to be no commitment and hold-out problems, we must show that there exists a degree of differentiation k such that both k_{min} and k_{ho} are less than k_c. It is easy to show that $k_{ho} < k_c$ if and only if $W(2) - W(1) > 5W(1)/6$. However, for this range of the parameter space, $k_{min} > k_c$. *Q.E.D.*

Proof of Proposition 6:

Suppose that firm A has integrated and foreclosed. We first show that whenever $k^{FI} < k < k^{FU}$ firm B will find it optimal to integrate rather than remain unintegrated.

Consider the profitability of integrating. The stand-alone profits for firm B and the independent software firm consist only of software profits equal to $\Pi_B^{FU} = W(2) - W(1)$, since in the absence of a response, hardware firm B will have zero sales. The profits of firm B from integrating are

$$\Pi_B^{FI} = p_B^{FI}(k)\left(1 - t^{FI}(k)\right) + W(1)\left(1 - t^{FI}(k)\right) + [W(2)-W(1)]t^{FI}(k), \tag{26}$$

where, from Proposition 3, $\frac{\partial p_B^{FI}(k)}{\partial k} > 0$ and $\frac{\partial t^{FI}(k)}{\partial k} < 0$. It follows immediately that $\frac{\partial \Pi_B^{FI}(k)}{\partial k} > 0$, since on a per consumer basis, monopoly software profits are greater than duopoly software profits. It also follows that if $\Pi_B^{FI} \geq \Pi_B^{FU}$ when $k = k^{FI}$, then $\Pi_B^{FI} > \Pi_B^{FU}$ for all $k^{FI} < k < k^{FU}$. When $k = k^{FI}$, $t^{FI} = 1$ and $\Pi_B^{FI} = \Pi_B^{FU} = W(2) - W(1)$.

Hence, if firm A forecloses, firm B will, at the very least respond by integrating. We show that if B responds by integrating, firm A will not foreclose. From Proposition 3,

$$\Pi_A^{FI} - \Pi_A^{UU} = \frac{(W(2) - W(1))^2}{18k} - \frac{2(W(2) - W(1))}{3},$$

which is clearly decreasing in k. Thus if $\Pi_A^{FI} - \Pi_A^{UU} < 0$ at $k = k^{FI}$, it will also be true for all $k^{FI} < k < k^{FU}$. At $k = k^{FI}$, $\Pi_A^{FI} - \Pi_A^{UU} = [W(1) - W(2)]/2$. Hence, $\Pi_A^{FI} - \Pi_A^{UU} < 0$ when $k = k^{FI}$.

Finally, if firm B responds by foreclosing, bilateral foreclosure profits ($\Pi_A^{FF} = k/2$) are less than profits earned in the unintegrated industry structure ($\Pi_A^{UU} = k/2 + W(2) - W(1)$). *Q.E.D.*

Proof of Proposition 7:

To establish the first part of this proposition we must establish:

i) that it is profit maximizing for firm A to foreclose;
ii) that firm B will not find it profitable to retaliate;
iii) that there is no hold-out problem.

We begin with i). The stand-alone profits for firm A if it does not foreclose are $\Pi_A^{UU} = k/2+W(2)-W(1)$. The profits earned by the merged firm when it forecloses are $\Pi_A^{FU} = W(1)-k$. Foreclosure profits exceed stand-alone profits whenever $k < k_{max}$. Note that k_{max} can be either greater than or less than k^{FI}.

For ii), note that integration yields the same profits as remaining unintegrated ($\Pi_B^{FI} = \Pi_B^{FU} = W(2) - W(1)$), while integration and foreclosure yield profits of $\Pi_B^{FF} = k/2$. $\Pi_B^{FU} > \Pi_B^{FF}$ if $k < 2[W(2) - W(1)] = k_2$. Note, however, that $k_2 > k^{FI}$. Since this proposition refers to $k < k^{FI}$, $\Pi_B^{FU} > \Pi_B^{FF}$ for all k in the relevant range.

For iii), $\Pi_B^{FU} = W(2) - W(1)$, since the profits of the unintegrated hardware firm are zero. The profits of the foreclosing firm are clearly greater by i). Hence there is no hold- out problem.

For the second part of the proposition, if $k > k_{max}$, it does not pay for firm A to unilaterally integrate. If $k > k^{FI}$ firm B will retaliate by integrating, in which case foreclosure by A is once again unprofitable. Hence the equilibrium industry structure is unintegrated. A simple comparison shows that $k^{FI} < (>) k_{max}$ whenever $\gamma < (>) \frac{2}{3} W(1)$. *Q.E.D.*

Proof of Proposition 9.

Using (5) and Proposition 1, consumer surplus in the unintegrated industry structure (CS^{UU}) is

$$\begin{aligned} CS^{UU} &= 2\int_0^{1/2}[2W(1) - W(2) + y + \alpha - k - kt]dt \\ &= 2W(1) - W(2) + y + \alpha - 5k/4. \end{aligned} \tag{27}$$

Using (5) consumer surplus in an interior foreclosure equilibrium (CS^{FUI})is

$$\begin{aligned} CS^{FUI} &= \int_0^{t^{FU}}[2W(1) - W(2) + y + \alpha - p_A^{FU} - kt]dt \\ &+ \int_{t^{FU}}^1[y + \alpha - p_B^{FU} - k(1-t)]d(1-t), \end{aligned} \tag{28}$$

where t^{FU}, p_A^{FU}, and p_B^{FU} are independent of t and given in Proposition 4.

After integration,

$$CS^{UU} - CS^{FUI} = \frac{[54kW(1) \ + \ 12W(1)W(2) \ - \ W(1)^2]}{36k},$$

which is clearly positive for $W(1) \leq W(2) \leq \frac{4}{3}W(1)$, the parameter range for which the interior foreclosure equilibrium exists. This proves the first part of the proposition.

Using (5) and Proposition 4, the consumer surplus in the *standardization* foreclosure equilibrium (CS^{FUS})is

$$\begin{aligned} CS^{FUS} &= \int_0^1[2W(1) - W(2) + y + \alpha - 2W(1) + W(2) + k - kt]dt \\ &= y + \alpha + k/2 \end{aligned} \tag{29}$$

Consumer surplus is higher when foreclosure leads to standardization ($CS^{FUS} > CS^{UU}$) if $k > 4[2W(1) - W(2)]/7$. Recall that a foreclosure equilibrium exists whenever $k < 2[2W(1) - W(2)]/3$. Hence for $4[2W(1) - W(2)]/7 < k < 2[2W(1) - W(2)]/3$, a foreclosure equilibrium exists and leads to higher consumer surplus than the unintegrated equilibrium. *Q.E.D.*